

# THE QRPA AND BEYOND \*)

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Resume of the "round the table" discussions on the proton-neutron QRPA approximation and on the extensions beyond the QRPA.

## 1 Summary of the talks presented during the second day of the workshop

- Comparison of realistic and schematic models (by O. Civitarese): The talk was devoted to the discussion of the calculation of nuclear wave-functions and matrix elements, in the framework of the proton-neutron QRPA approach, with particular attention to the non-perturbative aspects of the problem. In analogy to the description of space rotations, the need of the decoupling between collective and intrinsic degrees of freedom for rotations in isospin-space was discussed. The limits of vibrations and permanent deformations in isospin space were presented. Some evidences of permanent deformations, in isospin variables, induced by attractive proton-neutron interactions were discussed. Consequences of this induced "phase transition" upon the nuclear wave functions and nuclear matrix elements were shown. The failure of the renormalized QRPA approach was shown to be the natural consequence of this induced deformation.
- Proton-neutron pairing effects (by P. Vogel): The validity of the conventional BCS approach, with and without the inclusion of proton-neutron pairs, was discussed, both in schematic and in exact calculations. The role of the isospin symmetry and its breaking by renormalizations of the pairing strengths, in connection with proton-neutron pairing effects, was addressed. Generalized BCS transformations were introduced and the co-existence of different types of pairing channels was shown, as a function of renormalized pairing strengths. The method was applied to the calculation of proton-neutron pairing effects for  $N \approx Z$ . The second part of the talk was devoted to the study of these effects upon the proton-neutron QRPA approach and its applications to calculations of nuclear double-beta-decay matrix elements. The competition between isovector and isoscalar modes and the appearance of a phase transition was shown. It was shown that, while isovector proton-neutron pairing is likely irrelevant for double-beta decay, the isoscalar interaction is essential. It was pointed out that methods which eliminate the phase transition entirely cannot be appropriate for the full range of coupling constants.

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- QRPA and  $\beta$  decay physics: higher order approximations in a boson formalism (by M. Sambataro): The proton-neutron QRPA method in a boson formalism was introduced. Higher order approximations to the QRPA wave functions were defined by adding pair of bosons to the "first-order" QRPA boson operators. Applications of the formalism, for a schematic model were shown, by simultaneously mapping the original Hamiltonian into fermion and boson spaces. The convergence of the procedure was discussed with reference to energies, matrix elements and ground state correlations. The analysis in terms of two different bosons (with reference to the breakdown of the approximation) was advanced. A step by step procedure, to overcome the collapse of the standard pn-QRPA approach was introduced.
- Recent developments in theoretical descriptions of double-beta decay (F. Šimkovic): Different techniques, of evaluating two-neutrino and neutrinoless nuclear matrix elements, were presented in this talk. The consequences of some of the proposed theoretical scenarios for grand-unification schemes and also for SUSY inspired models were discussed. Examples of non-conventional final state interactions between electrons, and the subsequent effects on the electron sum-spectra were given. Also, a substantial part of the talk was devoted to the presentation of the Operator Expansion Method and its applications to nuclear double beta decay studies.
- RQRPA and related schemes (J. Suhonen, W. A. Kaminski, G. Pantis, J. Schwieger): This presentation of the renormalized QRPA method and related approximation consisted of four talks, given by the speakers in the order listed above. The talk by J. Suhonen gave an overview of the renormalized QRPA method, which also did open the way for the other talks following his own presentation. The essentials of the RQRPA method were discussed and compared with the self-consistent approach based on the use of coupled relations between BCS and QRPA expectation values. A possible explanation of the violation of Ikeda's Sum Rule based on the difference between BCS and QRPA vacuum correlations was presented. The presentation by W. A. Kaminski was devoted to the discussion of a QRPA equation of motion including scattering terms. The formalism includes quasi-particle pair creation and annihilation terms as well as quasiparticle scattering terms of the Hamiltonian. The QRPA matrices are enlarged in order to accommodate the new metric resulting from the inclusion of scattering terms. However, the calculation of the ground state values of the proton-neutron pairs is still an open problem. Further developments are expected along this formalism. The results of the Tuebingen non-collapsing QRPA method, for neutrinoless double-beta decay transitions, were presented by G. Pantis. The proposed method includes proton-neutron pairing interactions in large configuration spaces which resulted in very large effects upon the calculated matrix elements. The talk presented by J. Schwieger referred to the use of the renormalized QRPA method for the calculation of lepton-flavor violation transitions. With reference to the RQRPA method, the problems inherent to the violation of Ikeda's sum rule were addressed

and the lack of consistency in the inclusion of quasiparticle scattering terms was pointed out as the source of this violation.

## 2 Summary of the discussions

The main points of the discussion were the following:

- On the validity of the renormalized QRPA approach as a cure for the ‘collapsing QRPA’: the discussion was centered on the fact that the so-called ‘phase-transition’ demonstrated by the QRPA method and the total suppression of the matrix elements for two-neutrino double-beta decay transitions do not take place at the same point (in the range of values of  $g_{pp}$ ). The complete removal of the “phase transition” (as a signature of the change in the correlations governing the system of nucleons), as proposed by the renormalized QRPA approach was shown to be unphysical. However, the validity of the QRPA, for open-shell nuclei, was questioned by the shell model practitioners. Except for a crude criticism on the method not clear alternative was suggested to replace the QRPA approach. The fact that the renormalization scheme beyond the QRPA (RQRPA) violates the Ikeda Sum Rule and that it introduces new uncertainties in the values of  $g_{pp}$  (by shifting the point of collapse to values far away from physical values) was discussed. A certain degree of understanding upon the fact that the RQRPA was not presented in the first place as a cure but rather as an alternative to explore the validity of the QRPA approach was reached.
- About proton-neutron pairing effects in nuclear double beta decay studies: The application of these concepts to nuclear double beta decay studies was extensively discussed. The main questions, namely: a) how to evaluate proton-neutron pairing effects in a consistent way in the presence of large neutron excess and b) how to bring consistency between these results and the ones found for nuclei with  $N = Z$ , were formulated to the defenders of the inclusion of proton-neutron pairing effects in nuclei as heavy as  $^{130}\text{Te}$  and  $^{130}\text{Se}$ . It was shown that proton-neutron pairing gaps vanish for values of  $N - Z$  of the order of 4 to 6 nucleons. Thus, the inclusion of these correlations for, say, the description of the nuclear double beta decay of  $^{76}\text{Ge}$ ,  $^{82}\text{Se}$ ,  $^{130}\text{Te}$ , seems to be not sufficiently justified. The procedure to determine the strength of proton-neutron pairing correlations, was not sufficiently justified either.
- On the self-consistent approaches with reference to the simultaneous treatment of fermionic and bosonic degrees of freedom and their corresponding mappings onto effective boson representations: The possibilities suggested in M. Sambataro’s talk were discussed in detail, particularly, in dealing with the fact that the orthogonality of the “effective” bosons is not guaranteed by the expansions. Another question referred to the validity of the higher order expansion around the initial QRPA bosons. If the vicinity of a symmetry break-up is affecting the QRPA results, then higher order bosons will

certainly be strongly dependent upon the structure of the lowest order solutions. The analogy with the deformed case was mentioned (deformations cannot be attained by superposition of vibrations). Thus, the possibility of having different bosons at each side of the “phase-transition” was mentioned as the main difficulty to be dealt with by the approximation.

- About new “photonic” contributions to nuclear double beta decay transitions: the possibility mentioned in Dr. Simkovic’s talk, of a non-conventional photonic mechanism where final state interactions between emitted electrons are introduced in the analysis of the electrons sum-spectrum was discussed.

### 3 Conclusions

In the opinion of this moderator the talks presented by the speakers do show the variety of views about the physics governing double beta decay transitions. Therefore, these conclusions can also show a strong bias and they should be taken as such.

The predictive power of the QRPA approximation, in its conventional form, seems to be more firmly established. The physical meaning of the collapse of the QRPA, as induced by a change in the two-body correlations mostly of isoscalar nature, was discussed. Other approximations which aimed at the removal of the “phase-instability” (like the RQRPA) apparently do not pay-off since they violate the Ikeda Sum Rule as much as the consistency of the interactions. More realistic treatments based on the simultaneous description of fermionic and bosonic couplings are, for the moment, at some embryonic level of development and more theoretical work has to be done in order to see which kind of self-consistency can be achieved and how the question of the QRPA “phase-transition” can be overcome. The inclusion of proton-neutron pairing correlations for nuclei with large neutron excess is not justified, as seen by the results obtained in the calculations of the structure of pairing effects in light mass nuclei. The use of spectroscopic information and single-beta decay data, as experimental tools to test the validity of the models, is strongly recommended.