Beyond Bekenstein's Theory

L. Kraiselburd, M. Miller Bertolami, P. Sisterna, and H. Vucetich

Abstract There are several very different motivations for studying the variation of fundamental constants. They may provide a connection between cosmology and particle physics due to the coincidence of large dimensionless numbers arising from the combination of different physical constants. Bekenstein's variable charge model is very attractive because it is based on very general assumptions: covariance, gauge invariance, causality and time-reversal invariance of electromagnetism. The generality of its assumptions guarantee the applicability of the scheme to other gauge interactions such as the strong forces. Besides, it introduces a useful simplifying assumption; namely, that the gravitational sector is unaffected by the scalar field introduced to vary the coupling constant. That is why it is interesting to explore first this simplified model, before a similar exploration of more general theories.

1 Introduction

Since the proposal due to Gamow [11], the possible time variation of the fine structure constant has been analyzed by many authors. There are many publications

L. Kraiselburd
Grupo de Gravitación, Astrofísica y Cosmología, Facultad de Ciencias Astronómicas y Geofísicas, Universidad Nacional de La Plata, Paseo del Bosque s/n (1900) La Plata, Argentina
e-mail: lkrai@fcaglp.fcaglp.unlp.edu.ar

M.M. Bertolami
Grupo de Evolución estelar y pulsaciones, Facultad de Ciencias Astronómicas y Geofísicas, Universidad Nacional de La Plata, Paseo del Bosque s/n (1900) La Plata, Argentina
e-mail: mmiller@fcaglp.fcaglp.unlp.edu.ar

P. Sisterna
Facultad de Ciencias Exactas y Naturales, Universidad Nacional de Mar del Plata, Funes 3350 (7600) Mar del Plata, Argentina
e-mail: sisterna@mdp.edu.ar

H. Vucetich
Grupo de Gravitación, Astrofísica y Cosmología, Facultad de Ciencias Astronómicas y Geofísicas, Universidad Nacional de La Plata, Paseo del Bosque s/n (1900) La Plata, Argentina
e-mail: vucetich@fcaglp.fcaglp.unlp.edu.ar

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on observational upper bounds on its time variation as well as several theoretical frameworks (see [19,26] and references there in). It's very motivating to think about the possibility that alpha has had a different value to the current, although this is a subject of great debate and more research must be done about it [21].

Bekenstein's theory [1], resting on a number of minimal hypothesis based on highly accepted physical principles, is in a sense representative of many low energy theories inspired on grand unification schemes. In this work we will derive equations that govern the energy exchange between matter, the scalar field and the electromagnetic field. Although we do not analyze the precise mechanism of energy release, we assume that the work done by the scalar field is radiated away in an efficient way, as is the case in the rotochemical heating of neutron stars due to the spin down of the star [7,23].

In section 2 we make a brief review of Beckenstein's theoretical model. In section 3 we derive a generalized version of Poynting theorem for the electromagnetic field and we find how the energy flow of matter is modified by the scalar field. In section 4 we describe the magnetic energy of matter using "sum rules techniques". In section 5, we study the thermal history of the Earth in the presence of the scalar field. Finally in section 6 we present our conclusions.[1]

2 Bekenstein's Theory

Here we review Bekenstein's theory and its prediction for the cosmological time variation of α. Although we will consider galactic as well as terrestrial phenomena, we nevertheless can confidently assume that they track the cosmological evolution of α [25].

Bekenstein [1] modifies Maxwell's theory by introducing a field ϵ that dynamically describes the variation of α. The hypothesis are [1,19]

1. The theory must reduce to Maxwell's when $\alpha = \text{Cte}$.
2. The changes in α are dynamical (i.e. generated by a dynamical field) ϵ.
3. The dynamics of the electromagnetic field as well as ϵ's can be obtained from a variational principle.
4. The theory must be local gauge invariant.
5. The theory must preserve causality.
6. The action must be time reversal invariant.
7. Planck's scale ℓ_P is the smallest length available in the theory.
8. Einstein's equations describe gravitation.

String theories and the like in which there are other fundamental length scales, force us to set aside condition 7. These hypothesis uniquely lead to the following action:

$$S = S_{\text{em}} + S_\epsilon + S_m + S_G \tag{1}$$

[1] This contribution is a summary of our article "Energy production in varying α theories", which will be published in Astronomy and Astrophysics.

where

$$S_{\text{em}} = -\frac{1}{16\pi}\int F^{\mu\nu}F_{\mu\nu}\sqrt{-g}d^4x, \tag{2}$$

$$S_{\epsilon} = -\frac{\hbar c}{2\ell_B}\int \frac{\epsilon^{,\mu}\epsilon_{,\mu}}{\epsilon^2}\sqrt{-g}d^4x, \tag{3}$$

S_m and S_G are the matter and gravitational field actions respectively, and the metric here is $(-1, 1, 1, 1)$.

Bekenstein modifies the connection between the vector potential and the electromagnetic field that comes from Maxwell's.

$$F_{\mu\nu} = \frac{1}{\epsilon}\left[(\epsilon A_\nu)_{,\mu} - (\epsilon A_\mu)_{,\nu}\right] \tag{4}$$

and the (second kind) local gauge invariance implies

$$\epsilon A'_\mu = \epsilon A_\mu + \chi_{,\mu} \tag{5}$$

$$\nabla_\mu = \partial_\mu - e_0\epsilon A_\mu \tag{6}$$

as the gauge transformation and covariant derivative of the theory respectively. The last equation defines the local value of the elementary electric charge (coupling constant)

$$e(\boldsymbol{r}, t) = e_0\epsilon(\boldsymbol{r}, t) \tag{7}$$

that is

$$\epsilon = \left(\frac{\alpha}{\alpha_0}\right)^{\frac{1}{2}} \tag{8}$$

In what follows we will neglect the small spatial variations of α and focus on the cosmological variation, as we will be interested on any secular energy injection of the scalar field on a planet such as the Earth. In our approximation it is also enough to work in flat space-time.

The field equations for the electromagnetic field and for ϵ are

$$\left(\frac{1}{\epsilon}F^{\mu\nu}\right)_{,\nu} = 4\pi j^\mu \tag{9a}$$

$$\begin{aligned}\Box \ln\epsilon &= \frac{\ell_B^2}{\hbar c}\left[\epsilon\frac{\partial\sigma}{\partial\epsilon} - \epsilon j^\mu A_\mu + \frac{1}{4\pi}\left(A_\mu F^{\mu\nu}\right)_{,\nu}\right] \\ &= \frac{\ell_B^2}{\hbar c}\left(\epsilon\frac{\partial\sigma}{\partial\epsilon} - \frac{F^{\mu\nu}F_{\mu\nu}}{8\pi}\right)\end{aligned} \tag{9b}$$

where $j^{\mu} = \sum(e_0/c\gamma)u^{\mu}(-g)^{-1/2}\delta^3[x^i - x^i(\tau)]$ and σ is the energy density of matter [1]. $\Box$ is the covariant flat d'Alambertian

$$\Box\phi = \phi^{,\mu}{}_{,\mu} = \eta^{\mu\nu}\phi_{,\mu,\nu}. \tag{10}$$

A note regarding the matter lagrangian is in order: in [1, 2] Bekenstein represents matter as an ensemble of classical particles. However, wherever quantum phenomena become important, as in white dwarfs or condensed matter physics, this is not a realistic description. It is neither a good picture at large energy scales (or small length scales) because fermions have a "natural length scale", the particle Compton wave length $\lambda_C = \hbar/mc$, that makes quite unrealistic any classical model at higher energies. In particular several conclusions of reference [2] have to be reconsidered.

In reference [1] it is shown that the cosmological equation of motion for ϵ is

$$\frac{d}{dt}\left(a^3\frac{\dot{\epsilon}}{\epsilon}\right) = -a^3\frac{\ell_B^2}{\hbar c}\left[\epsilon\frac{\partial\sigma}{\partial\epsilon} - \frac{1}{4\pi}\left(\mathbf{E}^2 - \mathbf{B}^2\right)\right]. \tag{11}$$

In the non relativistic regime $\mathbf{E}^2 \gg \mathbf{B}^2$ and $\sigma \propto \epsilon^2$, hence

$$\frac{d}{dt}\left(a^3\frac{\dot{\epsilon}}{\epsilon}\right) = -a^3\zeta_c\frac{\ell_B^2}{\hbar c}\rho_m c^2 \tag{12}$$

where ρ_m is the total rest mass density of electromagnetically interacting matter and ζ_c is a parameter describing its "electromagnetic content", which is essentially the ratio of the energy-momentum trace and the total mass. A first estimation is

$$\zeta_c \sim 1.2 \times 10^{-3}. \tag{13}$$

Following the standard cosmological model, we assume dark matter to be electromagnetically neutral.

Given that $\rho_m \propto a^{-3}$ we can integrate Eq. (12) and use the usual cosmological notation obtaining

$$\frac{\dot{\epsilon}}{\epsilon} = -\frac{3\zeta_c}{8\pi}\left(\frac{\ell_B}{\ell_P}\right)^2 H_0^2\Omega_B\left[\frac{a_0}{a(t)}\right]^3(t - t_c). \tag{14}$$

Primordial nucleosynthesis standard model tell us that the integration constant t_c must be very small in order not to spoil the agreement between theory and observation. Using WMAP values we obtain the following prediction for $(\dot{\alpha}/\alpha)_0$

$$\left(\frac{\dot{\alpha}}{H_0\alpha}\right)_0 = 1.3 \times 10^{-5}\left(\frac{\ell_B}{\ell_P}\right)^2. \tag{15}$$

Any measurement with a precision such as $\sigma(\dot{\alpha}/H_0\alpha) \sim 10^{-5}$ is difficult to achieve, so the comparison between theory and experiment is a difficult task.

The same arguments can be applied to many theories with varying α, such as Kaluza-Klein [19] or string inspired theories as Damour-Polyakov's [4, 5].

3 Energy Transfer in Bekenstein's Formalism

We will study how energy is injected and then released in varying α theories, in order to look for observable consequences in the emissions of astrophysical as well as geophysical systems. According to Bekenstein and using $c = 1$, the electromagnetic contribution has the same form as in Maxwell's theory

$$T^{\text{em}}_{\mu\nu} = \frac{1}{4\pi}\left[F_{\mu\lambda}F_{\nu}{}^{\lambda} - \frac{g_{\mu\nu}}{4}F_{\lambda\sigma}F^{\lambda\sigma}\right] \tag{16}$$

the difference lying in the connections between the vector potential and the field Eq. (4).

On the other hand, the energy-momentum tensor of the scalar field ϵ is:

$$T^{\mu\nu}_{\epsilon} = \frac{\hbar}{\ell_B^2}\left(\frac{\epsilon^{,\mu}\epsilon^{,\nu}}{\epsilon^2} - \frac{1}{2}g^{\mu\nu}\frac{\epsilon^{,\alpha}\epsilon_{,\alpha}}{\epsilon^2}\right). \tag{17}$$

In what follows we use the redefined field as $\psi = \ln\epsilon$. As we will consider local phenomena, we can work in a locally inertial coordinate system. We denote the "field part of the energy-momentum tensor" as the scalar plus electromagnetic energy momentum tensor:

$$T^{\mu\nu}_{\text{f}} = T^{\mu\nu}_{\text{em}} + T^{\mu\nu}_{\epsilon} \tag{18}$$

In terms of ψ and replacing $g^{\mu\nu}$ with $\eta^{\mu\nu}$, we obtain that the divergence of T_{f} is

$$\begin{aligned} T_{\text{f}}^{\mu\nu}{}_{,\nu} = & \frac{1}{4\pi}\left[F^{\mu\alpha}{}_{,\nu}F^{\nu}{}_{\alpha} + F^{\mu\alpha}F^{\nu}{}_{\alpha,\nu} - \frac{1}{2}\eta^{\mu\nu}F^{\alpha\beta}F_{\alpha\beta,\nu}\right] \\ & + \frac{\hbar}{\ell_B^2}\left(\psi^{,\mu}{}_{,\nu}\psi^{,\nu} + \psi^{,\mu}\psi^{,\nu}{}_{,\nu} - \eta^{\mu\nu}\psi_{,\alpha,\nu}\psi^{,\alpha}\right). \end{aligned} \tag{19}$$

Putting the equations of motion (9) inside Eq. (19) and simplifying the result using the homogeneous Maxwell equation, we obtain the following expression

$$T_{\text{f}}^{\mu\nu}{}_{,\nu} = -e^{\psi}j^{\alpha}F^{\mu}{}_{\alpha} + \psi_{,\nu}\left(\eta^{\mu\nu}\frac{\partial\sigma}{\partial\psi} + T^{\mu\nu}_{\text{em}} - \frac{1}{16\pi}\eta^{\mu\nu}F_{\alpha\beta}F^{\alpha\beta}\right). \tag{20}$$

Let us add to both sides of the equation the divergence of the energy momentum tensor of matter $T_{\text{m}}^{\mu\nu}{}_{,\nu}$ in order to find the energy transfer (according to hypothesis 8 we assume that Einstein's equations hold unmodified for the gravitational field and hence the total energy momentum tensor is conserved) $T_{\text{f}}^{\mu\nu}{}_{,\nu} + T_{\text{m}}^{\mu\nu}{}_{,\nu} = 0$. So, this equation explicitly shows the energy transfer from the field ψ to matter

$$T_{\rm m}^{\mu\nu}{}_{,\nu} = e^{\psi} j^{\alpha} F^{\mu}{}_{\alpha} - \psi_{,\nu}\left(\eta^{\mu\nu}\frac{\partial\sigma}{\partial\psi} + T_{\rm em}^{\mu\nu} - \frac{1}{16\pi}\eta^{\mu\nu}F_{\alpha\beta}F^{\alpha\beta}\right) \tag{21}$$

which is the source of any observable effect. From

$$\psi_{,\nu} = \frac{\epsilon_{,\nu}}{\epsilon} = \frac{1}{2}\frac{\alpha_{,\nu}}{\alpha} \tag{22}$$

we find the "machian" contribution to energy transfer

$$T_{\rm m}^{\mu\nu}{}_{,\nu}{}^{(machian)} = \frac{1}{2}\frac{\alpha_{,\nu}}{\alpha}\left(\eta^{\mu\nu}\frac{\partial\sigma}{\partial\psi} + T_{\rm em}^{\mu\nu} - \frac{1}{16\pi}\eta^{\mu\nu}F_{\alpha\beta}F^{\alpha\beta}\right) \tag{23}$$

We use Bekenstein's notation, that is, the time-space components of $e^{\psi}F^{\mu\nu}$ are identified with **E** while space-space components are identified with **B**, and for us $\mathbf{S} = \frac{\mathbf{E}\times\mathbf{B}}{4\pi}$. Then, the component 0 of Eq. (21) reads

$$T_{\rm m}^{0\nu}{}_{,\nu} = \mathbf{j}.\mathbf{E} - e^{-2\psi}\frac{\mathbf{B}^2\dot{\psi}}{4\pi} - e^{-2\psi}\nabla\psi.\mathbf{S} + \dot{\psi}\frac{\partial\sigma}{\partial\psi} \tag{24}$$

Implicit in our previous analysis and algebra stands the generalized Poynting theorem. In its standard version it involves only electromagnetic terms, while in our case it will also involve the interaction between the electromagnetic and scalar fields.

$$T_{em}{}^{0\rho}{}_{,\rho} = \frac{\partial u_{em}}{\partial t} + \nabla.e^{-2\psi}(\frac{\mathbf{E}\times\mathbf{B}}{4\pi}) = -\mathbf{E}\cdot\mathbf{j} + \frac{e^{-2\psi}\mathbf{E}^2}{4\pi}\dot{\psi} + e^{-2\psi}\mathbf{S}.\nabla\psi \tag{25}$$

where $T_{em}{}^{00}{}_{,0} = (\partial u_{em})/\partial t$; the electromagnetic energy is $u_{em} = e^{-2\psi}(\mathbf{E}^2 + \mathbf{B}^2)/(8\pi)$ and $T_{em}{}^{0i}{}_{,i} = \nabla.e^{-2\psi}(\frac{\mathbf{E}\times\mathbf{B}}{4\pi}) = \nabla.e^{-2\psi}\mathbf{S}$; being **S** the Poynting vector. We note that this result is independent of the details of the gravitational and matter lagrangians, besides their interacting terms with the electromagnetic field. In particular it holds independently of the details of the interaction of matter with the scalar field. We recall that the usual interpretation of the first term in the right hand side of Eq. (25) is the work done by the electromagnetic field on matter. In the same fashion we may interpret the second and last term as the work done by the electromagnetic field on the scalar field. An analog phenomenon could be given by the work done by an increasing Newton constant G on a planet augmenting the pressure and thus compressing it [15].

Let us estimate the electrostatic contribution to the matter energy. In a non relativistic system such as a light atom or nuclei, the electromagnetic energy is given by the electrostatic field which satisfies the equation

$$\nabla\cdot\boldsymbol{E}e^{-2\psi} = 4\pi\rho^{0}_{\rm em} \tag{26}$$

where ρ^0_{em} is the reference charge density. In the limit when α varies only cosmologically the solution is

$$\boldsymbol{E} = e^{2\psi}\boldsymbol{E}_0 \tag{27}$$

where $\boldsymbol{E}_0$ is the electrostatic reference field defined for $e^{\psi} = 1$. The electromagnetic energy density results

$$u_{em} = e^{-2\psi}\frac{(\mathbf{B}^2 + \mathbf{E}^2)}{8\pi} = e^{2\psi}u^0_{em} \tag{28}$$

and the temporal variation

$$\dot{u}_{em} = 2\dot{\psi}u_{em} + e^{2\psi}\dot{u}^0_{em} = \frac{\dot{\alpha}}{\alpha}u_{em} + e^{2\psi}\dot{u}^0_{em}. \tag{29}$$

If there were no scalar injection of energy and $\dot{u}^0_{em} \approx 0$, the Poynting theorem Eq. (25) together with the expression for the energy variation Eq. (29) would lead to

$$\boldsymbol{j}\cdot\boldsymbol{E} = -\frac{\mathbf{B}^2}{4\pi}\dot{\psi}e^{-2\psi}. \tag{30}$$

As we will consider phenomena where the motion of matter is negligible, taking the first index as 0 is equivalent to project along the fluid four-velocity. Also the total time derivative $d/dt = \partial/\partial t + \mathbf{v}.\nabla$ will be equal to the partial time derivative $\partial/\partial t$. In the general case when there is viscosity and heat transfer, the right-hand side can be written, in the non relativistic limit, as

$$T^{0\nu}_{\mathrm{m}\ \ ,\nu} = \frac{\partial}{\partial t}(\frac{1}{2}\rho v^2 + u) + \nabla.[\rho\mathbf{v}(\frac{1}{2}v^2 + w) - \mathbf{v}.\sigma' + \mathbf{J}] \tag{31}$$

where w is the specific enthalpy, u is the internal energy density, $\mathbf{J}$ is the heat flux, which can generally be written as $-\kappa\nabla T$, being T the temperature and κ the thermal conductivity. Finally, $(\mathbf{v}.\sigma')_k$ stands for $v_i\sigma'_{ik}$, with σ' being the viscous stress tensor [18]. As we said above, we neglect the velocity of the fluid, so we obtain

$$T^{0\nu}_{\mathrm{m}\ \ ,\nu} = \frac{\partial u}{\partial t} + \nabla\mathbf{J} \tag{32}$$

A note of caution regarding the internal energy is in order. We understand, as usual, "internal energy" as the energy that can be exchanged by the system in the processes considered (heat exchange, radiative transfer, etc.), which will differ from what we understand by "rest mass", which is the "non convertible energy". If the scalar field can change the effective electric charge, then it can alter the electromagnetic contribution to the rest mass, and consequently, this contribution will be no longer "rest mass", but "internal energy".

The time variation of the internal energy u will have two contributions: one corresponding to the cooling process $\frac{\partial u}{\partial t}|_{cooling}$ and another one related to the interaction with the scalar field $\frac{\partial \sigma_\mu}{\partial t}$. This last term accounts for the dependence of the bulk of matter on the scalar field, which is mainly given by the electromagnetic contribution to the nuclear mass. Then equation (24) will finally read

$$\frac{\partial u}{\partial t}|_{cooling} + \frac{\partial \sigma_\mu}{\partial t} + \nabla \mathbf{J} = -\frac{\boldsymbol{B}^2}{4\pi}\dot{\psi}e^{-2\psi} - \frac{e^{-2\psi}\mathbf{B}^2\dot{\psi}}{4\pi} - e^{-2\psi}\nabla\psi.\mathbf{S} - \dot{\psi}\frac{\partial \sigma}{\partial \psi}. \quad (33)$$

Since the scalar field is space independent, and given that the electromagnetic energy of matter is mainly accounted by the nuclear content, we assume that the following condition $\frac{\partial \sigma}{\partial \psi} - \frac{\partial \sigma_\mu}{\partial \psi} \approx 0$ is fulfilled. Consequently, we obtain

$$\nabla \mathbf{J} = -\frac{e^{-2\psi}\mathbf{B}^2\dot{\psi}}{2\pi} - \frac{\partial u}{\partial t}|_{cooling}. \quad (34)$$

We define

$$\xi_a = 2\frac{e^{-2\psi}\mathbf{B}^2\dot{\psi}}{M_a 4\pi} \approx 2\frac{\dot{\alpha}}{\alpha}\frac{\mathbf{B}^2}{8\pi M_a} \quad (35)$$

as two times the energy production per mass unit of any material substance a (using the approximation, $e^{-2\psi} \to 1$ when $\psi << 1$).

Now follows our main physical assumption: *the cooling term is not modified by the scalar field*. The reasons for this assumption are two: 1) as we just showed, the electrostatic energy "injected" by the scalar field stays within the matter bulk (the cancellation of terms as seen in Eq. (34)) and 2) the thermal evolution should not change given the high thermal conductivity of the Earth and white dwarfs considered in this work. Thus we expect the magnetic energy excess to be radiated away, increasing the heat flux **J** as shown in Eq. (34).

4 The Electromagnetic Energy of Matter

As we have mentioned in the previous section, the only "input" we have is that which comes from the magnetic field. Stationary electric currents which are generated by charged particles and their static magnetic moments, and quantum fluctuations of the number density are the responsible of the generation of magnetic fields in quantum mechanics. Such contributions have been studied and calculated by [13, 28] from a minimal nuclear shell model using the following analysis (for more details see [16]).

The total magnetic energy of the nucleus can be written as,

$$E_m \simeq \frac{1}{2c^2}\sum_\alpha \int d\boldsymbol{x}d\boldsymbol{x}' \frac{\langle 0|\boldsymbol{j}(\boldsymbol{x}')\,|\alpha\rangle \cdot \langle \alpha|\boldsymbol{j}(\boldsymbol{x})\,|0\rangle}{|\boldsymbol{x}-\boldsymbol{x}'|}, \quad (36)$$

where α runs over a complete set of eigenstates of the nuclear hamiltonian H. We neglect the momentum dependence of the nuclear potential and assume a constant density within the nucleus. Making some calculations we finally obtain,

$$E_m = \int d^3x \frac{B^2}{8\pi} \simeq \frac{1}{2c^2} \int d^3x d^3x' \frac{\boldsymbol{j}(\boldsymbol{x}) \cdot \boldsymbol{j}(\boldsymbol{x}')}{\mid \boldsymbol{x} - \boldsymbol{x}' \mid} \simeq \frac{3}{20\pi} \frac{\bar{E}}{R(A)\hbar c} \int \sigma dE, \quad (37)$$

where $R(A)$ is the nuclear radius, A number of nucleons. These quantities have the following approximate representation

$$R(A) = 1.2A^{\frac{1}{3}} \text{ fm}, \qquad \int \sigma dE \simeq 1.6A \, \text{MeV fm}^2. \quad (38)$$

Then, the fractional contribution of the magnetic energy to rest mass energy is

$$\zeta(A) \simeq \frac{E_{m_A}}{m_A c^2} \approx 8.60 \times 10^{-6} A^{-1/3} \quad (39)$$

5 The Earth Heat Flux

The contribution of $\dot{\alpha}/\alpha$ to the heat flux can be calculated using the global heat balance for the Earth [17], assuming that the *machian* contribution H_C is the only extra energy production,

$$M_E C_p \frac{dT_m}{dt} = -Q_{tot} + H_C + H_G \quad (40)$$

where M_E is the Earth's mass; $C_p \approx 1200\,\text{J}/\text{Kg} - \text{K}$ is the average heat capacity of the planet and T_m is the mantle potential temperature. H_G represents the heat generated by radioactive isotopes. The total heat loss Q_{tot} can be written as the sum of two terms, one that comes from the loss of heat in the oceans Q_{oc}, and the other by continental heat loss Q_{cont}. Using the results obtained by Labrosse and Jaupart [17], we rewrite the total heat loss as $Q_{tot} \approx MC_p\lambda_G T_m$ where $\lambda_G \approx 0.1\,\text{Gyr}^{-1}$ is the timescale constant for the secular Earth's cooling. Assuming that the most abundant elements of the Earth are oxygen, silica and iron $\bar{\zeta} \approx 2.75 \times 10^{-6}$ and using $H_0 \approx 2.5 \times 10^{-18}\,\text{s}^{-1}$, the "extra" energy contribution can be written as,

$$H_C = \bar{\zeta} c^2 H_0 \frac{\dot{\alpha}}{\alpha H_0} \quad (41)$$

From (14), we can describe the extra contribution as a function of time, writing $\frac{a(t)}{a_0}$ as a power series [27],

$$\frac{a(t)}{a_0} \approx 1 + H_0 dt - \frac{q_0}{2}(H_0 dt)^2 + \frac{j_0}{6}(H_0 dt)^3 + \cdots \quad (42)$$

and then making a Taylor series expansion up to third order of H_C. Replacing this *machian* contribution in Eq. (40) and solving it, we find an expression for the cosmological perturbation of the mantle's temperature ΔT_m in terms of the time interval Δt and $\frac{\dot{\alpha}}{\alpha H_0}$.

$$\begin{aligned} \Delta T_m(t) =& 2.43 \times 10^5\,\mathrm{K/Gyr} \frac{\dot{\alpha}}{H_0 \alpha} (\Delta t)^3 - 3.78 \times 10^6\,\mathrm{K/Gyr} \frac{\dot{\alpha}}{H_0 \alpha} (\Delta t)^2 \\ &+ 3.05 \times 10^7\,\mathrm{K/Gyr} \frac{\dot{\alpha}}{H_0 \alpha} \Delta t \end{aligned} \quad (43)$$

According to [17], *the total amount of cooling experienced by the Earth after an initial magma ocean phase cannot exceed* 200 K. So, in the last 2.5 Gyr, $\Delta T_m <$ 200 K. With these restrictions we obtain a bound for the time variation of α,

$$\left| \frac{\dot{\alpha}}{H_0 \alpha} \right|_0 < 1.93 \times 10^{-6} \quad (44)$$

Using this result into Eq.(15) we find that,

$$\left(\frac{\ell_B}{\ell_P} \right)^2 < 0.15 \qquad \frac{\ell_B}{\ell_P} < 0.39 \quad (45)$$

A different bound can be obtained observing that the total radiated power of the Earth Q_{tot} can be explained by radioactive decay within twenty per cent [17]. The most recent data was estimated from an adjustment made with 38347 measurements. The methodology was to use a half-space cooling approximation for hydrothermal circulation in young oceanic crust; and for the rest of the Earth surface, the average heat flow of various geological domains was estimated as defined by global digital maps of geology, and then made a global estimate by multiplying the total global area of the geological domain [6].

The result shows that $Q_{tot} \approx 47$ TW (see [6] fore more details). Therefore,

$$|Q_{mach}| = |M_E C_P \lambda_G T_m(t)| < 0.2 Q_{tot} \quad (46)$$

Then, in an interval of 2.5 Gyr we find

$$\left| \frac{\dot{\alpha}}{H_0 \alpha} \right|_0 < 3.98 \times 10^{-6} \quad (47)$$

and

$$\left(\frac{\ell_B}{\ell_P} \right)^2 < 0.31 \qquad \frac{\ell_B}{\ell_P} < 0.55 \quad (48)$$

6 Conclusions

The energy exchange with ordinary matter in alternative theories with new fields such as Beckenstein's theory is a delicate subject. Using the field equations and general hypothesis of the theory we derived the energy transfer between matter and fields. Hypothesis 8 is key, as states that the matter energy momentum tensor is the quantity that has to be added to the field sector in order to make the total tensor divergence free. We also assumed that dark matter is electrically neutral, neglected the motion of matter in the bodies considered, and found that the dynamical feature of the electric charge makes the atomic electromagnetic energy part of the internal energy of the system. Eq. (34) shows that there is an extra contribution to the heat current besides the cooling of matter, which is given by the time variation of the scalar field and by the magnetic content of matter. We also justified our assumption that the matter cooling rate is not modified by the scalar field. Finally using a minimal nuclear shell model we estimated the magnetic energy content of matter, thus permitting us to quantify the anomalous heat flux in terms of the fundamental parameters of the theory and the chemical composition of the body.

Our best bound was obtained analyzing the geothermal aspects of the Earth, as those are naturally the best understood and measured of our solar system, and the surface heat flux is very low. Our bounds ($1.52 \times 10^{-16}\,\mathrm{yr}^{-1}$ and $3.14 \times 10^{-16}\,\mathrm{yr}^{-1}$) are comparable with that obtained in laboratory combining measurements of the frequencies of Sr [1], Hg+ [9], Yb+ [22] and H [8] relative to Caesium ($(3.3 \pm 3.0) \times 10^{-16}\,\mathrm{yr}^{-1}$) [20]; only one order of magnitude weaker than Oklo's ($(2.50 \pm 0.83) \times 10^{-17}\,\mathrm{yr}^{-1}$) (the theory independent most stringent bound on α time variation up to date [10]) and another found from measurements of the ratio of Al+ and Hg+ optical clock frequencies over a period of a year ($(5.3 \pm 7.9) \times 10^{-17}\,\mathrm{yr}^{-1}$) [20, 24]. The constraints we found depend on the cooling model of the Earth, but there is a general agreement on the mechanisms behind it [14]. The data set is redundant putting solid constraints on the theory. This analysis may be applied to other theories with extra fields that introduce extra "internal energies" to matter. We will report further work on future publications.

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