TEMPORAL EVOLUTION OF FLUCTUATIONS

N. Canosa*, A. Plastino* and R. Rossignoli**

Physics Department, National University of La Plata
(1900) La Plata, Argentina

INTRODUCTION

The Time Dependent Hartree-Fock (TDHF) approximation[1,2] constitutes the basic tool for dealing with the evolution of an uncorrelated many-body wave function. The approach has been extensively applied to a wide variety of many-fermion problems, yielding in general a reliable description of single particle (s.p.) expectation values. However, in situations where fluctuations become relevant, as for instance in the prediction of spreading widths of s.p. operators, TDHF fails to provide an accurate picture, due to the inherent neglect of correlations. The addition of involved collisional terms becomes thus necessary[3-5].

CORRECTED MEAN FIELD APPROACH (CMFA)

In ref. 6 a systematic and tractable procedure for improving TDHF predictions was developed, based on a suitable approximate closure of the semialgebra formed by the Hamiltonian $H = H_o + V$ with the observables of interest. Starting with a set of one-body observables $O_i^{(1)}$, the ensuing scheme can be cast as (we set $\hbar = 1$)

$$-i\,d\langle O_i^{(j)}\rangle/dt = \langle[H,O_i^{(j)}]\rangle, \quad j = 1,\dots,m-1 \tag{1a}$$

$$-i\,d\langle O_i^{(m)}\rangle/dt = \langle[H_o,O_i^{(m)}]\rangle + \langle[V,O_i^{(m)}]\rangle_{hf} \tag{1b}$$

*Supported by Consejo Nacional de Investigaciones Científicas y Técnicas (CONICET) of Argentina.
**Supported by Comisión de Investigaciones Científicas de la Provincia de Buenos Aires (CIC).

Condensed Matter Theories, Volume 5
Edited by V.C. Aguilera-Navarro
Plenum Press, New York, 1990

where $O_i^{(j)}$, $j\geq 2$, denote the j-body operators arising in the commutators with H in the j-1 step. The subindex hf indicates an uncorrelated evaluation (i.e., by means of Wick's theorem). H_0 denotes an unperturbed s.p. Hamiltonian (it can be for instance a static HF Hamiltonian) and V the corresponding residual interaction. Thus, the semialgebra with H is exactly closed up to m-1 body operators, while in the last step, it is closed just with H_0. System (1) is then complete if all one-body operators entering the uncorrelated evaluation are included in the original set.

For m=1, the scheme becomes equivalent to TDHF. For m>1, we attain thus an improved description, which yields exact (m-j-1)th order temporal derivatives at t=0 for operators $O_i^{(j)}$. In TDHF, only the initial first order time derivatives of the one-body observables $O_i^{(1)}$ are exactly evaluated, due to the violation of Ehrenfest theorem for higher order observables.

If the non-linear evaluation in the r.h.s. of (1b) is omitted, we attain a linear perturbative scheme [7], in which the mth power of V is discarded in the time evolution of $O_i^{(1)}$. In this case, it is necessary to go up to m+1 order in (1) to obtain the correct mth order time derivatives of observables $O_i^{(1)}$ at t=0.

THE EVOLUTION OF FLUCTUATIONS

In view of what has been said above, it becomes clear that the temporal evolution of the fluctuation of a one-body observable cannot be correctly described, even for short times, using TDHF. The exact equation of motion for the fluctuation F of an operator O_i, $F = \langle O_i^2\rangle - \langle O_i\rangle^2$, can be cast as

$$-i dF^{ex}/dt = \langle [H, O_i^2 - 2\langle O_i\rangle O_i]\rangle = 2C\{[H,O_i],O_i\} \qquad (2)$$

with $C\{O_i,O_j\} = \frac{1}{2}\langle O_iO_j + O_jO_i\rangle - \langle O_i\rangle\langle O_j\rangle$ (quantum covariance), whereas in TDHF,

$$-i dF^{hf}/dt = \langle [h, O_i^2 - 2\langle O_i\rangle O_i]\rangle_{hf} \qquad (3)$$

since $-i d\langle O_i^2\rangle_{hf}/dt = \langle [h,O_i^2]\rangle_{hf}$, where $h = \sum_i (\partial\langle H\rangle_{hf}/\partial\langle O_i^{(1)}\rangle)O_i^{(1)}$ is the s.p. mean field effective Hamiltonian (with the sum running over all one-body observables appearing in $\langle H\rangle_{hf}$). Thus, even at t=0 we attain a non-vanishing difference between both evolutions, given by

$$-i d(F^{ex} - F^{hf})/dt|_{t=0} = \langle [V_{res}, O_i^2]\rangle_{hf} \qquad (4)$$

where $V_{res} = H - h$ is the residual interaction. In fact, the l.h.s. of (4) is identical with the initial rate of increase of the correlation $\langle O_i^2\rangle_c = \langle O_i^2\rangle - \langle O_i^2\rangle_{hf}$.

On the other hand, the CMFA yields exact initial temporal derivatives of fluctuations already for m=2, providing at least the correct initial trend. Perturbative treatments require m=3. Higher quality predictions of s.p. fluctuations can be obtained in CMFA for m=3, in which case the second temporal derivative of F coincides with the exact value at t=0.

As a specific example, we have calculated the temporal evolution of the fluctuation of the operator J_x, under the action of the Hamiltonian $H = \varepsilon J_z - V(J_x^2 - J_y^2)$, within the framework of the Lipkin model[8], where

$$J_z = \tfrac{1}{2}\sum_{p,\nu}^{n} \nu c_{p\nu}^{+} c_{p\nu} \quad , \quad J_+ = \sum_{p}^{n} c_{p+}^{+} c_{p-} = J_-^{+} \quad , \tag{5}$$

with p = 1,...,N, $\nu=\pm 1$ (N is the number of particles). The s.p. density matrix $\langle c_{q\nu}^{+} c_{p\nu'}\rangle = \delta_{pq} x_{\nu} x_{\nu'}^{*}$, $\sum_{\nu}|x_{\nu}|^2 = 1$, provides the allowed initial conditions to solve the system (1).

It can be easily seen that in this case the initial difference (4) is given by

$$\langle [V_{res}, J_x^2]\rangle_{hf} = 4v\langle J_x\rangle\langle J_y\rangle\langle J_z\rangle/[N(N-1)] \tag{6}$$

(where v = V(N-1)), which is of the same order of magnitude of the fluctuation (O(N)), for fixed v (in (6) we have neglected terms of order 1). The evolution of the fluctuation of J_x in TDHF is given by

$$-i dF^{hf}/dt = -2\varepsilon\langle J_x\rangle\langle J_y\rangle/N + 4v\langle J_x\rangle\langle J_y\rangle\langle J_z\rangle/N^2, \tag{7}$$

so that a situation in which the sign of the exact initial derivative differs from that given by (7) may occur. A typical situation is illustrated in Fig. 1, for real initial values of x_{ν}. In this case, (6) and (7) vanish at t=0, but nevertheless TDHF fails to provide the correct initial trend. It also predicts a wrong amplitude, and the extrema are out of phase with the exact ones.

Results obtained with CMFA and the perturbative closure (for m = 2 and m = 3 respectively) are also shown, and are of similar quality, yielding both an acceptable agreement with exact results, at least for short times. Time is given in units of $\hbar/\varepsilon$, which for ε = 500 Kev yields 1.3×10^{-21}s, larger than the nucleon transversal time ($\cong 10^{-22}$s).

In conclusion, we have shown that TDHF predictions for fluctuations are in general inadequate, even for short times. However, a corrected mean field approach which does not violate Ehrenfest theorem for two-body operators, provides a reliable picture, being at the same time sufficiently tractable and simple. This fact allows possible applications to more complex and realistic systems.

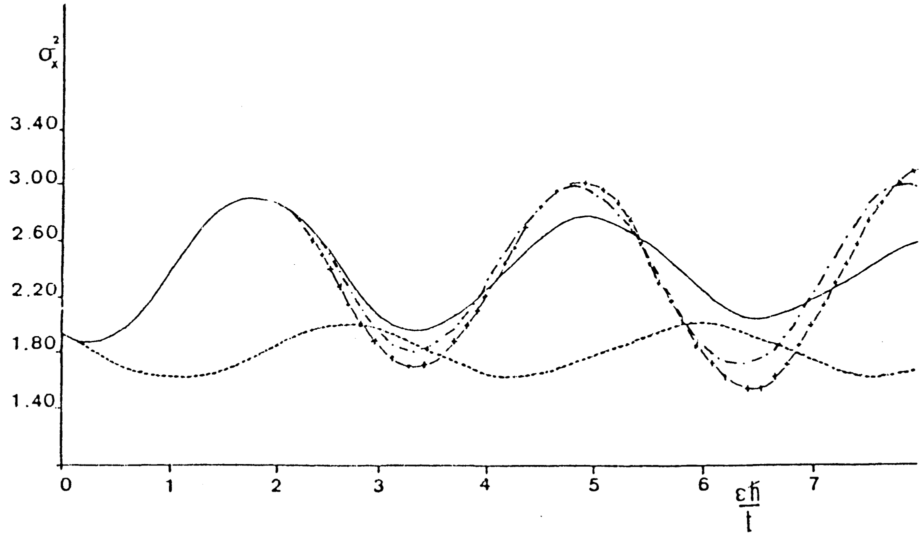

Fig. 1. Temporal evolution of the fluctuation of the operator J_x for N=8 and v/ε = -0.30. The initial conditions correspond to x_μ real, with x_-^2 = 0.034. Exact results,———; TDHF, ...; CMFA,._._._.; perturbative treatment,-+-+-+-.

References

1. P. Ring and P. Schuck, "The Nuclear Many-Body Problem", Springer Verlag, New York (1980).
2. J. Negele, Rev. Mod. Phys. 54:913 (1982).
3. H. Reinhardt, R. Balian, and Y. Alhassid, Nucl. Phys. A422:349 (1984); R. Balian, H. Reinhardt and Y. Alhassid, Phys. Rep. 131 1&2 (1986).
4. S. Ayik and M. Dworzecka, Phys. Rev. Lett. 54:534 (1985); Nucl. Phys. A440:424 (1985).
5. H. Reinhardt, P. Reinhard, and K. Goeke, Phys. Lett. 151B:177 (1985).
6. N. Canosa, A. Lõpez, A. Plastino, and R. Rossignoli, Phys. Rev. C37:320 (1988).
7. N. Canosa, A. Lõpez, D. Otero, A. Plastino, and A. Proto, Z. Phys. A326 :195 (1987).
8. H. Lipkin, N. Meshkov, and A. Glick, Nucl. Phys. 62:188 (1965).