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Nonlocality Effects in the Phase Diagram of Neutral Quark Matter¶

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Abstract—We construct phase diagrams of two-flavor quark matter under compact star constraints for two nonlocal, covariant quark models (instanton liquid and one-gluon exchange (OGE) inspired) within the mean field approximation and compare the results for different diquark coupling strengths. For the OGE model, stable hybrid stars with quark cores are obtained.

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1. INTRODUCTION

In the past few years much effort was focused on the construction and understanding of the QCD phase diagram. On the one hand, at low chemical potential and high temperatures, one expects a so-called quark–gluon plasma (QGP) phase, in which chiral symmetry is restored. The signatures of that phase are being investigated in relativistic heavy ion collisions. On the other hand, in the region of low temperatures and chemical potential, the chiral symmetry is broken due to the presence of a quark–antiquark condensate. But when increasing the chemical potential, the quark–antiquark channel is expected to vanish giving rise to a diquark condensate. For two light flavors u and d, the color symmetry might be spontaneously broken resulting in a two flavor color superconductivity (2SC) phase. When the *strange* flavor is also taken into account, a color flavor locking (CFL) phase appears. These phase regions are of great interest in astrophysics, in particular in connection with the physics of compact stars [1].

Due to difficulties when dealing with large baryon densities, lattice QCD calculations are not yet able to provide a detailed knowledge of that particular region of the QCD phase diagram. Thus, most theoretical approaches are based on the use of effective models of QCD. In the present work, we study the competition between chiral symmetry restoration and two flavor color superconductivity in the framework of two covariant nonlocal chiral quark models, under compact star conditions of electric and color charge neutrality. The first one is inspired by the instanton liquid model (ILM) [2], and the second one arises from an effective one-gluon exchange (OGE) [3] in a separable form.

¶ The text was submitted by the authors in English.

2. THE FORMALISM

Let us write the Euclidean action for the nonlocal chiral quark model in the case of two light flavors and antitriplet diquark interactions,

$$S_E = \int d^4x \Big\{ \bar{\psi}(x)(-i\partial\!\!\!/ + m)\psi(x) - \frac{G}{2} j_M^f(x) j_M^f(x) - \frac{H}{2}[j_D^a(x)]^\dagger j_D^a(x) \Big\}. \quad (1)$$

Here, m is the current quark mass, which is assumed to be equal for u and d quarks. As we mentioned above, we introduce nonlocality arising from two alternative scenarios called "Model I" and "Model II," where the effective interactions are based on ILM and OGE, respectively. The currents $j_{M,D}(x)$ in Eq. (1) are given by the following nonlocal operators:

$$\begin{aligned} j_M^f(x) &= \int d^4y d^4z r(y-x) r(x-z) \bar{\psi}(y) \Gamma_f \psi(z), \\ j_D^a(x) &= \int d^4y d^4z r(y-x) r(x-z) \\ &\quad \times \bar{\psi}_C(y) i\gamma_5 \tau_2 \lambda_a \psi(z), \end{aligned} \tag{2}$$

for Model I, and

$$\begin{aligned} j_M^f(x) &= \int d^4z g(z) \bar{\psi}\left(x+\frac{z}{2}\right) \Gamma_f \psi\left(x-\frac{z}{2}\right), \\ j_D^a(x) &= \int d^4z g(z) \bar{\psi}_C\left(x+\frac{z}{2}\right) i\gamma_5 \tau_2 \lambda_a \psi\left(x-\frac{z}{2}\right) \end{aligned} \tag{3}$$

for Model II. Here we have defined $\psi_C(x) = \gamma_2\gamma_4\bar{\psi}^T(x)$ and $\Gamma_f = (\mathbb{1},\ i\gamma_5\vec{\tau})$, while $\vec{\tau}$ and λ_a, with a = 2, 5, 7, stand for Pauli and Gell–Mann matrices acting on flavor and color spaces, respectively. The functions $r(x-y)$ and $g(z)$ in Eqs. (2) and (3) are nonlocal form factors characterizing the corresponding interactions.

The effective action (1) might come from a more fundamental interaction via Fierz transformations (in the case of OGE or ILM interactions, the coupling ratio H/G would be equal to 3/4). However, since the precise derivation of the effective couplings from QCD is not known, we will leave the ratio as a free parameter, analyzing our results for the range from $H/G = 0.5$ to 1.

After a proper bosonization of the theory, in the mean field approximation, the thermodynamical potential per unit volume reads

$$\Omega^{\mathrm{MFA}} = \frac{\bar{\sigma}^2}{2G} + \frac{|\bar{\Delta}|^2}{2H} - \frac{T}{2}\sum_{n=-\infty}^{\infty}\int\frac{d^3\vec{p}}{(2\pi)^3}\ln\det\left[\frac{S^{-1}}{T}\right], \tag{4}$$

where the inverse propagator $S^{-1}(\bar{\sigma},\ \bar{\Delta})$ is a 48×48 matrix in the Dirac, flavor, color, and Nambu–Gorkov spaces (details can be found in [4]). The mean field values $\bar{\sigma}$ (scalar meson field) and $\bar{\Delta}$ (scalar diquark field) are obtained by solving the coupled pair of gap equations $d\Omega^{\mathrm{MFA}}/d\bar{\sigma} = 0$ and $d\Omega^{\mathrm{MFA}}/d\bar{\Delta} = 0$. In general, we consider different chemical potentials μ_{fc} for each quark flavor f and color c. However, in our case [4] they can be written in terms of only three independent quantities: the baryon chemical potential μ_B ($\mu_B = 3\mu$), a quark electric chemical potential μ_Q, and a quark color chemical potential μ_8. The corresponding relations read

$$\begin{aligned} \mu_{qr} = \mu_{qg} &= \mu + Q_q\mu_Q + \mu_8/3, \\ \mu_{qb} &= \mu + Q_q\mu_Q - 2\mu_8/3, \end{aligned} \tag{5}$$

where $q = u, d$, and Q_q are quark electric charges. Now in compact stars, in addition to quark matter, we have electrons. Under β-equilibrium, and assuming that antineutrinos escape from the stellar core, we must have

$$\mu_{dc} - \mu_{uc} = -\mu_Q = \mu_e. \tag{6}$$

As we are interested in studying the phase diagram under compact star conditions, we impose color and electric charge neutrality, i.e.,

$$\begin{aligned} \rho_{Q_{\mathrm{tot}}} &= \rho_Q - \rho_e = \sum_{c=r,g,b}\left(\frac{2}{3}\rho_{uc} - \frac{1}{3}\rho_{dc}\right) - \rho_e = 0, \\ \rho_8 &= \frac{1}{\sqrt{3}}\sum_{f=u,d}(\rho_{fr} + \rho_{fg} - 2\rho_{fb}) = 0, \end{aligned} \tag{7}$$

where ρ_e, ρ_{fc} stand for fermion densities. In this way, for each value of T and μ, we should find the values of $\bar{\Delta}$, $\bar{\sigma}$, μ_e, and μ_8 that solve the pair of gap equations, supplemented by Eqs. (6) and (7).

3. NUMERICAL RESULTS AND CONCLUSIONS

In the present study we use the Gaussian form factor for both models I and II (see [4] for details). The input parameters for both models are those which reproduce the empirical values for the pion mass m_π and decay constant f_π, and lead to a phenomenologically acceptable value $\langle \bar{q}q \rangle = -250$ MeV for the chiral condensates at vanishing T and μ_B. The parameters considered here for Model I are $m = 5.14$ MeV, $\Lambda = 971$ MeV, and $G\Lambda^2 = 15.41$, while for Model II we have taken $m = 5.12$ MeV, $\Lambda = 827$ MeV, and $G\Lambda^2 = 18.78$ [5]. As stated, we considered different values for the coupling ratio H/G between 0.5 and 1. The corresponding phase diagrams for Models I and II are shown in Fig. 1, in which we can see the regions corresponding to different phases as well as the position of triple points (3P) and end points (EP). Besides the low $T-\mu$ region, in which the chiral symmetry is broken (CSB), one finds normal quark matter (NQM) and two-flavor superconducting (2SC) phases. Between the CSB and NQM phases, one has first order and crossover transitions, represented by solid and dotted lines, respectively. Between the NQM and 2SC regions, in all cases a second order phase transition appears (dashed lines in the diagrams of Fig. 1). Close to this phase border, the dashed–dotted lines in the graphs delimit a narrow band that corresponds to the gapless 2SC (g2SC) phase. In some cases we find a 2SC-NQM mixed phase in which the system realizes the constraint of electric neutrality globally: the coexisting phases have opposite electric charges which neutralize each other at a common equilibrium pressure. For both models the 2SC phase region becomes larger when the ratio H/G is increased.

We also have found that Model II predicts a larger quark mass gap and a chiral symmetry breaking (CSB) phase transition line compared to Model I. The critical

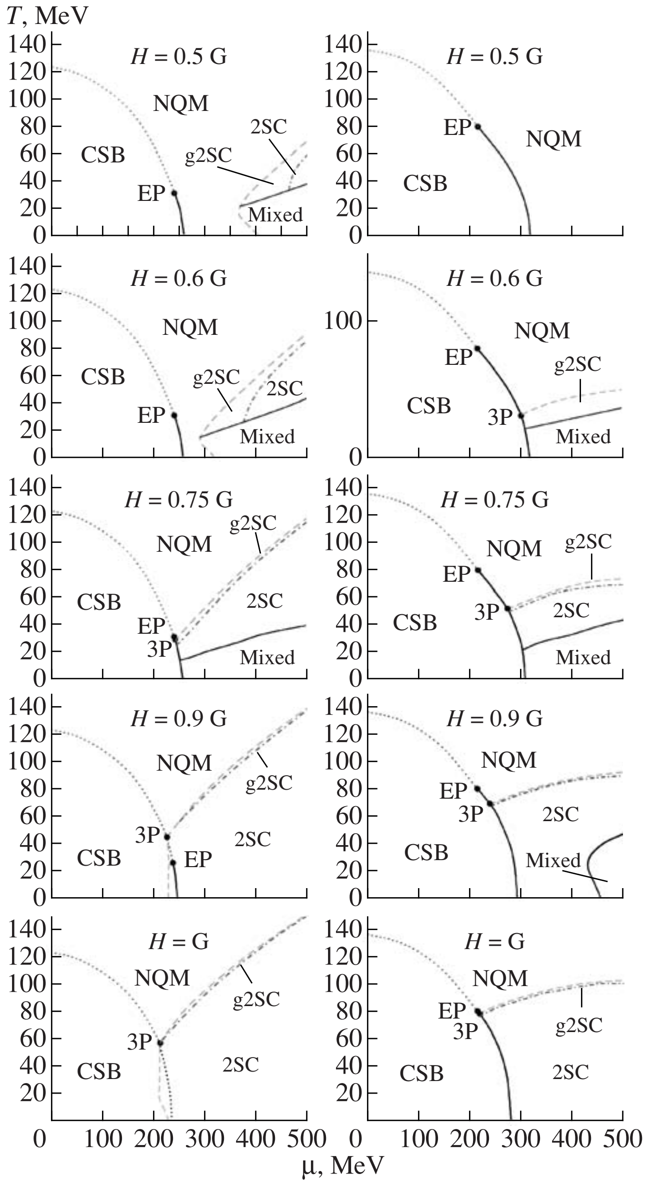

Fig. 1. Phase diagrams for Models I (left) and II (right) and different values of the ratio H/G, compact star constraints.

temperature for the 2SC phase transition is a rising function of μ for Model I, whereas it is rather μ independent in Model II due to the different μ dependences in the form factors associated with the scalar diquark gaps. The prediction for the critical temperature at $\mu = 0$ in Model II, $T_{\mathrm{CSB}} \sim 140$ MeV, is closer to the results of recent lattice QCD simulations.

As an application of the developed approach, we consider the question whether a phase transition to quark matter is likely to occur in neutron star interiors. We apply a two-phase description with a low-density hadronic matter phase described within the Dirac–Brueckner–Hartree–Fock (DBHF) approach using the Bonn-A nucleon–nucleon potential [8]. The quark–hadron phase transition is obtained by applying Gibbs criteria of phase equilibrium, in particular, equality of pressures and chemical potentials of the coexisting phases (see Fig. 2, left panel). In the right panel of Fig. 2, we show the mass–radius relationships for hadronic and hybrid star configurations obtained from a solution of the Tolman–Oppenheimer–Volkoff equations with the corresponding EosS. Increasing the diquark coupling lowers the phase transition density and leads to a lower critical star mass for the formation of a quark matter core. For details concerning the astrophysical constraints on the equation of state, see [9, 10, 11].

As a general conclusion, it can be stated that even under compact star constraints, provided the ratio H/G is not too low, the nonlocal schemes favor the existence of color superconducting phases at low temperatures and moderate chemical potentials. This is in contrast with the situation in the NJL model [6], where the existence of a 2SC phase turns out to be rather dependent on the input parameters. Our results are also qualitatively different from those obtained in the case of noncovariant nonlocal models [7], where above the chiral phase transition the NQM phase is preferable for values of the coupling ratio $H/G \lesssim 0.75$, and a color superconducting phase can be found only for $H/G \approx 1$.

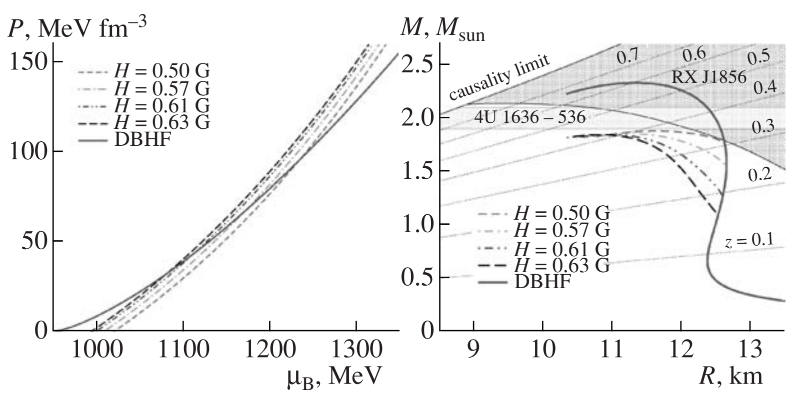

Fig. 2. (Left panel) Pressure vs. baryochemical potential for the hadronic phase (DBHF) and the quark matter phase (Model II—OGE approach) for different values of the diquark coupling constant H. The intersection defines the critical chemical potential for the deconfinement transition. (Right panel) Compact star configurations with and without deconfinement corresponding to the equations of state given in the left panel.

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