The pseudo SU(3) model is shown to be a powerful scheme for describing the excitation spectra as well as $B(E2)$ and $B(M1)$ transition strengths in heavy deformed nuclei. It is also useful for describing double beta decay amplitudes for transitions from the ground state of an even-even nucleus to the ground and excited states of the daughter nucleus, both for the two and zero neutrino emitting modes. The existence of selection rules which strongly restricts the decays is discussed. Anti-correlations between the quadrupole deformation and the Gamow-Teller (GT⁺) strength are found in an extension of the pseudo SU(3) model which explicitly includes pairing, which is also able to describe the fragmentation of the scissors mode. The projected shell model is introduced and proposed as an alternate means for studying single and double beta decay processes.

1 Introduction

In the last few years the combination of refined many-body techniques and powerful computers has allowed for impressive achievements in the microscopic description of heavy deformed nuclei. Here we will mention two of them: the pseudo SU(3) model [1, 2] and the projected shell model (PSM) [3]. Both use microscopic

hamiltonians, including single-particle energies as well as pairing and quadrupole-quadrupole interactions; and both work in a many-particle basis with good angular momentum.

The calculation of two neutrino double beta decay matrix elements has proven to be extremely sensitive to the details of the wave functions of the initial and final nuclei. For this reason our goal has been to describe both the even-even and odd-odd nuclei within the same formalism, asking it to reproduce the excitation spectra as well as $B(E2)$ and $B(M1)$ values of all the nuclei of interest, leaving a minimum number or no free parameters.

In this contribution we review previous results obtained with the simplest pseudo SU(3) scheme. In [4] we used the pseudo SU(3) shell model to evaluate two neutrino double beta half lives of eleven heavy deformed double beta emitters. We found good agreement with the available experimental information. We also calculated the zero neutrino matrix elements of six heavy deformed double beta emitters [5]. Using the upper limit for the neutrino mass we estimated their $\beta\beta_{2\nu}$ half-lives. In the case of $^{238}$U we found the zero neutrino half-life at least three orders of magnitude greater than the two neutrino one, giving strong support to the identification of the observed half-life as being two neutrino double beta decay [6]. The $\beta\beta_{2\nu}$ of $^{150}$Nd to the ground and excited states of $^{156}$Sm has also been studied [7].

The ability of the pseudo SU(3) model with pairing to describing the spectra of deformed and triaxial nuclei is also shown. The fragmentation of the scissors mode in the Gd and Dy isotopes is nicely reproduced by the model, which we expect will provide us reliable double beta decay matrix elements. Within this model an anti-correlation between $B(E2)$ values and Gamow-Teller (GT+) strengths is found.

The PSM is also presented. With its impressive description of the spectra of heavy deformed nuclei and a detailed understanding of the backbending phenomena, we expect similar good results in the description of single and double beta transition amplitudes.

2 The pseudo SU(3) formalism

In the pseudo SU(3) shell-model coupling scheme [1], normal parity orbitals $(\eta, l, j)$ are identified with orbitals of a harmonic oscillator of one quanta less $\tilde{\eta} = \eta - 1$. The set of orbitals with $\tilde{j} = j = \tilde{l} + \tilde{s}$, pseudo spin $\tilde{s} = 1/2$, and pseudo orbital angular momentum $\tilde{l}$, define the so-called pseudo space. The orbitals with $j = \tilde{l} \pm 1/2$ are nearly degenerate. For configurations of identical particles occupying a single $j$ orbital of abnormal parity, a convenient characterization of states is made by means of the seniority coupling scheme.

The many-particle states of $n_\alpha$ nucleons in a given shell $\eta_\alpha$, $\alpha = \nu$ or $\pi$, can be defined by the totally anti-symmetric irreducible representations $\{1^{n_\nu}\}$ and $\{1^{n_\pi}\}$ of unitary groups. The dimensions of the normal (N) parity space is $\Omega^N_\alpha = (\tilde{\eta}_\alpha + 1)$ ($\tilde{\eta}_\alpha + 2$) and that of the unique (A) space is $\Omega^A_\alpha = 2(\eta_\alpha + 1)$ with the constraint $n_\alpha = n_\nu^A + n_\pi^N$. Proton and neutron states are coupled to angular momentum $J^N_\alpha$ and $J^A_\alpha$ in both the normal and unique parity sectors, respectively. The wave function of
the many-particle state with angular momentum $J$ and projection $M$ is expressed as a direct product of the normal and unique parity ones, as:

$$|J M\rangle = \sum_{J_N J_A} ([|J_N\rangle \otimes |J_A\rangle])_M.$$  \hspace{1cm} (1)

Since we are interested in describing low-lying energy states, only pseudo spin zero configurations are taken into account in the normal parity space and only seniority zero configurations in the abnormal parity space. This simplification implies that $J_A^N = J_A^A = 0$. This is a strong assumption, but one that is physically motivated and very useful for simplifying the calculations.

Double beta decay, when described in the pseudo SU(3) scheme, is strongly dependent on the occupation numbers for protons and neutrons in the normal and abnormal parity states: $n_N^N, n_N^A, n_A^N, n_A^A$ [4]. These numbers are determined by filling the Nilsson levels from below, as discussed in [4].

In the first series of papers we selected the simplest version of the pseudo SU(3) hamiltonian [2]. It consisted of a spherical Nilsson hamiltonian which describes the single-particle motion of neutrons or protons, a quadrupole-quadrupole interaction, and a residual rotor term. The latter allows for fine tuning to low-lying spectral features like K-band splitting and the effective moments of inertia.

With the occupation numbers and the hamiltonian discussed above, the wave function of the deformed ground state of $^{150}$Nd can be written as: [4]

$$|^{150}Nd, 0^+\rangle = |(h_{11/2})_N^A, J_A^N = M_A^N = 0; (i_{13/2})_N^A, J_A^N = M_A^N = 0)\rangle$$

$$|\{16\}_N \{25\}_N \{12, 0\}_N; \{10\}_N \{23\}_N \{18, 0\}_N; 1(30, 0)K = 1, J = M = 0\rangle_N.$$  \hspace{1cm} (2)

3 Results

3.1 Forbidden decays

In all of the calculations only one active shell was allowed for protons, and likewise, only one for neutrons. This is a very strong truncation. For the $\beta\beta_{2\nu}$ decay this implies that only one uncorrelated Gamow-Teller transition is allowed: that which removes a neutron from a normal parity state with maximum angular momentum and creates a proton in the intruder shell ($h_{9/2}^N \rightarrow h_{11/2}^N$ in rare earth nuclei, $i_{11/2}^N \rightarrow i_{13/2}^N$ in actinides). This unique Gamow-Teller transition controls the $\beta\beta_{2\nu}$ decay. Under these assumptions, if the occupation of the Nilsson levels is such that the number of protons in the abnormal states does not change for the initial and final state configurations, the decay is forbidden.

It follows that the present version of theory predicts the complete suppression of the $\beta\beta_{2\nu}$ decay for the following five nuclei: $^{154}$Sm, $^{160}$Gd, $^{176}$Yb, $^{232}$Th and $^{244}$Pu [8]. In particular it predicts a null result for the present search in $^{244}$Pu [8], a potentially strong test of our model. Inclusion of pairing within the same truncation scheme will allow mixing of different pseudo SU(3) irreps. However if the main part
of the wave function is well represented by the pseudo SU(3) model those forbidden
decays will have, in the best case, matrix elements that will be no greater than 20% of
the allowed ones, resulting in at least one order of magnitude reduction in the
predicted half-life.

3.2 The g.s. → g.s. double beta decay

The results for six $\beta\beta$ emitters are given in Table 1. In the second and third
columns the theoretical [4] and experimental [9] $\beta\beta_{2\nu}$ half lives are given. The agree-
ment with the available data for $^{150}\text{Nd}$ and $^{238}\text{U}$ is good. In the last two columns the
theoretical predictions [5] and experimental lower limits [9] of the $\beta\beta_{0\nu}$ half lives are
given. In order to calculate the zero-neutrino half life we assumed $\langle m_\nu \rangle = 1$ eV. In

Table 1. The calculated (thy) and experimental (exp) double beta half-lives for the two-
neutrino and the zero-neutrino modes.

<table>
<thead>
<tr>
<th>Transition</th>
<th>$\tau_{2\nu}^{1/2}$ (yr)</th>
<th>$\tau_{0\nu}^{1/2}$ (yr)</th>
</tr>
</thead>
<tbody>
<tr>
<td>$^{146}\text{Nd} \rightarrow ^{146}\text{Sm}$</td>
<td>$2.1 \times 10^{21}$</td>
<td>$1.18 \times 10^{28}$</td>
</tr>
<tr>
<td>$^{148}\text{Nd} \rightarrow ^{148}\text{Sm}$</td>
<td>$6.0 \times 10^{20}$</td>
<td>$6.75 \times 10^{24}$</td>
</tr>
<tr>
<td>$^{150}\text{Nd} \rightarrow ^{150}\text{Sm}$</td>
<td>$6.0 \times 10^{18}$</td>
<td>$1.05 \times 10^{24}$</td>
</tr>
<tr>
<td>$^{186}\text{W} \rightarrow ^{186}\text{Os}$</td>
<td>$6.1 \times 10^{24}$</td>
<td>$5.13 \times 10^{25}$</td>
</tr>
<tr>
<td>$^{192}\text{Os} \rightarrow ^{192}\text{Pt}$</td>
<td>$9.0 \times 10^{25}$</td>
<td>$3.28 \times 10^{26}$</td>
</tr>
<tr>
<td>$^{238}\text{U} \rightarrow ^{238}\text{Pu}$</td>
<td>$1.4 \times 10^{21}$</td>
<td>$1.03 \times 10^{24}$</td>
</tr>
</tbody>
</table>

the case of $^{238}\text{U}$ the predicted $0\nu$ half life is three orders of magnitude greater than
the predicted $2\nu$ half life, which essentially agrees with the experimental one [6],
confirming that the observed $\beta\beta$ decay of $^{238}\text{U}$ has to be the two neutrino mode.
In the case of $^{150}\text{Nd}$, the pseudo SU(3) $0\nu$ matrix element reported here is about
a factor four smaller than the QRPA estimations [10, 11]. This is a very relevant
result. First, it exhibits the stability of the neutrinoless double beta decay matrix
elements evaluated in quite different nuclear models, in the case of deformed nuclei.
Second, this factor of four, which is small compared with the order of magnitude
variations in the $2\nu$ theoretical estimations, is still important in order to extract
the parameter $\langle m_\nu \rangle$.

As can be seen in the last two columns of Table 1, the $\tau_{0\nu}^{1/2}$ predicted for
$\langle m_\nu \rangle = 1$ eV are at least three order of magnitude greater than the experimental
limits. These results reflect the fact that, at the present stage of the experimental
$\beta\beta$ research, the limits $\langle m_\nu \rangle \leq 1.1$ eV obtained by the Heidelberg–Moscow collabora-
tion [12] using significative volumes of ultrapure $^{76}\text{Ge}$ are the most sensitive.
But, if the $\beta\beta_{0\nu}$ decay is observed in $^{76}\text{Ge}$, at least a second observation will be
essential, and $^{150}\text{Nd}$ is a likely candidate to do this job. In the next few years the
limit for $\langle m_\nu \rangle$ extracted from $\beta\beta_{0\nu}$ experiments is expected to be improved up to
0.1 eV and $^{150}\text{Nd}$ is one of the selected isotopes [9].
3.3 The double beta decay to excited states

We also studied the $\beta\beta_{2\nu}$ decay mode of $^{150}$Nd to the ground and excited states of $^{150}$Sm [7]. In Table 2 the matrix elements and predicted half-lives for the $\beta\beta_{2\nu}$ decay of $^{150}$Nd to the ground state, the first $2^+$ and the first and second excited $0^+$ states of $^{150}$Sm are presented. The matrix elements are given in units of $(m_e c^2)^{-1}$.

Table 2

<table>
<thead>
<tr>
<th>Transition</th>
<th>$M^{GT}<em>{2\nu}(J</em>\pm)$</th>
<th>$\tau^{1/2}<em>{2\nu}(0^+ \rightarrow J^+</em>\pm)$ (yr)</th>
</tr>
</thead>
<tbody>
<tr>
<td>$0^+ \rightarrow 0^+ (g.s.)$</td>
<td>0.0549</td>
<td>6.73 $\times 10^{18}$</td>
</tr>
<tr>
<td>$0^+ \rightarrow 0^+ (1)$</td>
<td>0</td>
<td>$\infty$</td>
</tr>
<tr>
<td>$0^+ \rightarrow 0^+ (2)$</td>
<td>0.00499</td>
<td>4.31 $\times 10^{22}$</td>
</tr>
<tr>
<td>$0^+ \rightarrow 2^+$</td>
<td>5.38 $\times 10^{-5}$</td>
<td>7.21 $\times 10^{24}$</td>
</tr>
</tbody>
</table>

The $\beta\beta_{2\nu}$ decay to the first excited $0^+$ state was found forbidden in the model and the decay to the second excited $0^+$ state has a half-life four orders of magnitude greater than that to the g.s. The decay to the $2^+$ state is strongly inhibited due to the energy dependence of its matrix elements.

The present results differ from those previously published [13] where it was speculated that the $\beta\beta_{2\nu}$ decay of $^{150}$Nd to the g.s. and the first excited $0^+$ state of $^{150}$Sm could have similar intensity. The reduction of the matrix element of the $\beta\beta_{2\nu}$ decay to the excited $0^+$ state as compared with the decay to the g.s. is not a general result of the pseudo SU(3) scheme. An analysis of the case of $^{150}$Mo [14] shows that both matrix elements are very similar and that they are in agreement with the experimental information. The appearance of selection rules which can produce the suppression of the matrix elements governing a $\beta\beta_{2\nu}$ transition is a consequence of the details of the irreps involved.

4 Pairing in the pseudo SU(3) model

The development of powerful computer codes for evaluating reduced matrix elements of generic two-body operators in the SU(3) basis [15] has opened up the possibility of working in the SU(3) (and pseudo SU(3)) basis with realistic hamiltonians that reach beyond those used in most algebraic analyses. In recent calculations the hamiltonian

$$H = \sum_\pi \epsilon_\pi a^\dagger_\pi a_\pi + \sum_\nu \epsilon_\nu a^\dagger_\nu a_\nu - \frac{1}{2} \chi : Q^a \cdot Q^a :$$

$$+ a K^2_\gamma + b \gamma^2 - G_\gamma H_{\gamma}^{\text{pair}} - G_H H_{\nu}^{\text{pair}}$$

has been used [16]. It contains proton and neutron single-particle energies, a quadrupole-quadrupole term, a rotor interaction that includes a $K^2_\gamma$ term for shifting the gamma band, and a monopole pairing interaction for protons and neutrons which strongly mixes the SU(3) irreps.
Using this hamiltonian it has been shown that ground state deformation induced by pairing are triaxial and soft, whereas the quadrupole-quadrupole interaction favors prolate or oblate shapes which are sharp [16]. The model has been used to reproduce spectra and $B(E2)$ values for heavy triaxial nuclei. The experimentally observed fragmentation of M1 scissors modes in Gd and Dy isotopes have also been shown to be described well within this framework [17]. This fragmentation reflects the subtle interplay of the quadrupole-quadrupole interaction and the SU(3) irreps mixing induced by the single-particle energies and the pairing interaction.

This hamiltonian gives a very good fit to the energies and $B(E2)$ strengths for $^{20}\text{Ne}$, comparable to results generated in traditional shell-model calculations [18]. Furthermore, in this case it was found that changes in the $\beta$ deformation track with $B(E2)$ values while the total Gamow-Teller ($\text{GT}^+$) transition strengths follows the $\gamma$ degree of freedom; the latter two quantities being anti-correlated with the former two. Given the close relation between the $\text{GT}^+$ strengths and the double beta decay amplitudes, these results suggest the presence of deformation should reduce double beta decay processes. This could be the reason for the smaller matrix elements obtained for $^{150}\text{Nd}$ in the pseudo SU(3) model compared with the QRPA calculation which assumes that $^{150}\text{Nd}$ is a spherical nuclei.

5 The Projected Shell Model

The Projected Shell Model (PSM) [3] gives a different extension of the Elliott SU(3) model for heavy nuclei. It starts from a deformed mean field, based on the Nilsson and BCS schemes, and takes strong pairing correlations into account explicitly, using numerical techniques to project the deformed, multi-quasiparticle basis onto states of good angular momenta. The deformed basis provides a very efficient way of truncating the shell model, and allows a simple physical interpretation of the results in terms of rotational bands and their mixing. The PSM is a microscopic theory that is able to explain quantitatively the finest features of the observed data, and provides a unified understanding of high-spin experimental information.

The subtle and different phenomena of back-bending and signature splitting observed in high-spin spectroscopy can be understood in terms of specific orbital motion of particles occupying the intruder subshell, which appears as a consequence of the very strong spin-orbit force. With the use of schematic interactions ($Q \cdot Q + \text{monopole pairing} + \text{quadrupole pairing}$), the spectroscopy of many heavy deformed, axially symmetric even-even, odd-odd and even-odd nuclei can be accurately reproduced. Also the $\Delta I = 4$ bifurcation at high spin is explained without invoking any $C_4$ type symmetry. Efforts have been made to include in this description single and double beta transitions, with the possible inclusion of a $\beta^+\beta^-$ residual interaction. The first results based on this scheme should be reported soon [19].
6 Conclusions

There are at least two refined theoretical models which can be used to describe quantitatively the large amount of experimental information available for heavy deformed nuclei. Both are many-particle formalisms with good angular momentum and explicitly take into account the presence of a strong spin-orbit coupling in the mean field.

The simplest version of the pseudo SU(3) model uses a quite restrictive Hilbert space. This model has been improved by incorporating mixing between different irreps via pairing. In some cases the selection rules that impose such strong restrictions on the $\beta\beta$ decays of some nuclei can be suspended. However, if the main part of the wave function is well represented by the pseudo SU(3) model those forbidden decays will have, in the best case, matrix elements that will be no greater than about 20% of the allowed ones, resulting in at least one order of magnitude suppression in half-lives. On the other hand, the anti-correlation between $B(E2)$ and $G^{+}$ strengths reinforce the idea that double beta decays are hindered by the presence of deformations.

The use of the pseudo SU(3) model with pairing and the Projected Shell Model to study single and double beta decay processes will shed light on an understanding of the nuclear structure of heavy deformed nuclei. The fact that models as different as the pseudo SU(3) and QRPA predict neutrinoless double beta decay matrix elements which are consistent within a factor of four for $^{150}$Nd exhibit, the consistency of the results, putting an upper limit to the theoretical uncertainty. On the other hand, it also points out the need of more detailed calculations, using the most refined techniques which are well-proven in this region, to provide nuclear matrix elements which are reliable enough to assist experimentalists in their work and to allow for the extraction of Majorana neutrino masses if a positive measurement of the neutrinoless double beta decay is obtained.

This work was supported in part by CONACyT (México), CONICET (Argentina), NSF and DOE (U.S.A.).

References