On the Self-organization of Migrating Individuals

E. Albano

Instituto de Investigaciones Fisicoquímicas Teóricas y Aplicadas, (INIFTA),
Facultad de Ciencias Exactas, Universidad Nacional de La Plata.
Sucursal 4, Casilla de Correo 16, (1900) La Plata - ARGENTINA.
FAX : 0054-21-254642. E-mail: ealbano@isis.unlp.edu.ar

1 Introduction

This lecture is concerned with a simple model aimed to describe the self-organized displacements of self-driving individuals. Processes such as growth, death, survival, self-propagation, competition and communication are considered. Starting from either small colonies or a randomly distributed population, the system evolves towards a stationary state where the population density is conserved. It is shown that the stationary state exhibits self-organized critical behavior.

It is known that a variety of biological objects frequently exhibit tendency to clustering and migration (herds of quadrupeds, flocks of birds, bacterial growth, etc.). Motivated by this observation, Vicsek *et al.* 1995, Csahók *et al.* 1995 and Toner *et al.* 1995 have very recently presented models aimed to describe the collective motion of living individuals. These models are quite simple but they retain basic facts characteristic of biological objects exhibiting collective behavior : i) the individuals are self-driven, i.e. transforming energy gained from food into mechanical energy they are able to perform displacements. ii) The motion of an individual is, on the one hand, perturbated by enviromental conditions and on the other hand, conditioned by communications among their neighbours Vicsek *et al.* 1995, Csahók *et al.* 1995.

The aim of this work is to further extend these models in order to account for two relevant facts characteristic of the actual cooperative behaviour of the individuals : the first one is to allow the onset of self-organization via communications between neighboring individuals, while the second one is to explicitly consider the dynamic evolution of the population.

2 Definition of the Model

The model is based on the following rules :

Rule 1 : *The displacements.* All individuals have the same absolute velocity $\mid \mathbf{v} \mid$. At each time step all individuals assume the average direction

P. L. Garrido et al. (eds.), *Fourth Granada Lectures in Computational Physics*

of motion of the neighboring individuals within a range R with some random perturbation added. So, the location of the $j-th$ individual is updated according to

$$\mathbf{x}_j(t+1) = \mathbf{x}_j(t) + \mathbf{v}_j \Delta t. \tag{1}$$

The direction of motion is given by the angle $\theta_j(t+1)$ according to

$$\theta_j(t+1) = < \theta_j(t) >_R + \pi Q N_R^{-\alpha}, \tag{2}$$

where the first term of eq.(2) is the average direction of the velocities of the N_R individuals (including the $j-th$ one) within a circle of radii R surrounding the $j-th$ individual. The second term introduces a random noise, where Q is a random number in the interval $(-1, 1)$ and α is an exponent. So, neighboring individuals may self-organize in order to minimize the noise being α a measure of the strength of such ability. This rule implies communications between the individuals (e.g. via sensing of chemicals, visual, verbal, etc.).

Rule 2 : *The population dynamics.* A live individual such as $N_R > N3$ will died in the next step (decease by overcrowding). Also, a live individual will died in the next step if $N_R \leq N1$ (decease by isolation). Individuals survive if the neighborhood is not too crowded $(N2 \leq N_R \leq N3)$ and birth also occurs if N_R satisfies some stringent constrains $(N1 < N_R < N2)$. This rule, inspired on Life (Berlekamp *et al.* 1982), allows the population to self-regulate its density.

The model is simulated in a two dimensional off-lattice cell of linear size L with periodic boundary conditions. All individuals are updated simultaneously at each time step (cellular automata updating). Simulations are made taking $| \mathbf{v} | = 0.03$, $R = 1$, $N1 = 2$, $N2 = 6$, and $N3 = 9$.

3 Results and Discussion

3.1 Dynamics of Population Spreading

Population spreading is studied starting, at $t = 0$, with a small colony of N_o individuals placed in the center of the lattice. Then the colony is allowed to evolve according to the rules of the model and the following quantities are computed: (i) The average number of living individuals $N(t)$ and (ii) the survival probability $P(t)$ (i.e. the probability that the system had not evolved to extinction at time t). Figure 1 shows plots of $P(t)$ *vs* t obtained for lattices of different size and $\alpha = 0$. For large lattices $(L \geq 8)$ the average survivability of the colonies is of about 70%, while in smaller lattices $(L = 5)$ the system evolves to extinction. The insert of figure 1, which shows plots of $N_S(t) = N(t)/P(t)$ *vs* t, indicates that after same transient period, the population of the surviving colonies becomes stabilized, i.e. the systems reach a stationary state of constant density of living individuals.

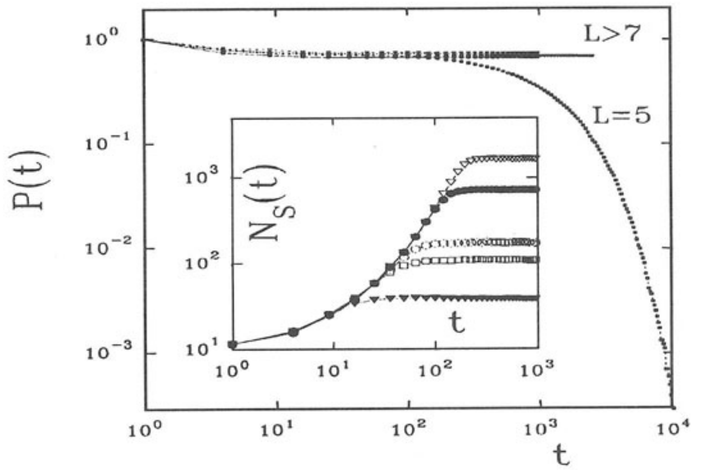

Fig. 1. Log-log plots of $P(t)$ *vs* t. The insert shows the corresponding plots of $N_S(t)$ *vs* t, obtained for lattices of size (from top to bottom) $L = 30, 20, 10, 8$ and 5.

Figure 2 shows plots of $N_S(t)$ *vs* t obtained for lattices which are large enough so that the spreading colonies never reach the boundary. In the asymptotic regime the behavior $N_S(t) \propto t^{\eta}$ is found to hold, whith $\eta = 2.13 \pm 0.03$, independent of α and N_o.

3.2 The Stationary State

Starting either from a random distribution of living individuals or due to the spreading of small colonies, it is found that the system evolves towards a stationary state (SS). The global density (ρ_g) is defined as the number of individuals over the total area of the sample. Also, the local density (ρ_l) is measured within the neigborhood (circle of radii R) of each individual. In the SS the system self-organizes in order to keep both the global and the local density constant independent of α, $\rho_g \cong 1.825(8)$ and $\rho_l \cong 2.511(8)$, respectively. The observed enhancement of the local density reflects a tendency to clustering ("flocking behavior"). This behavior is mostly due to the dynamics of the population, but it is not a consequence of the operation of an attractive potential as observed in most physical systems. However, the behaviour of the clusters ("flocks") as a whole, depends on α. In fact, using the absolute value of the normalized average velocity $|< v >|$ as a meassure of the flocking behavior, the crossover between two distinct regimes is observed : for larger α values (e.g. $\alpha \geq 4$) one has $|< v >| \rightarrow 1$, that is individuals self-organize in

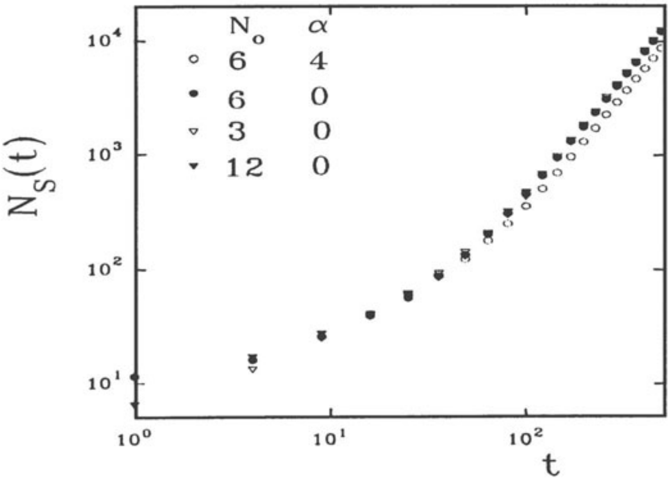

Fig. 2. Log-log plots of $N_S(t)$ *vs* t.

a single flock with a well defined direction of migration; however for $\alpha \to 0$ also $|< v >| \to 0$, that is many flocks move in random directions.

A relevant feature of the SS is that it exhibits **self-organized criticality (SOC)** behavior. SOC is a concept proposed by Bak *et al.* 1987-88 to describe the dynamics of a class of non-linear spatio-temporal systems, which evolve spontaneously toward a critical state (i.e. without having to tune a control parameter). Systems exhibiting SOC have attracted much attention since they might explain part of the abundance of $1/f$ noise, fractal structures and Lévy distributions in Nature (Bak *et al.* 1987-88), for examples of systems exhibiting SOC see also Bunde *et al.* 1995. In order to test for SOC behavior, the stationary state is perturbed by randomly adding a single individual. The evolutionary change triggered by this small perturbation is called an avalanche. The fate of the added individual depends on the enviroment: some individuals may die while other may succed to survive and reproduce generating avalanches of all sizes, i.e. a higly complex branching process. The life-time of an avalanche t is defined as the time elapsed between the introduction of the perturbation and the extinction of the perturbative individual itself and all its descendants. The size of the avalanche (s) is then computed by counting the number of descendants originated by the perturbative individual during the life-time of the avalanche.

For the stationary state to be SOC the distributions of life-time ($D(t)$) and size ($D(s)$) must exhibit power law behavior, i.e. $D(t) \propto t^{-a}$ and $D(s) \propto s^{-b}$

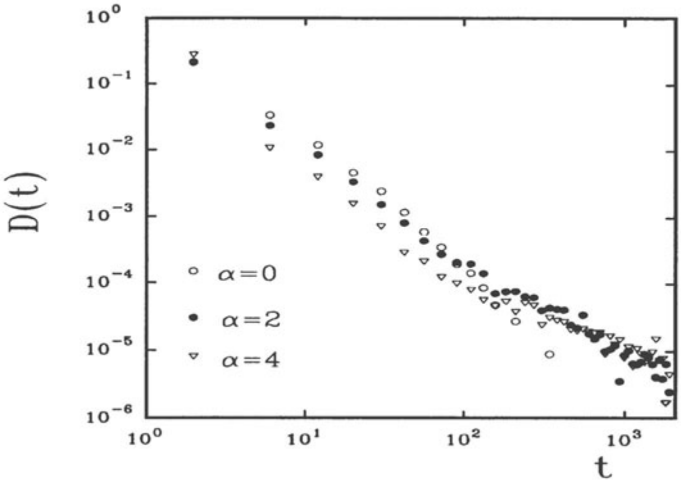

Fig. 3. Log-log plots of the distribution of life-time $D(t)$ *vs* t of the avalanches within the SOC regime. $L = 30$, results averaged over $2x10^4$ avalanches.

(see e.g. figure 3). The estimates for the exponents are $a \cong 1.7 \pm 0.1$ and $b \cong 1.6 \pm 0.1$, respectively. Due to the highly complex non-linear branching process involved in the present model, one expects $a \neq b$ and the scaling relationship $t \propto s^x$, with $x = (b-1)/(a-1)$, holds. Meassurements give $x = 0.82 \pm 0.1$ in agreement with the scaling value obtained from the exponents a and b, i.e. $x \approx 0.86$.

It has to be stressed that, while the flocking behaviour of the individuals strongly depends on α (i.e. single flock behaviour for large α and random displacements of individuals for $\alpha \to 0$), all power laws describing the distribution of avalanches and spatial correlations are independent of α; pointing out that the model exhibits robust critical behaviour which is achieved spontaneously without the necesity of tuning any external parameter, i.e. SOC.

4 Conclusions

A model which describes the self-organized cooperative displacement of self-driving and self-replicating individuals is proposed and studied. The spreading of small colonies in an otherwise empty landscape is investigated. If the available space for spreading is large enough, the colonies have a high average survivability (roughly 70%) while extinction, which takes place at early times, is mostly due to unfavorable initial conditions. In finite spaces, however, a stationary state is achieved such as the population self-organize to keep constant both, the local and the global density. The addition of an individual (i.e. a small perturbation) in the stationary state triggers avalanches of all sizes, i.e. the system lacks of any characteristic time- and size-scale. This behavior is the signature that the system self-organize in a critical state. So, the emergency of a very rich and complex critical behaviour at global scale, originated in simple local rules, is observed.

ACKNOWLEDGEMENTS. This work was financially supported by the CONICET (Argentina), the National University of La Plata and the Commission of European Communities under contract ITDC-122.

References

Bak P., Tang C. and Wiesenfeld K. (1987): Phys. Rev. Lett. **59**, 381;(1988): Phys. Rev. A. **38**, 364.

Berlekamp E. R., Conway J. H. and Guy R. K. (1982): *Winning Ways for your Mathematical Plays.* Vol. 2, Academic.

Bunde A. and Havlin S. (Eds) (1995): in *Fractals in Science*, (Springer Verlag), Berlin.

Csahók Z. and Vicsek T. (1995): Phys. Rev. E., **52**, 5295.

Toner J. and Tu Y. (1995): Phys. Rev. Lett., **75**, 4326.

Vicsek T., *et al.* (1995): Phys. Rev. Lett. **75**, 1226.