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Octupole Vibrations and Ground State Correlations

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Abstract. The relationship between ground-state correlations and collectiveness is investigated for the case of low-lying nonrotational states in the rare earth nuclei. Both octupole and quadrupole modes of excitation are studied and the quasiparticle virtual populations associated with each of them are discussed. The relative importance of particle-particle and particle-hole interaction matrix elements is also analyzed in connection with the shape of the correlation patterns. The fundamental role of the Nilsson + BCS scheme is emphasized and the consistency of the quasiparticle random-phase approximation is established.

1. Introduction

In recent years, the analysis of the ground state correlations (g.s.c.) associated with low-lying excited states has received a great deal of attention [1, 2]. These investigations have proved to be useful in that they have shed light on some properties of the random-phase approximation [1-3] (RPA), and in particular, on the validity of the quasiboson approach [4].

In this connection, the nonrotational collective states of the even-even rare earth nuclei offer one a suitable field of research. In these cases, however, one must work with a rather large 2 quasiparticle (q.p.) basis (see for example Ref. 5), a fact which would present one with considerable numerical problems, should one attempt a straightforward calculation of the correlation matrix [1] (see also Section 2). These difficulties can be surmounted by recourse to an elegant method developed by Rowe [2], which gives the ground state density matrix as a function of the backward-going amplitudes of the quasiparticle random-phase approximation (QRPA). In this way, the g.s.c. associated with gamma vibrations in rare earth nuclei have been examined in Ref. 6.

The purpose of this paper is to make a comparison between the g.s.c. associated with different excitation modes in deformed heavy nuclei. In this respect, quadrupole and octupole vibrations are discussed. The following tasks are undertaken, 1) to ascertain whether the consistency of the QRPA [2, 4], already verified for the case of gamma vibrations [6], still holds for other excitation modes, 2) to shed light on the effects of the nuclear shape upon the structure of the Fermi surface, 3) to determine to what an extent the single q.p. field predetermines the properties of the c.g.s., 4) to study the influence of the different interaction terms that appear in the QRPA matrices on the g.s.c.

The last point deserves special comment. Collective vibrational states in deformed heavy nuclei are usually

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described with the quasi-particle version of Brown and Bolsterli's schematic model [7]. Rather successful results have been obtained in this way (see for example Ref. 5 and works cited therein). The method entails the neglect of particle-particle and exchange-particle-hole interaction terms, which arise as a consequence of the q.p. transformation [8]. Only the direct particle-hole diagrams are retained.

At this point it should be stressed that the use of Brown and Bolsterli's model within the framework of the QRPA formalism cannot be regarded in the same light as its application to the particle-hole RPA for near closed-shell nuclei. In the latter case one would neglect only exchange particle-hole terms. In the former one, however, one would not take into account particle-particle terms either. As a matter of fact, of the three antisymmetrized diagrams which enter a QRPA calculation, the schematic model keeps only one [8].

Recently, some amount of work has been devoted to investigate the role of those terms which in the schematic treatment (s.t.) are dropped [9]. In this respect, so-called nonschematic (n.s.t.) calculations have been performed, for the case of gamma vibrations in the rare earth nuclei [10]. The most important conclusion to be drawn therefrom can be summarized by the assertion that the s.t. overestimates the coherence of $B(E2)$ transition rates [10].

These should in fact be regarded as generalized schematic calculations, in which advantage was taken of the fact that for certain residual interactions, the conventional P and Q matrices of the QRPA formalism can be rewritten as a sum of separable terms [9]. All the diagrams referred to above are then taken into account, but each is, of course, affected by different pairing factors [8].

In Section 2, a brief sketch of the formalism is given. The main results are presented and discussed in Sections 3 and 4. In Section 5 the beta vibrational case is considered and an overall summary is given in Section 6.

2. Formalism

We are interested in the microscopic description of collective states of parity π and angular momentum projection K on the nuclear symmetry axis. These states are generated by boson operators [2, 4] (as usual, a bar over a symbol denotes time-reversal).

$$\Omega^+_{nK^\pi} = \sum_{ij} \{X_{ij}[b_i^+ b_j^+]^{K^\pi} - Y_{ij}[b_i b_j]^{\bar{K}^\pi}\}. \tag{1}$$

In (1) we sum over all s.p. orbitals which can be coupled to K^π. The b's are q.p. operators originating from a Bogoliubov-Valatin transformation on the s.p. ones. The operators Ω^+ are to be applied to a correlated g.s. that within the QRPA formalism is expressed as [1, 2]

$$|\mathrm{QRPA}\rangle = \mathcal{N}_0 \exp\{-(1/2) \sum_{nK^\pi} S_{nK^\pi}\} |\mathrm{BCS}\rangle. \tag{2}$$

with

$$S_{nK^\pi} = \sum_{ijkl} C_{ijkl} [b_i^+ b_j^+]^{K^\pi} [b_k^+ b_l^+]^{\bar{K}^\pi}. \tag{3}$$

In (3) we sum over all pairs (ij), (kl) which span the coupled 2 q.p. basis. The C's are the so-called correlation coefficients. They relate the X and Y amplitudes of Eq. (1). In matrix notation we can write [1, 2]

$$Y = CX. \tag{4}$$

In order to obtain the amplitudes X and Y the approximation is usually made that the commutator $[[b_i^+ b_j^+]^{K^\pi}, [b_k b_l]^{K^\pi}]$, can be replaced by its expectation value with respect to the q.p. vacuum $|\mathrm{BCS}\rangle$ or $|\mathrm{HFB}\rangle$ [1, 2, 4]. This replacement is known as the quasi-boson approximation and constitutes an essential feature of the QRPA [4]. It can only be valid if the g.s. (2) does not appreciably differ from the BCS wave function. Thus the X and Y amplitudes can be computed after assuming the legitimacy of the quasi-boson approach. It is then possible afterwards to utilize (3) and (4) to build up explicitely the QRPA g.s. and verify whether the original hypothesis remains true. In this way, the consistency of the QRPA can be tested [1, 2].

The essential ingredients of such a probe are the q.p. densities defined as

$$\rho_{ji} = \langle \mathrm{QRPA} | b_i^+ b_j | \mathrm{QRPA} \rangle, \tag{5}$$

since it can be shown that first order corrections to the quasi-boson approximation are linear in the ρ_{ij} [6].

The q.p. densities are evaluated by the procedure described in Ref. 6, which is just the q.p. version of the method originally developed by Rowe [2]. The final expression is seen to be [6]

$$\rho_{ij} = \sum_{nK^\pi} \sum_l Y_{il}^{*nK^\pi} Y_{jl}^{nK^\pi}. \tag{6}$$

As useful figure is that provided by the diagonal terms of the density matrix (5), which corresponds to the q.p. occupation numbers in the correlated g.s. and measures the departure from the q.p. vacuum.

3. The SDI Description

One of the standard interactions usually adopted for the description of collective vibrations in deformed heavy nuclei is the well-known Surface Delta Interaction (SDI) [5, 8, 10]. All the calculations reported here have been performed with the methods described in Ref. 10. The details are similar to the ones mentioned therein.

It is worthwhile to point out that for an SDI the antisymmetrized particle-particle and the direct particle-hole term are of the same order of magnitude [8]. They appear in the QRPA equations, however, affected by different pairing factors. It is then natural to ask for the effect that neglect of the former diagram has upon the description of collective vibrations.

The overestimation of collective effects introduced by the s.t. can be easily appreciated from inspection of Table 1, in which the quantities

$$\eta(E\lambda)=\frac{B(E\lambda)(\text{s.t.})-B(E\lambda)(\text{n.s.t.})}{B(E\lambda)(\text{n.s.t.})}, \tag{7}$$

are listed. Here $B(E\lambda)$ is the reduced transition rate from the g.s. into either the gamma ($K^\pi=2^+$, $\lambda=2$) or the octupole ($K^\pi=0^-$, $\lambda=3$) band-heads. They have been evaluated within the framework of both the q.p. Tamm-Dancoff approximation (QTDA) and the QRPA. It is clearly seen that s.t. values are larger than n.s.t. ones. The result is thus obtained that, also for the case of octupole excitations, discarding particle-particle and exchange particle-hole diagrams originates an enhancement of the corresponding transition rates. The difference between s.t. and n.s.t. predictions is much larger, however, in the QRPA case than in the QTDA one.

For a proper appreciation of these facts, it should be borne in mind that the main physical distinction between the QTDA and the QRPA resides in the neglect of g.s.c. by the former. On the other hand, the n.s.t. and s.t. are just different ways of handling the residual interaction within the framework of either method. A glance at Table 1 should enable one to assert that there ought to exist a connection between the amount of collectiveness and the manner in which g.s.c. are generated by the residual interaction.

It is interesting to notice that $[\eta_{\text{QRPA}}(E\lambda)-\eta_{\text{QTDA}}(E\lambda)]$ is larger for $\lambda=2$ than for $\lambda=3$. Simultaneously, $[\eta_{\text{QRPA}}(E2)-\eta_{\text{QRPA}}(E3)]$ constitutes always an appreciable percentage of $\eta_{\text{QRPA}}(E2)$. In other words, comparison of results corresponding to diverse excitation modes allows one to conclude that, when the extent to which the transition rates are enhanced becomes greater, also the influence of the g.s.c. turns out to be more important. One reaches then a sort of self-consistency which confirms previous findings [6], in the sense that the responsability for the overestimation of collective effects within the s.t. lies squarely with the precise way of dealing with the g.s.c.

More details concerning the relationship between g.s.c. and collectivity can be appreciated by examining Figs. 1 and 2. Both display, for the case of the protons of the nucleus ^{154}Sm, as a typical example, the diagonal terms of the q.p. density operator ρ (i.e., the q.p. population) against the q.p. energy. These plots depict the departure of the correlated g.s. from the q.p. vacuum. Fig. 1 corresponds to correlations induced by gamma vibrations while in Fig. 2 the octupole case is considered.

In these pictures only the contribution of the collective bandhead has been taken into account, on the ground that higher bands lie at energies at which a 2 q.p. description cannot be expected to be an accurate one. In addition, it has been shown in Ref. 6 that, for the case of $K^\pi=2^+$ excitations, the pattern of the q.p. population is already given by the gamma band-head.

Let us now discuss Fig. 1. First of all one can notice that the s.t. distribution exhibits a kind of "Maxwellian tail" (i.e., the highest peaks lie near the Fermi level). On the other hand, the n.s.t. population is more

Table 1. The quantities $\eta(E\lambda)$ defined in Eq. (7) are shown for several rare-earth nuclei. The residual interaction is the SDI. The effective charge [5] has been chosen as 0.7. Both QRPA and QTDA results are displayed

Nucleus	$\eta(E2)$		$\eta(E3)$	
	QTDA	QRPA	QTDA	QRPA
^{152}Sm	0.16	1.16	0.26	0.86
^{154}Sm	0.14	1.36	0.29	0.72
^{154}Gd	0.12	1.40	0.31	0.74
^{156}Gd	0.11	1.42	0.34	0.74
^{158}Gd	0.12	1.51	0.34	0.67
^{160}Gd	0.09	1.43	0.46	0.66
^{162}Er	0.06	1.61	0.48	0.82
^{164}Er	0.05	1.61	0.53	0.73
^{166}Er	0.02	1.43	0.65	0.74
^{170}Er	0.11	1.47	0.26	0.47

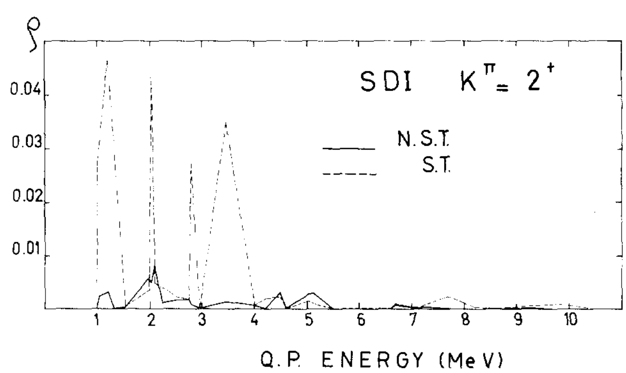

Fig. 1. The virtual proton quasiparticle population induced by the gamma band-head of ^{154}Sm into its ground state is plotted against the quasiparticle energy (in MeV). The residual interaction is the SDI

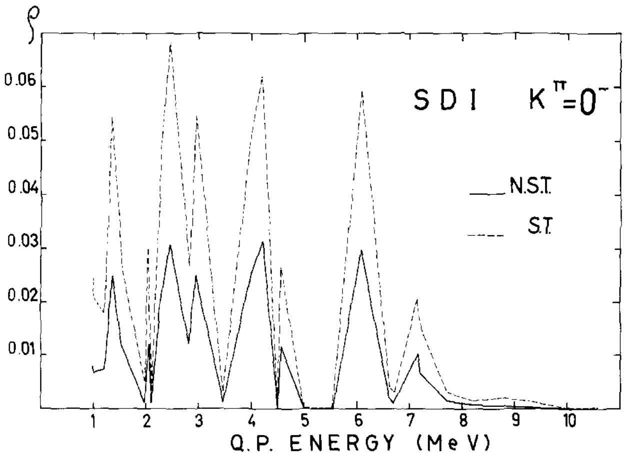

Fig. 2. The virtual proton quasiparticle population induced by the octupole ($K^\pi=0^-$) band-head of ^{154}Sm into its ground-state is plotted against the quasiparticle energy (in MeV). The residual interaction is the SDI

uniformely spread over the whole energy scale. The octupole pattern, however, looks different. The s.t. and n.s.t. distributions are rather similar (Fig. 2), and in both cases we find occupation peaks at high q.p. energies. The only common feature to be extracted from Figs. 1 and 2 turns out to be the larger amount of correlations induced by the s.t.

One can suggest then that there are two factors which contribute to the exaggeration of collective effects within the s.t. The first one is the number of excited q.p. within the correlated g.s. The second and more important one, is the distribution of the population peaks. The closer they lie to the Fermi level, the greater the corresponding degree of collectivity.

Finally, it should be noticed that q.p. numbers in the g.s. are smaller than 0.1. It can be then concluded that both gamma and octupole vibrations are consistently described by the QRPA, in the sense discussed in Section 2.

4. The SIF Description

The State Independent Force (SIF) has proved to be a valuable tool in gaining a first insight into the properties of low-lying collective excitations in deformed heavy nuclei [11, 12]. Since it acts with the same strength, independently of the s.p. orbitals involved, within the QRPA framework only the HFB field (replaced, as usual, by a Nilsson+BCS scheme) determines the features of the nuclear dynamics, as evidenced by the nonrotational modes of collective excitations [12].

In Table 2 we present, for the case of $E3$ transitions, the quantities η defined by Eq. (7), corresponding to both the QRPA and the QTDA formalisms. The calculations for the gamma vibrational case have been already reported in Ref. 12.

It is seen that for both modes of oscillation the s.t. values are much larger that the n.s.t. ones in the QRPA case. In the QTDA scheme, s.t. and n.s.t. predictions do not differ noticeably.

Thus, one finds that, as far as transition rates are concerned, quadrupole and octupole vibrations behave in the same way. This feature of the SIF description should be confronted with the one discussed above in connection with the SDI results. It is then reasonable to examine the corresponding q.p. population patterns. They are displayed in Figs. 3 and 4.

Table 2. The quantity $\eta(E3)$ defined in Eq. (7) is shown for several rare-earth nuclei. The residual interaction is the SIF. The effective charge is 0.4. Both QRPA and QTDA results are displayed

Nucleus	$\eta(E3)$	
	QTDA	QRPA
^{152}Sm	0.45	2.59
^{154}Sm	0.44	2.22
^{154}Gd	0.46	1.85
^{156}Gd	0.47	1.88
^{158}Gd	0.51	2.25
^{160}Gd	0.58	2.02
^{162}Er	0.49	2.80
^{164}Er	0.62	1.70
^{166}Er	0.77	1.77
^{170}Er	0.29	0.89

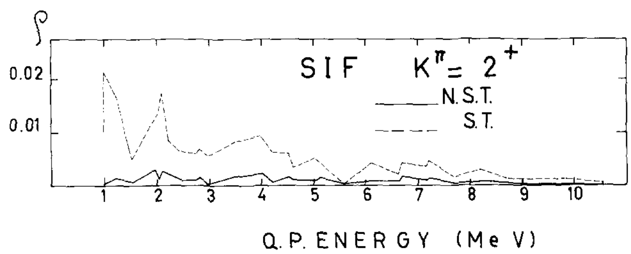

Fig. 3. The same as in Fig. 1, the residual interaction being the SIF

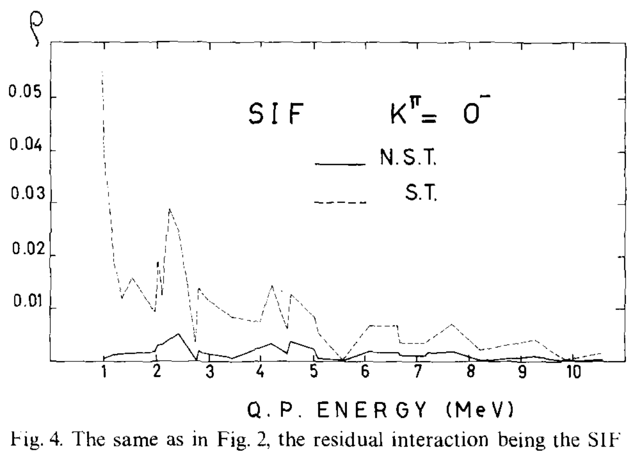

Fig. 4. The same as in Fig. 2, the residual interaction being the SIF

It is to be noticed that both plots exhibit the same behaviour, which appears then to be insensitive to the difference in multipolarity. Consequently, the structure of the correlated g.s. would be independent of the excitation mode that induces it. Schematic values are always larger than nonschematic ones, and, moreover, the former clearly display a Maxwellian tail, in contrast with the rather uniform n.s.t. pattern.

5. Beta Vibrations

For the sake of completeness one should also discuss the other kind of quadrupole vibrations found in deformed nuclei, namely the so-called beta vibrations, characterized by a positive parity and zero projection of the total angular momentum on the symmetry axis.

In doing so one finds, however, the difficulty of removing the spurious state associated with the nonconservation of the number of particles [15, 16]. A prescription for surmounting this trouble has been given in the literature [15, 16]. The corresponding treatment is associated with the schematic approach, so we have limited ourselves to the s.t. in dealing with beta vibrations.

We have studied the g.s.c. induced by $K=0^+$ excitations in four different cases, namely: 1) the SDI as the residual interaction and projecting out the spurious state; 2) likewise, but with the SIF; 3) with an SDI without projecting out the spurious state; and 4) likewise, but with the SIF.

The q.p. occupation patterns resulting from these calculations are quite similar to those obtained in the case of gamma vibrations. A more detailed picture can be found by glancing at Table 3. As in Figs. 1–4, the diagonal elements of the q.p. density matrix are listed for the case of the protons in ^{154}Sm, up to an energy of 2.92 MeV corresponding to q.p. orbitals.

It is seen that there is only one prominent peak, near the Fermi surface. The SIF results display a smoother behaviour than the SDI ones, just as it happens when gamma vibrations are concerned.

The removal of the spurious state leads to similar occupation numbers, and thus to a more consistent QRPA description. In the four cases, however, the corresponding figures are small enough as not to affect the consistency of the QRPA.

Table 3. The q.p. density matrix (beta vibrational case) for the protons of ^{154}Sm in the vecinity of the Fermi surface. The schematic approach has been employed

Q.P. energy (MeV)	Without projection of spurious state		With projection of spurious state	
	SDI	SIF	SDI	SIF
0.98	0.021	0.027	0.016	0.006
1.01	0.001	0.028	0.000	0.007
1.21	0.003	0.022	0.000	0.006
1.36	0.011	0.009	0.002	0.001
1.58	0.000	0.006	0.001	0.000
1.98	0.002	0.012	0.003	0.004
2.04	0.002	0.011	0.000	0.001
2.11	0.001	0.010	0.002	0.003
2.25	0.005	0.004	0.005	0.000
2.45	0.006	0.004	0.006	0.001
2.74	0.001	0.004	0.000	0.001
2.96	0.000	0.000	0.000	0.000

6. Conclusions

The results of the present work allow one to assert that to a good approximation, the QRPA can be regarded as consistent, even when several modes of excitation are jointly considered. There is a close connection between the differences in collectivity, as predicted by either the s.t. or the n.s.t., and the differences in the corresponding correlation patterns. As a general feature, the larger the relative amount of virtual q.p. population near the Fermi level, the higher the degree of collectiveness.

The bearing of the nuclear shape upon the structure of the Fermi surface can be appreciated in the fact that q.p. occupation patterns for octupole modes of oscillation are very distinct from the quadrupole ones (see Figs. 1 and 2). This characteristic does not appear when the interaction is state-independent. In this last case, the s.p. basis and the pairing factors pre-establish the general trend. In this respect, it is worth noting that the Nilsson basis is obtained from a Hamiltonian which explicitely includes a quadrupole deformation [13]. The multipolarity of the nuclear excitation, in a SIF description, cannot add any peculiarity of the nuclear shape not already contained in the chosen s.p. basis. In contrast, a state-dependent force gives information about the way in which the residual interaction between nucleons is influenced by the shape of the nucleus. This fact turns, of course, to be especially relevant for an SDI [14].

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