

FINITE TEMPERATURE CORRELATED MEAN FIELD TREATMENTS AND INFORMATION THEORY

R. Rossignoli, A. Plastino

Departamento de Física, Universidad Nacional de La Plata
1900 La Plata, Argentina

Abstract

A general self-consistent scheme for approximating statistical operators is discussed within the context of Information Theory. As an application, a special correlated finite temperature mean field approximation is derived and applied to many-fermion systems. A substantial improvement over conventional approaches such as finite temperature Hartree-Fock and finite temperature BCS is obtained in finite systems.

Introduction

Finite temperature (FT) mean field approaches such as FT Hartree-Fock (FTHF) [1], FTBCS and FT Hartree-Fock-Bogoliubov (FTHFB) [2], constitute the basic microscopic methods for dealing with many-body systems at finite temperature. They provide the first step upon which further approximations and also higher order treatments (such as FTRPA [3]) are constructed. The essential ingredient in these theories is the replacement of the full hamiltonian by an effective temperature dependent single particle (sp) or single quasiparticle Hamiltonian in the exponent of the statistical operator and the use of a Grand Canonical (GC) ensemble for calculating the pertinent traces. As a consequence of these approximations, the Fermi-Dirac expression for the average sp occupation number and the finite temperature version of Wick's theorem for calculating averages of m -body operators, hold.

However, important shortcomings are exhibited by these theories, especially in finite systems and in the so called transitional regions. In particular, the vanishing of the order parameter, such as the quadrupole moment or the pairing gap in systems like finite nuclei, at the corresponding mean field critical temperature, is not actually seen in exact canonical calculations [4]-[6].

In the present contribution our aim is first to make a short review of general self consistent approximations for statistical operators [7]-[8], within the general framework of Information Theory [9]-[11]. As a particular case, the usual FT mean field

approximations are derived. Based on this general formalism, we derive next a correlated mean-field approximation, including a Correlated FTHF approach [12,14] (suitable also for *canonical* and projected statistics calculations), and also a general correlated FTHFB scheme [13] suitable for systems exhibiting pairing-like interactions in addition to long range forces. Finally, a particular application in a finite pairing model is developed.

General Statistical Formalism

Let us consider a quantum system about which the only available information consists of the expectation values O_i of n linearly independent observables \hat{O}_i . According to Information Theory, the least biased statistical operator $\hat{\rho}$ describing the system is that which maximizes the entropy (we set Boltzmann constant $k = 1$)

$$S = -\text{Tr}\hat{\rho}\ln\hat{\rho}, \quad (1)$$

subject to the constraints

$$\text{Tr}\hat{\rho}\hat{O}_i = O_i, \quad i = 1, \dots, n. \quad (2)$$

The result can be cast as

$$\hat{\rho} = \exp\{-\lambda_0 - \sum_i \lambda_i \hat{O}_i\}, \quad (3)$$

where λ_0 is a normalization constant ($\text{Tr}\hat{\rho} = 1$) and λ_i a set of Lagrange parameters to be determined by means of the constraints (2).

Ordinary equilibrium finite temperature descriptions are obtained when one of the operators \hat{O}_i is the Hamiltonian \hat{H} of the system and the remaining ones commute with \hat{H} (for instance, one of the \hat{O}_i 's is the particle number operator in GC treatments). Nevertheless, the present formalism is completely arbitrary and allows for general non-commuting operators \hat{O}_i , being thus suitable for off equilibrium statistical descriptions.

The ensuing maximum entropy reads

$$S = \lambda_0 + \sum_i \lambda_i O_i. \quad (4)$$

It is easy to show that S and λ_0 , as functions of O_i and λ_i respectively, satisfy the conjugate relationships

$$\partial S / \partial O_i = \lambda_i, \quad \partial \lambda_0 / \partial \lambda_i = -O_i. \quad (5)$$

Generalized Self-Consistent Approximation

In many-body systems, the previous exact statistical operator (3) is not tractable in general and one is forced to employ approximate schemes for effectively computing the pertinent traces. We shall now describe a general self-consistent approximation for the density (3) [7]–[8]. Starting with a set of (in principle) m arbitrary operators \hat{P}_j , $m \geq n$, we set up a trial approximate statistical operator possessing the form

$$\hat{\rho}_{ap} = \exp\{-\lambda_0 - \sum_j \lambda_j \hat{P}_j\}, \quad (6)$$

where now λ_j are a set of variational parameters to be determined by maximizing the approximate entropy

$$S_{ap} = -\text{Tr} \hat{\rho}_{ap} \ln \hat{\rho}_{ap}, \quad (7)$$

subject to the constraints

$$\text{Tr} \hat{\rho}_{ap} \hat{O}_i = O_i. \quad (8)$$

In other words, the exponent of the statistical operator is approximately expanded in the (undercomplete) space spanned by the operators \hat{P}_j .

The optimum $\hat{\rho}_{ap}$ can be obtained by introducing n additional Lagrange parameters β_i and maximizing the quantity

$$S' = S_{ap} - \sum_i \beta_i O_i, \quad (9)$$

with respect to the yet unknown expectation values $P_j = \text{Tr} \hat{\rho}_{ap} \hat{P}_j$. Using equations (5) [applied to the operator (6)], we obtain the fundamental relations

$$\lambda_j = \sum_i \beta_i \partial O_i / \partial P_j, \quad (10)$$

which represent a *self-consistent* set of equations for the parameters λ_j (they are given by a function of the expectation values they determine). By recourse to the Kubo transforms [15] of the operators \hat{P}_j , defined here as

$$\hat{P}_j^* = \int_0^1 \hat{\rho}_{ap}^{-u} \hat{P}_j \hat{\rho}_{ap}^u du - \text{Tr}(\hat{\rho}_{ap} \hat{P}_j), \quad (11)$$

it is possible to explicitly express (10) as

$$\lambda_j = \sum_{i,l} \beta_i A_{il} B_{lj}^{-1}, \quad (12)$$

where

$$A_{il} = \partial O_i / \partial \lambda_l = -\langle \hat{P}_l^* \hat{O}_i \rangle = -\langle \hat{O}_i^* \hat{P}_l \rangle, \quad (13)$$

and B is the $m \times m$ generalized covariance matrix

$$B_{jl} = \partial P_j / \partial \lambda_l = -\partial^2 \lambda_0 / \partial \lambda_j \partial \lambda_l = -\langle \hat{P}_l^* \hat{P}_j \rangle. \quad (14)$$

The approximate density operator can finally be written as

$$\hat{\rho}_{ap} = \exp\{-\lambda_0 - \sum_i \beta_i \hat{o}_i\}, \quad (15)$$

with

$$\hat{o}_i = \sum_j (\partial O_i / \partial P_j) \hat{P}_j. \quad (16)$$

The effective operator (16) can be considered as the (density dependent) *projection* of \hat{O}_i onto the space spanned by the operators \hat{P}_j .

We are thus led to a formal self consistent solution for the density operator. In case all operators \hat{O}_i are linearly related to the \hat{P}_j 's we obviously recover from (10) the exact statistical operator. Otherwise, the operator (15) provides obviously a lower bound to the exact entropy (4).

We can distinguish now two different applications: a) Those in which the expectation values of the operators \hat{O}_i are actually given; b) Those in which the Lagrange parameters β_i are supposed to be known and we are interested in the behavior of the

approximation as a function of the β_i 's. In case a), it is obvious that situations may exist in which we cannot fulfill the constraints (8) (i.e. we cannot obtain a solution for the β_i 's), since the range of expectation values spanned by the approximate density may be smaller than the exact range.

We shall consider in the present work case b) with the aim of applying the formalism to finite temperature problems. In this case, we shall take the operator \hat{O}_1 as the Hamiltonian \hat{H} of the system and the associated parameter $\beta \equiv \lambda_1$ will be the inverse of the temperature T . We can write in this situation (15) in the more familiar form

$$\hat{\rho}_{ap} = \exp(-\lambda_0' - \beta \hat{h}'), \quad (17)$$

where

$$\hat{h}' = \hat{h} - \sum_{i=2}^n \mu_i \hat{o}_i, \quad (18)$$

with \hat{h} the effective Hamiltonian constructed as in (16) and $\mu_i = -\lambda_i/\beta$ the chemical potentials.

As we shall see, the present formalism contains as a very particular case the usual FT mean field approaches. We shall proceed now to a rigorous and quite general rederivation of microscopic FT and statistical mean field theories.

Extended FTHF Formalism

In order to extract from the previous formalism a useful approximation for many-body systems, it is necessary to suitably select a set of operators \hat{P}_j such that the ensuing density operator becomes tractable. The most obvious choice is to restrict the \hat{P}_j 's to *single particle* operators of the type $c_i^\dagger c_j$ in which case the density becomes

$$\begin{aligned} \hat{\rho}_{ap} &= \exp\{-\lambda_0 - \sum_{i,j} \lambda_{ij} c_i^\dagger c_j\} \\ &= \exp\{-\lambda_0 - \sum_i \lambda_i a_i^\dagger a_i\}, \end{aligned} \quad (19)$$

where a_i^\dagger, a_i are related to the original operators by means of a unitary (HF-like [16]) transformation

$$c_i = \sum_j U_{ij} a_j, \quad UU^\dagger = I, \quad (20)$$

such that $U^\dagger \Lambda U = \Lambda'$, with Λ and Λ' the matrices of elements $\lambda_{ij}, \lambda_i \delta_{ij}$ respectively. In what follows we shall consider the fermion case [7], so that $[c_i^\dagger, c_j]_+ = \delta_{ij}$ (for the boson case see [8]).

The formalism becomes in this situation equivalent to a *generalized* statistical HF approximation. The ensuing variational equations can be cast in two different pieces: one involving the parameters λ_i and the other the transformation matrix U . We obtain, using (10),

$$\lambda_i = \text{Tr} \hat{\rho}_{ap} \sum_j [(a_j^\dagger a_j - f_j) \hat{O}'] B_{ji}^{-1}, \quad (21)$$

with $\hat{O}' = \sum_i \beta_i \hat{O}_i$, and where the covariance matrix B reduces now to

$$B_{ij} = f_{ij} - f_i(f_j - \delta_{ij}), \quad (22)$$

with f_j and f_{ij} the generalized average one and two-body occupation numbers,

$$f_i = \text{Tr} \hat{\rho}_{ap} a_i^\dagger a_i, \quad (23)$$

$$f_{ij} = \text{Tr} \hat{\rho}_{ap} a_i^\dagger a_j^\dagger a_j a_i, \quad (24)$$

such that

$$\text{Tr} \hat{\rho}_{ap} a_i^\dagger a_j = \delta_{ij} f_i, \quad \text{Tr} \hat{\rho}_{ap} a_i^\dagger a_k^\dagger a_j a_l = f_{ik} (\delta_{kj} \delta_{il} - \delta_{kl} \delta_{ij}). \quad (25)$$

Minimization with respect to the HF transformation leads to the fundamental equation

$$\text{Tr} \hat{\rho}_{ap} [a_i^\dagger a_j, \hat{O}'] = 0, \quad i \neq j. \quad (26)$$

Equations (21)–(26) are completely general and hold in *any* type of ensemble. The usual FTHF equations are recovered if the pertinent traces are taken in a *Grand Canonical* ensemble. Only in this case the familiar Fermi-Dirac expression for the one-body occupation numbers,

$$f_j = [1 + e^{\lambda_j}]^{-1}, \quad (27)$$

and the *finite temperature Wick's theorem* [17] for calculating averages of m -body operators ($m > 2$) hold. This implies in the present context

$$f_{ij} = f_i f_j (1 - \delta_{ij}). \quad (28)$$

In this case, $B_{ij} = f_i (1 - f_i) \delta_{ij}$, corresponding to a fully uncorrelated picture, and equations (21)–(26) reduce to the usual HF equations (see next section). We remark however that in other ensembles, expressions (27) and (28) do not necessarily hold and one should calculate them using the diagonal density operator (19).

Hence, the previous formalism allows us to extend the usual GC FTHF equations to any type of ensemble and to a general statistical context. We are thus in a position to perform *Canonical* FTHF calculations [12,14] (conserving exactly the number of particles). The formalism is also suitable for *projected* statistics [12] (ensembles which are microcanonical with respect to some set of operators), as for instance, angular momentum projected statistics, of much interest in finite temperature Nuclear Physics.

Nevertheless, the two-body occupation numbers are obviously still dependent on the one-body mean values, and do not constitute new degrees of freedom in the present formalism. Up to now, we have considered thus the possibility of introducing correlations just in the trace, but not in the density operator itself. We will now attempt to go beyond the statistical HF approximations (whether GC or not) by introducing special correlations in the density operator but preserving at the same time its tractability.

Correlated FTHF

We shall now consider the addition of *diagonal two-body* terms in the exponent of the density operator, such that

$$\hat{\rho}_{ap} = \exp\{-\lambda_0 - \sum_i \lambda_i a_i^\dagger a_i - \sum_{i,j} \lambda_{ij} a_i^\dagger a_j^\dagger a_j a_i\}, \quad (29)$$

with the operators a_i^\dagger , a_i given again by (20). In this way, (29) remains diagonal in an independent particle basis (i.e., consisting of Slater Determinants), so that no diagonalization is required for calculating traces.

The essential difference which is introduced now is that the two-body occupation numbers (24) are now *independent quantities*. In other words, they constitute new degrees of freedom. As we shall explicitly see, the density operator (29) is able to provide one with an essentially different probability distribution at finite temperature, in comparison with that given by ordinary or extended FTHF.

In order to determine the parameters λ_i , λ_{ij} , we employ now equations (10). We shall restrict ourselves to one and two-body operators \hat{O}_i , so that \hat{O}' can be written as

$$\hat{O}' = \sum_{i,j} \varepsilon_{ij} c_i^\dagger c_j + \sum_{i,j,k,l} \frac{1}{4} V_{ijkl} c_i^\dagger c_j^\dagger c_l c_k \quad (30)$$

$$= \sum_{i,j} \varepsilon'_{ij} a_i^\dagger a_j + \frac{1}{4} \sum_{i,j,k,l} V'_{ijkl} a_i^\dagger a_j^\dagger a_l a_k, \quad (31)$$

with

$$\varepsilon'_{ij} = [U^\dagger \varepsilon U]_{ij}, \quad V'_{ijkl} = [U^\dagger U^\dagger V U U]_{ijkl}, \quad (32)$$

in obvious notation.

In this case, the expectation value of \hat{O}' with respect to (29) will be a *linear* function of the elements f_i , f_{ij} . Hence, eqs. (10) lead to

$$\lambda_i = \varepsilon'_i, \quad \lambda_{ij} = \frac{1}{2} V'_{ijij}, \quad (33)$$

so that we obtain the expected result

$$\hat{\rho}_{op} = \exp(-\lambda_0 - \hat{O}'_d), \quad (34)$$

where

$$\hat{O}'_d = \sum_i \varepsilon'_i a_i^\dagger a_i + \frac{1}{2} \sum_{i,j} V'_{ijij} a_i^\dagger a_j^\dagger a_j a_i \quad (35)$$

is just the diagonal part of \hat{O}' in the independent particle basis.

The general equation determining the HF matrix U is still given by (26). For the case of a two-body \hat{O}' , it can be cast as

$$\varepsilon'_{ij}(f_j - f_i) + \sum_k V'_{ikjk}(f_{jk} - f_{ik}) = 0, \quad (36)$$

which is the fundamental equation of the CFTHF approach. We note that if Wick's theorem is applied in eqs. (36) [i.e. eq. (28)], we recover the ordinary FTHF equations

$$\varepsilon'_{ij} + \sum_k V'_{ikjk} f_k = 0, \quad (37)$$

with f_k given by expression (27). However, this does not obviously hold any longer in the correlated approach (whatever the ensemble) and equations (36) must be solved self-consistently with respect to the (U -dependent) diagonal operator (34).

We should bear in mind that in the statistical HF approximation, recovered in the present scheme for $\lambda_{ij} = 0$, we have in addition to solve the non linear eqs. (21) (i.e., we must solve for the sp energies), which represent the *projection* of \hat{O}'_d onto the space spanned by the diagonal sp operators. On the other hand, in the present CFTHF treatment, eqs. (36) are the *only* ones to be solved, since the λ_i 's and the

λ_{ij} 's are already known [eqs. (33)]. In this sense, the CFTHF eqs. are structurally simpler than the ordinary FTHF equations.

Hence, the central point we have considered is the addition of diagonal two-body correlations in the density operator. Equation (36) can be considered now as the extension of the usual FTHF eqs. to density operators *diagonal* in an independent particle basis. In some sense, this CFTHF treatment can be regarded as the 'true' or correct statistical extension of HF. For the operator (29) is, for one and two-body operators \hat{O}_i , the *best* statistical operator diagonal in this type of basis, that can be constructed. The extension of the present scheme to arbitrary m -body operators \hat{O}_i is straightforward (one should just include in (29) diagonal m -body terms).

Finally, for thermal applications, it seems more familiar to cast the density (34) as

$$\hat{\rho}_{ap} = \exp(-\lambda_0 - \beta \hat{H}'_d), \quad (38)$$

where \hat{H}'_d is the *diagonal* part of the generalized Hamiltonian [see (18)] $\hat{H}' = \hat{H} - \sum_{i=2}^n \mu_i \hat{O}_i$.

Extension to Pairing Interactions; Correlated FTHFB

The extension of the approximations of the previous sections to systems containing pairing-like interactions is straightforward. The standard FTBCS and FTHFB theories can be derived by considering just the set $\hat{P}_j = \{c_i^\dagger c_j, c_i^\dagger c_j^\dagger, c_i c_j\}$, i.e., by restricting the exponent of $\hat{\rho}_{ap}$ to a general quadratic function of fermion operators. The ensuing pairing extension of the correlated FTHF, leading in general to a correlated FTHFB (CFTHFB) [13] approximation, can be obtained again from a trial density of the form (29) but with the operators $a_i^\dagger a_i$ related to the original operators by means of a Bogoliubov transformation

$$c_i = \sum_j (U_{ij} a_j + V_{ij} a_j^\dagger), \quad (39)$$

with

$$UV^{tr} + VU^{tr} = 0, \quad UU^\dagger + VV^\dagger = I, \quad (40)$$

in order to ensure fermion commutation relationships for the operators a_i, c_i . In this way, the density operator (29) is diagonal now in an *independent quasiparticle* basis. The formalism obviously reduces to a correlated FTBCS (CFTBCS) approach if (39) is restricted to a BCS transformation [13]. We recover the ordinary FTBCS and FTHFB approaches by setting $\lambda_{ij} = 0$ in (29).

We again obtain for $\hat{\rho}_{ap}$ an expression similar to (34), where \hat{O}'_d is now the full diagonal part of \hat{O}' in the quasiparticle representation. In order to perform simple calculations, it is necessary to take the traces now in a GC ensemble. The fundamental equations that determine the transformation (39) are

$$\text{Tr} \hat{\rho}_{ap} [a_i^\dagger a_j, \hat{O}'] = 0, \quad \text{Tr} \hat{\rho}_{ap} [a_i^\dagger a_j^\dagger, \hat{O}'] = 0, \quad i \neq j, \quad (41)$$

which, for two-body operators \hat{O}_i , can be cast as the equation (36) plus

$$\varepsilon_{ij}^{02} (f_i + f_j - 1) + \sum_k V_{kki j}^{13} (f_{ik} + f_{jk} - f_k) = 0, \quad (42)$$

where now ε'_{ij} , V'_{ijkl} , ε_{ij}^{02} and $V_{kki j}^{13}$ represent, respectively, the (suitably antisymmetrized) coefficients of $a_i^\dagger a_j$, $a_i^\dagger a_j^\dagger a_l a_k$, $a_j a_i$ and $a_k^\dagger a_l a_j a_i$ in the quasiparticle expansion of \hat{O}' , and are obviously transformation dependent. Thus, equations (36) and

(42) must be solved self-consistently. Again, we can recover the conventional FTBCS and FTHFB equations (or more adequately, the general statistical extensions of these approximations) if Wick's theorem and the Fermi-Dirac expression are employed in (36) and (42).

Application

As a particular illustration of the previous formalism, we shall consider a schematic two-level pairing model. We shall label the sp states as $|p, \nu\rangle$, $p = 1, \dots, \Omega$, $\nu = \pm 1$. We shall examine the quasispin pairing Hamiltonian [18]

$$\hat{H} = \varepsilon \hat{J}_z - \frac{1}{2} G \hat{Q}_+ \hat{Q}_-, \quad (43)$$

where

$$\hat{Q}_+ = \sum_p c_{p+}^\dagger c_{p-}^\dagger, \quad \hat{J}_z = \frac{1}{2} \sum_{p,\nu} \nu c_{p\nu}^\dagger c_{p\nu}, \quad (44)$$

are collective operators satisfying, together with $\hat{Q}_z = \frac{1}{2}(\hat{N} - \Omega)$ and $\hat{J}_\nu = \sum_p c_{p\nu}^\dagger c_{p-\nu}$ an $SU(2) \times SU(2)$ algebra [18]. The present Hamiltonian represents a pairing interaction between levels separated by an energy ε , and reduces to the degenerate pairing model [2] for $\varepsilon = 0$. We shall consider the case $N = \Omega$.

The relevant Bogoliubov transformation reduces here to the BCS transformation

$$c_{p\nu} = a_{p\nu} \cos \frac{1}{2}\theta + \nu e^{i\phi} a_{p-\nu}^\dagger \sin \frac{1}{2}\theta, \quad (45)$$

which gives rise, in the conventional FTBCS approach [5,6], to a non vanishing pairing tensor represented here by

$$|\langle \hat{Q}_+ \rangle| = \Omega q \sin \theta = 2\Delta/G, \quad (46)$$

where $q = \frac{1}{2}(1 - f_+ - f_-)$, with $f_\nu = \langle a_{p\nu}^\dagger a_{p\nu} \rangle$, and Δ the FTBCS pairing gap. This particle number symmetry breaking tensor is the *essential* ingredient in the FTBCS theory of superconductivity. The mean value of the interaction becomes $-2\Delta^2/G$ in this approximation (neglecting small contributions of relative order $1/\Omega$ to the HF potential). For $G\Omega > 4\varepsilon$, the systems starts at $T = 0$ in a superconducting state characterized by $q = \frac{1}{2}$, $\theta = \frac{1}{2}\pi$, and, as T increases, the gap decreases approaching 0 at the critical temperature determined by $8T_c \cosh^2(\varepsilon/4T_c) = \Omega G$ [5], where there is a transition to the 'normal' state characterized by $q = 0$. For $T > T_c$, no effect of the interaction is seen in FTBCS ($\Delta = 0$). On the other hand, the exact canonical results exhibit no sharp transition for finite values of Ω , and the expectation value of the interaction decreases smoothly with temperature but does not vanish.

When considering the correlated FTBCS treatment based on the operator (29) and using the transformation (45) [13], the thermal description of the pairing interaction is substantially different from that given by ordinary FTBCS. One finds in CFTBCS a smooth thermal evolution of the system, with no sharp transitions and a smooth decrease of the mean value of the interaction as T increases, in agreement with exact results. Moreover, one finds also $\langle \hat{Q}_+ \rangle = 0$, implying thus a *vanishing* pairing tensor and hence an *approximate symmetry restoration* [13] (actually the particle number fluctuations are still non vanishing).

This behaviour is depicted in figure 1, for a typical situation. Since the gap is essentially a mean field parameter, we have defined the gap for the exact and

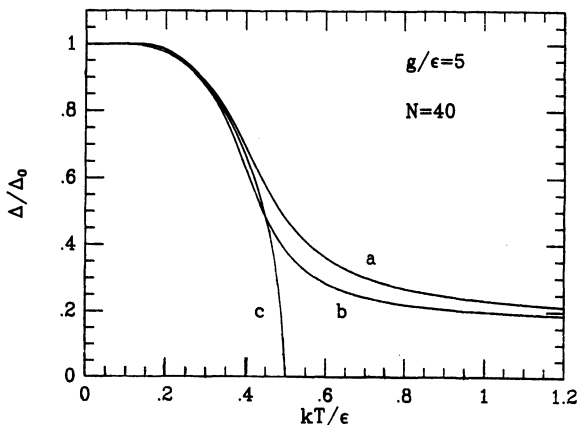


Figure 1. The pairing gap Δ as a function of the temperature T in the quasispin pairing model (see text). Δ_0 denotes the gap at $T = 0$, N is the number of particles and $g = \Omega G$. (a) denotes exact results, (b) results corresponding to the present correlated FTBCS treatment and (c) the conventional FTBCS results.

correlated treatment as $\Delta = \frac{1}{2}G\sqrt{\langle\hat{Q}_+\hat{Q}_-\rangle}$, which coincides with the gap defined in (46) in case we use the FTBCS expectation value. It is clearly seen that a substantial improvement over FTBCS results is obtained in the correlated approach for finite values of Ω .

Results in other finite systems containing different interactions, including realistic nuclei such as ^{20}Ne , are qualitatively similar (see [12]–[14]). A much better qualitative description of the pertinent physics is found in the correlated approach, in comparison with that given by ordinary FT mean field treatments.

Conclusions

Based on a general self-consistent approximation for statistical operators derived within the framework of Information Theory, we have developed a general correlated finite temperature mean field approximation. The method is based on the inclusion of special diagonal correlations in the density operator, and can be easily applied in arbitrary ensembles and general statistical contexts.

It is seen that the method provides a substantial improvement over the conventional FT mean field treatments, especially in finite systems and in transitional regions. Moreover, it is able to approximate restore some of the symmetries broken in mean field calculations, providing one with a much better global description of the pertinent physics.

In particular, we have seen in the example considered that a correct description of the pairing interaction is obtained in the correlated FTBCS treatment with a *vanishing* value of the pairing tensor. Furthermore, the interaction effects do not completely vanish at high temperatures, in contrast with ordinary FTBCS results. No sharp transition is obtained at finite temperatures for a finite value of the number of particles, in agreement with exact canonical results.

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