# TESTING GALACTIC OSCILLATIONS

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**Abstract.** Recent numerical simulations using an $N$-body code suggest that galaxies may oscillate in a very regular and long lasting way. Here we investigate galactic oscillations using a different approach: the perturbation particle method. Our results confirm the computational results given by Miller and Smith (1994).

**Key words:** Galaxies: structure, stars: stellar dynamics

## 1. Introduction

Research on stellar dynamics frequently involves dealing with analytic distribution functions that represent the state of a system. These functions are mostly *equilibrium* configurations that have to be tested for their stability properties. Significant efforts are devoted to finding suitable new distribution functions (see, e.g., Gerhard, 1991; Louis, 1993) which provide the basis to analytically represent a large variety of stellar systems.

Stability analyses are, however, much more challenging than the study of equilibrium configurations. Since the first studies published by Antonov (Antonov, 1960, 1962a, b) it has become clear that attempts to attack stability problems from a purely theoretical point of view leads to an 'exponential' increase of the difficulties as long as we try to include more realism into the systems investigated. As a result, numerical simulations have become the favored method to try to understand the behavior of even moderately complex configurations. Extensive work has been done in this direction (see, e.g., Merritt and Aguilar, 1985; Barnes et al., 1986; Merritt and Hernquist, 1991) but there is still a long way to go before we will be able to explain all the instabilities that affect clusters, galaxies and other objects.

Over the past few years several authors have been investigating the properties of stable systems and a new understanding is arising. As an example, we may mention the work of Mathur (1990), Weinberg (1991), Vandervoort (1991) and Sweatman (1993), who centered their attention on the possibility of finding long lasting oscillations in analytically stable stellar systems. Miller (1993) and Miller and Smith (1994) have reported the existence of what they call *galatic oscillations* in numerical simulations using different models to represent the stable galaxy, and their finding may have important consequences for stellar dynamics.

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The present paper aims to check the existence of those galatic oscillations running experiments similar to those of Miller and Smith, 1994 (MS hereafter). Nevertheless, instead of using a full $N$-body code as they did, we use the perturbation particle method (Wachlin et al., 1993; Leeuwin et al., 1993) which is a completely different technique and, therefore, provides an independent way to test the work of MS.

In Section 2 we present our numerical experiments, with the particular choice of constants and parameters for each case. Section 3 gives the results obtained from our simulations, and Section 4 discusses the meaning of those results for the theory of galactic oscillations.

## 2. Numerical Experiments

The idea of the perturbation particle method, used for our experiments, was developed many years ago by G. Rybicki, but remained unpublished until recently. The importance of the perturbation particle method for the present work arises from its independence from traditional $N$-body treatments, because it thus provides an alternate way to check the results obtained by MS. We will take advantage of the spherical symmetry of the unperturbed model, studying the problem in the radial direction only. This limitation is not important, since the *fundamental mode* oscillation found by MS is a 'breathing mode', an homologous expansion and contraction of the entire galaxy in the radial direction; alternatively, we get a much better resolution in this particular direction when we reduce the problem to one dimension.

We adopted a polytropic distribution for our unperturbed system:

$$f[\mathbf{r}(\mathcal{E}), \mathbf{v}(\mathcal{E})] = \begin{cases} F\mathcal{E}^{n-3/2}, & (\mathcal{E} > 0), \\ 0, & (\mathcal{E} \leqslant 0), \end{cases} \tag{1}$$

where $\mathcal{E} = \Psi - v^2/2$ is the relative energy, $\Psi = -\Phi + \Phi_0$ the relative potential, $F$ a normalization constant and $n$ is the polytropic index (see, e.g., Binney and Tremaine, 1987). We chose a polytropic index $n = 3$ and, thus, we had to solve numerically the Lane–Emden equation to obtain the potential of the distribution.

In our system of units the gravitational constant $(G)$, the total mass $(M)$ of the polytrope and its half mass radius $(r_h)$ are all equal to one. With this choice of constants the total phase space volume occupied by the unperturbed system is $\Gamma_t = 0.89$. The distribution of volumes among the perturbation particles was done according to one of the two criteria applied by Wachlin et al. (1993): we performed it in such a way that all particles had initially the same mass, that is, the volumes become proportional to the inverse of the value of the perturbation in the distribution function at the initial position of the particle in question.

Now arises the problem of how to perturb the equilibrium system in order to mimic the perturbations present in the $N$-body simulations run by MS. Clearly, the representation of a distribution function (such as a polytrope) with a finite number of particles is an approximation that improves as the number of particles grows larger. There is also an effect due to the way the distribution is generated: pseudo-random number techniques are commonly used at this stage, and this implies the presence of poissonian noise in the numerical distribution. These effects can be emulated by the perturbation particle method, and in all the experiments we run we adopted an initial perturbation resembling the poissonian noise present in $N$-body systems of 10 000 particles.

The integration of the equations of motion was performed using a Runge–Kutta algorithm described by Fehlberg (1968) which incorporates a variable time step in order to maintain a specified accuracy from one integration step to another. Since, as mentioned above, we solve the radial problem only, the perturbation particles are then perturbation *shells*. The number of shells adopted to represent the evolution of the system was 1000 (model A), 2000 (model B) and 4000 (model C). All the integrations were followed for 128 crossing times, defining a crossing time in the usual way

$$t_{cr} \equiv \frac{GM^{5/2}}{(2|E|)^{3/2}}, \tag{2}$$

where $E$ is the total energy of the system. In our case $t_{cr} = 1.52$ time units. We also set the time interval between outputs, $\Delta t$, to $0.25t_{cr}$ so that we can have a good temporal resolution of the changes that take place in the properties of the system. The results from different experiments are summarized in the next section.

## 3. Results

The galatic oscillations described by MS were detected studying the evolution of the total kinetic energy $T$ of the system, which showed a very regular oscillation pattern along their whole integration run. Thus, we decided to adopt the same indicator, and our results are summarized in Figures 1, 2 and 3, which show the evolution of the relative kinetic energy deviations $(\Delta T)$ as a function of time (in crossing times $t_{cr}$ units), for models A, B and C, respectively. It is evident from the figures that the total kinetic energy does not remain constant along the integrations, but it keeps osillating around some value near $\Delta T = 0$. Let us concentrate separately on each model, in order to analyze their differences and similarities.

Our first experiment was model A, the one with fewest perturbation shells and, consequently, lowest resolution. The oscillations of kinetic energy in this model (Figure 1) certainly do not seem to be just pure noise. There is some regular pattern, although the amplitude varies along the whole integration interval. Nevertheless, while the amplitude is not constant, it is clear from Figure 1 that it neither grows nor decreases monotonically with time, suggesting the existence of long lasting oscillation modes for the underlying dynamical system.

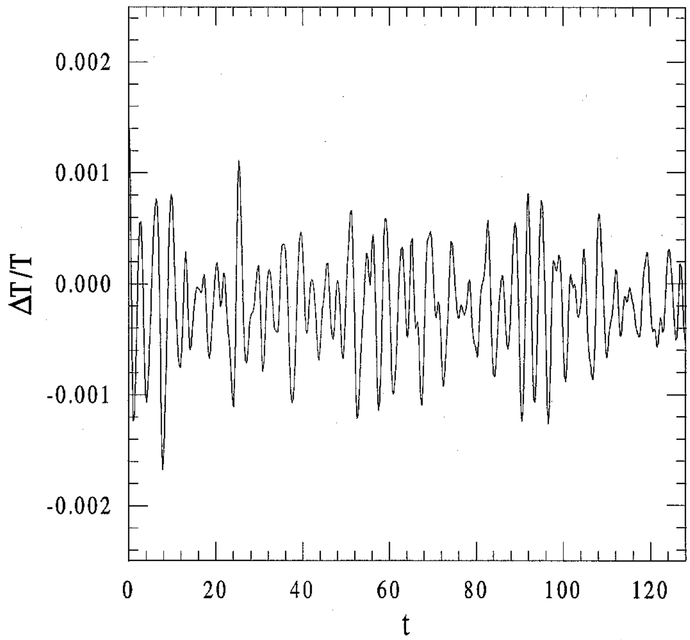


*Figure 1.* Evolution of the relative variation of the total kinetic energy of the system $(\Delta T)/T$ as a function of time for model A. The timescale is in crossing times $t_{cr}$.

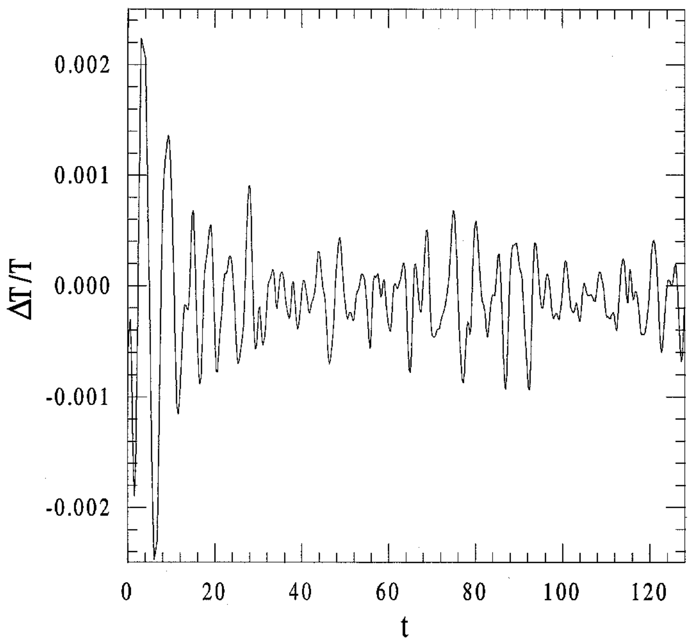


*Figure 2.* Same as Figure 1 for model B.

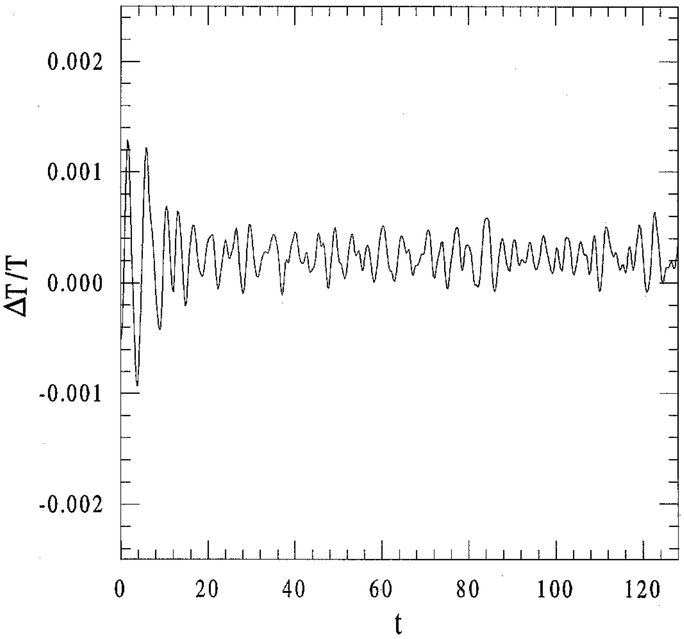


*Figure 3*. Same as Figure 1 for model C.

In order to improve the accuracy of our results, we run a second experiment (model B) using 2000 perturbation shells, and the results are plotted in Figure 2. The behavior is now somewhat different from that shown in Figure 1: some oscillation seems to be present, but the amplitude does not remain constant anymore. At the beginning of the integration, the amplitude shows a sudden growth well beyond the maximum amplitude reached by model A, but after the first ten crossing times it decays until achieving a value to that of the former case. Comparing Figures 1 and 2, it is possible to notice a small difference between the 'equilibrium' amplitudes of oscillation in models A and B, which suggests a possible dependence with the number of perturbation shells used to follow the evolution of the system. In order to ascertain whether that is the case, we decided to run the third numerical experiment (model C) using 4000 perturbation shells. Figure 3 shows the results obtained for the evolution of the kinetic energy in this model, and it becomes clear that the 'equilibrium' amplitude is appreciably smaller than in the two cases described before. The consequences of this dependence on the number of shells will be discussed in the next section.

Thus far, the signals shown in Figure 1, 2 and 3 have been presented and analyzed in a very qualitative way. We need now to study the pattern of oscillation in more detail, and we found that Fourier analysis (see, e.g., Press et al., 1989) can provide a good tool for that purpose. Here we have a function $T(t)$ that is sampled at evenly spaced time intervals. Let $\Delta$ denote the interval between consecutive

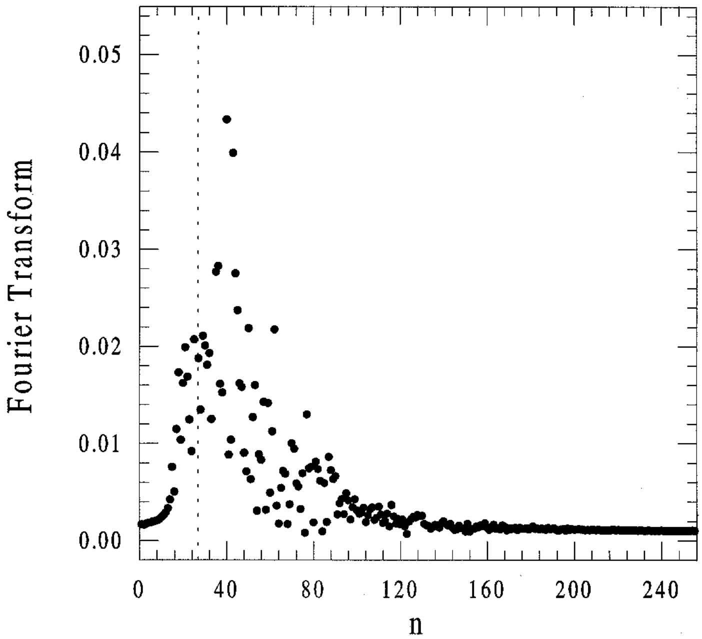


*Figure 4.* Discrete Fourier transform as a function of index $n$ (see text) corresponding to the oscillation shown in Figure 1. The vertical dashed line corresponds to the frequency predicted by the model of Chandrasekhar and Elbert (1972).

samples, $0.25t_{cr}$ in our case, and let $N$ be the total number of consecutive sampled values. Thus

$$T_k \equiv T(t_k), \quad t_k \equiv k\Delta, \quad k = 0, 1, 2, \ldots, N-1 \tag{3}$$

and it is possible to obtain the discrete Fourier transform $H_n$ of the $N$ points $T_k$ at the frequencies

$$f_n \equiv \frac{n}{N\Delta}, \quad n = -\frac{N}{2}, \ldots, \frac{N}{2}. \tag{4}$$

Figures 4, 5 and 6 summarize the results obtained for the Fourier transform as a function of the $n$ index for models A, B and C, respectively. The figures clearly show from the start that we are not in the presence of a purely noisy signal; on the contrary, there seems to exist some frequency at which the system *prefers* to oscillate. From the form of the spectra it is possible to estimate these frequencies for the different models, arriving to the following values: $f_{30} = 0.234t_{cr}^{-1}$ for model A, $f_{25} = 0.195t_{cr}^{-1}$ for model B and $f_{26} = 0.203t_{cr}^{-1}$ for model C. There is also a zero frequency component (that is, a constant) present in the spectral decomposition of the signal (see Figure 5) which means that the oscillation takes place around a value of total kinetic energy that is different from the value of this quantity in the

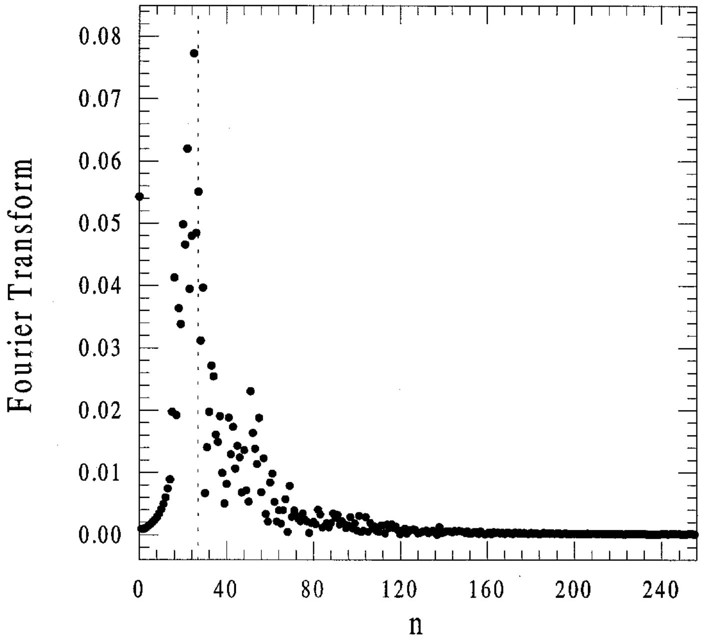


*Figure 5*. Same as Figure 4 but for the oscillation shown in Figure 2.

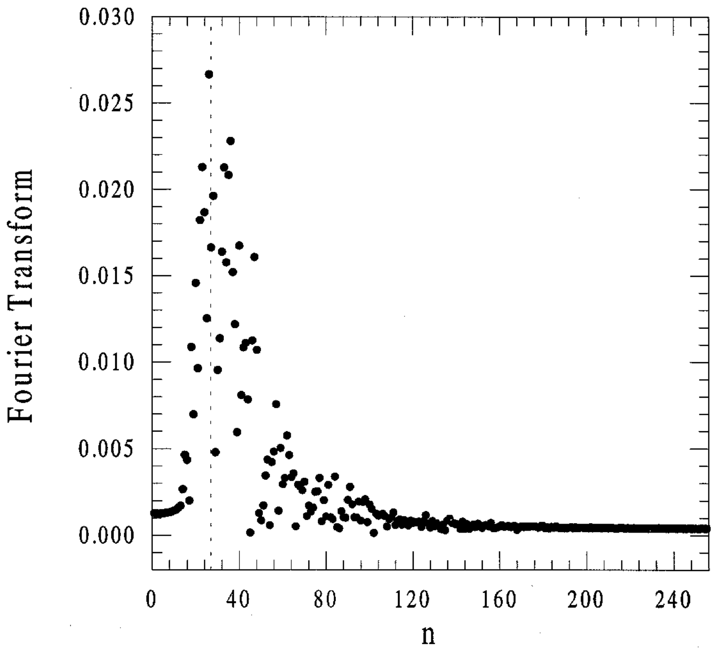


*Figure 6*. Same as Figure 4 but for the oscillation shown in Figure 3.

original unperturbed polytrope $(T_0)$. This behavior was also observed for models A and C but we have chosen the scales so as to emphasize the contribution of the non-zero frequencies leaving out of the graphs the values of the constant term in these cases.

The existence of an oscillating pattern that persists along the whole integration is very interesting and, therefore, it would be important to find a model that lets us understand the physical process that generates this phenomenon. We may apply to the perturbation the simple model described by Chandrasekhar and Elbert (1972). Let us consider the scalar virial theorem

$$\frac{1}{2}\frac{\mathrm{d}^2 I}{\mathrm{d}t^2} = 2T + W, \tag{5}$$

where $I$ is the moment of inertia, $T$ is the kinetic energy and $W$ is the potential energy of the system. For a spherically symmetric system we may write

$$W = -\frac{GM^2}{R_g}, \tag{6}$$

where $R_g$ is the gravitational radius, and

$$I = \alpha M R_g^2, \tag{7}$$

with $\alpha =$ constant (i.e., we assume that the system deforms self similarly, changing $R_g$, but not $\alpha$). Replacing expressions (6), (7) and the equation of the energy in the virial theorem we find that

$$\frac{1}{2}\frac{\mathrm{d}^2 I}{\mathrm{d}t^2} = 2E + \frac{C}{I^{1/2}}, \tag{8}$$

where

$$C = GM^{5/2}\alpha^{1/2}, \tag{9}$$

and $E$ is the total energy of the system. Consider now a stellar system that is very close to an equilibrium state. If we call $I_0$ the moment of inertia that the system has when it is in the equilibrium state, from Equation (8) it follows that

$$2E + \frac{C}{I_0^{1/2}} = 0. \tag{10}$$

Let us perturb slightly the system (at constant energy) in such a way that the mass is redistributed, modifying consequently the moment of inertia. The new moment of inertia is then written as

$$I = I_0 + \Delta I. \tag{11}$$

Replacing (11) in (8) we obtain

$$\frac{1}{2}\frac{\mathrm{d}^2\Delta I}{\mathrm{d}t^2} = 2E + \frac{C}{(I_0 + \Delta I)^{1/2}}, \tag{12}$$

which may be approximated if we consider just small perturbations ($\Delta I \ll I_0$) and make use of Equation (10), reaching the following harmonic oscillator equation

$$\frac{\mathrm{d}^2\Delta I}{\mathrm{d}t^2} + \frac{C}{I_0^{3/2}}\Delta I = 0. \tag{13}$$

Therefore the solution of this equation implies harmonic oscillations of angular frequency

$$\omega = \sqrt{\frac{C}{I_0^{3/2}}}. \tag{14}$$

In our case, we apply this expression to a polytrope of index $n = 3$, with the units chosen so that $G = 1, M = 1$ and $r_h = 1$, resulting in an angular frequency of $\omega = 0.868$ and a period of $P = 7.24$ time units. Transforming this result to the frequency $f$ and expressing its value in terms of the crossing time we obtain

$$f = 0.21 t_{cr}^{-1}. \tag{15}$$

We have represented this frequency in Figures 4, 5 and 6 with a vertical dashed line. In the first experiment (model A), the system shows a tendency to oscillate at some slightly greater frequency than that predicted by the theoretical model, but in the other two experiments (models B and C) the agreement is excellent. We will analyze these results in detail in the next section, discussing their consequences for the theory of stellar dynamics.

## 4. Discussion

The results presented in the last section suggest that there is some oscillation affecting the whole system and that it persists for very long periods of time. Our experiments, in accordance with those of MS, also led us to conclude that the frequency of the signal is accurately predicted by a simple model of homologous variations of the mass distribution of the system. There are, however, significant differences between the results obtained by us with the perturbation particle technique and those obtained by MS from the $N$-body treatment: for the polytropic model, MS found a nearly steady oscillation with a rms (root mean square) of about 0.7% of the mean value of $T$, while our experiments yielded amplitudes of

0.05%, 0.05% and 0.03%, respectively, for models A, B and C. Not only are our amplitudes smaller, but they decrease as we improve the sampling of phase space with larger numbers of particles.

Qualitatively, the signals resulting from models A, B and C are very similar to each other, not only from the spectral decomposition (compare Figures 4, 5 and 6) but also from the appearance of Figures 1, 2 and 3 when the ordinate ranges are chosen so that the oscillations have the same apparent amplitude. Consequently, we can conclude that the general form of the oscillation does not depend on the particular phase space sampling, although there is some relationship between this sampling and the amplitude of the signal.

What are the implications of our results in the galatic oscillations context? We think that the behavior revealed by the application of the perturbation particle method has an actual physical basis (recall the simple model of Chandrasekhar and Elbert, 1972), but it is influenced by the numerical technique used to simulate the evolution of the system. We see a proof of this statement in the paper by Gerber (1996), where the oscillation amplitude was shown to depend on the softening parameter. There is also a dependence of the amplitude and the algorithm used to integrate the system – e.g., Particle Mesh or Tree Codes – but this should be caused by the different application of the softening parameter.

The important point is: *the oscillation persists* for more than a hundred dynamical times. In the classical picture of galactic dynamics, one would expect the variations in kinetic energy to damp out, leaving a constant value (by some of the known damping processes like, e.g., phase mixing), but this does not happen. MS studied the possible importance of Landau damping or mode–mode coupling and they concluded that, if present, the effect of either must be very weak.

It is important to analyze how the number of particles influences the 'noise' in the virial ratio, in order to understand what to expect as natural fluctuations due to the finite number of particles involved. The study of the relation between half-power variation of the virial ratio and the number of particles was performed by Miller in the mid-1970s (Miller, 1973, 1974). He used the variance given in those papers to estimate the rms fluctuation in total kinetic energy and obtained a dependence with $2/\sqrt{3n-5}$. In our experiments we simulate the poissonian noise of 10 000 particles, so we obtain an expected value of rms of 1.16%. If we recall the results obtained by us for models A, B and C (0.05, 0.05 and 0.03, respectively) we see that they are contained in the range of expected noise. This is quite different from the results of MS, they found oscillations with amplitudes several times larger than expected. MS pointed out that these larger amplitudes had to be left over from initial conditions and Gerber (1996) demonstrated that they were related to the ratio of softening length to system dimensions. There is also a dependence on the algorithm used to evaluate the forces, e.g., Particle Mesh or Tree Codes. The 'effective softening' is different for each method.

Thus, we have reproduced the results of MS with a very different technique of integration. Galaxies seem to sustain regular oscillations for long periods of time, but the amplitude (of these oscillations) shown by numerical simulations are sensitive to softening present in each algorithm of integration. The model of Chandrasekhar and Elbert (1972) predicts very well the frequency of the 'fundamental mode' of oscillation.

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