On weighted clique graphs

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Abstract. Let $K(G)$ be the clique graph of a graph G. A *m-weighting of $K(G)$* consists on giving to each m-size subset of its vertices a weight equal to the size of the intersection of the m corresponding cliques of G. The 2-weighted clique graph was previously considered by McKee. In this work we obtain a characterization of weighted clique graphs similar to Roberts and Spencer's characterization for clique graphs.
Some graph classes can be naturally defined in terms of their weighted clique graphs, for example clique-Helly graphs and their generalizations, and diamond-free graphs. The main contribution of this work is to characterize several graph classes by means of their weighted clique graph: hereditary clique-Helly graphs, split graphs, chordal graphs, UV graphs, interval graphs, proper interval graphs, trees, and block graphs.

Keywords: weighted clique graphs, graph classes structural characterization.

1 Introduction

A *complete set* is a set of pairwise adjacent vertices. A *clique* is a complete set that is maximal under inclusion. We will denote by $M_1, \ldots, M_p$ the cliques of G, and by $\mathcal{C}_G(v)$ the set of cliques containing the vertex v in G.

Consider a finite family of non-empty sets. The *intersection graph* of this family is obtained by representing each set by a vertex, two vertices being connected by an edge if and only if the corresponding sets intersect.

The *clique graph* $K(G)$ of G is the intersection graph of the cliques of G.

Let $\mathcal{A}$ be a class of graphs. The notation $K(\mathcal{A})$ means the class of clique graphs of the graphs in $\mathcal{A}$, that is, $\mathcal{B} = K(\mathcal{A})$ if and only if for each G in $\mathcal{A}$, $K(G)$ belongs to $\mathcal{B}$ and for each H in $\mathcal{B}$, there exists G in $\mathcal{A}$ such that $K(G) = H$.

Given a graph G, the set of its cliques can be computed in $O(mnp)$ time [31], where n, m and p are the number of vertices, edges and cliques of G, respectively.

* Partially supported by ANPCyT PICT-2007-00518 and PICT-2007-00533, and UBACyT Grants X069 and 20020090300094 (Argentina), CAPES/SPU Project CAPG-BA 008/02 (Brazil-Argentina), and Math-AmSud Project 10MATH-04 (France-Argentina-Brazil).

So, the clique graph $K(G)$ can be computed in $O(mnp+np^2)$ time. Note that the number of cliques of a graph with n vertices can grow exponentially on n, so this time complexity is not necessarily polynomial in the size of G. In fact, deciding if the clique graph of a given graph G is a complete graph is a co-NP-complete problem [20].

The converse problem is also not easy to solve. Clique graphs have been characterized by Roberts and Spencer in [27], but the problem of deciding if a graph is a clique graph is NP-complete [1].

A family $\mathcal{F}$ of subsets of a set S is *separating* when for every pair of different elements x, y in S, there is a subset in $\mathcal{F}$ that contains x and does not contain y or, equivalently, when for each x in S, the intersection of all the subsets in $\mathcal{F}$ containing x is $\{x\}$.

A family of subsets of a set satisfies the *Helly property* when every subfamily of it consisting of pairwise intersecting subsets has a common element. A graph is *clique-Helly* when its cliques satisfy the Helly property.

Clique-Helly graphs are clique graphs [15]. In that case, given a graph H, the problem of building a graph G such that $K(G) = H$ can be solved with the same time complexity as building $K(H)$. Nevertheless, the problem of deciding if the clique graph of a given graph G is clique-Helly is NP-hard [6].

Given a graph H, a *weighting of* H *of size* m, or m*-weighting of* H, consists on giving a weight w to every complete set of H of size m. A *full weighting* of H consists on giving a weight w to every complete set of H.

A *weighting of* $K(G)$ *of size* m, or m*-weighting of* $K(G)$, consists on defining the weight w for a subset of its vertices $\{M_{i_1}, \ldots, M_{i_m}\}$ as $w(M_{i_1}, \ldots, M_{i_m}) = |M_{i_1} \cap \ldots \cap M_{i_m}|$. (In the right-hand side, we are considering $M_{i_1}, \ldots, M_{i_m}$ as cliques of G.) We will denote by $K^w_{m_1,\ldots,m_\ell}(G)$ the clique graph of G with weightings of sizes $m_1, \ldots, m_\ell$. Note that w should be non-decreasing with respect to inclusion relationship. Also by definition of $K(G)$, if 2 is one of the sizes considered, then $w(M_i, M_j) > 0$ for every edge M_iM_j of $K(G)$.

Weighted clique graphs with weightings restricted to size 2 were considered in [21, 22], and in [12–14, 23, 24], in the context of chordal graphs.

The organization of this paper is at follows. In Section 2, we introduce some definitions and results related to clique graphs. In Section 3, we give a characterization of weighted clique graphs similar to Roberts and Spencer's characterization for clique graphs. One of the contributions of this work is to characterize several classical and well known graph classes by means of their weighted clique graph, and is given in Section 4. We prove a characterization of hereditary clique-Helly graphs in terms of K^w_3 and show that $K^w_{1,2}$ is not sufficient to characterize neither hereditary clique-Helly graphs nor clique-Helly graphs. For chordal graphs and their subclass UV graphs, we obtain a characterization by means of $K^w_{2,3}$. We show furthermore that $K^w_{1,2}$ is not sufficient to characterize UV graphs. We describe also a characterization of interval graphs in terms of $K^w_{2,3}$ and of proper interval graphs in terms of $K^w_{1,2}$. Besides, we prove that $\{K^w_1, K^w_2\}$ is not sufficient to characterize proper interval graphs. For split graphs, we give a characterization by means of $K^w_{1,2}$, and prove that $\{K^w_1, K^w_2\}$ is not sufficient to characterize split graphs. Finally, we characterize trees in terms of K_1^w and block graphs in terms of K_2^w, and show that this last class cannot be characterized by means of their 1-weighted clique graph.

2 Preliminaries

We shall consider finite, simple, loopless, undirected graphs. Let G be a graph. Denote by $V(G)$ its vertex set and by $E(G)$ its edge set. Given a vertex v of G, denote by $N_G(v)$ the set of neighbors of v in G and by $N_G[v]$ the set $N_G(v) \cup \{v\}$. A vertex v of G is called *universal* if $N_G[v] = V(G)$. A *diamond* is the graph $K_4^w - \{e\}$, where e is an edge of the complete graph on four vertices K_4^w. A *claw* is the complete bipartite graph $K_{1,3}^w$. If H is a graph, a graph G is called *H-free* if G does not contain H as an induced subgraph.

A *stable set* in a graph is a set of pairwise non-adjacent vertices.

A graph is a *split graph* if its vertices can be partitioned into a clique and a stable set. A graph is a *star* if it has a universal vertex. In that case, the universal vertex is called the *center* of the star.

A graph G is an *interval graph* if G is the intersection graph of a finite family of intervals of the real line, and it is an *proper interval graph* if it is the intersection graph of a finite family of intervals of the real line, all of the same length. Proper interval graphs are exactly the claw-free interval graphs [28].

Theorem 1 (Fulkerson and Gross, 1965 [8]). *A graph G is an interval graph if and only if its cliques can be linearly ordered such that, for each vertex v_i of G, the cliques containing v_i are consecutive.*

Such an ordering is called a *canonical ordering* for the cliques.

Theorem 2 (Roberts, 1969 [28]). *A graph G is a proper interval graph if and only if its vertices can be linearly ordered such that, for each clique M_j of G, the vertices contained in M_j are consecutive.*

Such an ordering is called a *canonical ordering* for the vertices.

A graph G is a *tree* if it is connected and contains no cycle. A graph is *chordal* if it contains no chordless cycle of length at least 4. Equivalently, a graph is chordal if it is the intersection graph of subtrees of a tree [4, 9, 33]. A graph is a *UV graph* if it is the intersection graph of paths of a tree.

A graph is a *block graph* if each maximal 2-connected subgraph is a complete subgraph. Equivalently, a graph is a block graph if it is chordal and diamond-free.

A graph G is *domino* if all its vertices belong to at most two cliques. If, in addition, each of its edges belongs to at most one clique, then G is a *linear domino graph*. Linear domino graphs coincide with {claw,diamond}-free graphs [18].

A graph G is *dually chordal* if it admits a spanning tree T such that, for every edge vw of G, the vertices of the v—w path in T induce a complete subgraph in G [3, 30]. In that case, T is called a *canonical spanning tree* of G.

A graph is *hereditary clique-Helly* when H is clique-Helly for every induced subgraph H of G.

Class $\mathcal{A}$	$K(\mathcal{A})$	Reference
Block	Block	[16]
Clique-Helly	Clique-Helly	[7]
Chordal	Dually Chordal	[3, 11, 30]
Dually Chordal	Chordal $\cap$ Clique-Helly	[3, 11]
Hereditary clique-Helly	Hereditary clique-Helly	[26]
Interval	Proper interval	[17]
Proper interval	Proper interval	[17]
Diamond-free	Diamond-free	[5]
Split	Stars	
Trees	Block	[16]
Triangle-free	Linear domino	[25]
Linear domino	Triangle-free	[5]
UV	Dually Chordal	[30]

Table 1. Clique graphs of some graph classes

Clique graphs of many graph classes have been characterized. The known results involving the graph classes that will be considered in this paper are summarized in Table 1.

3 Characterization of weighted clique graphs

The characterization of clique graphs is as follows.

Theorem 3 (Roberts and Spencer, 1971 [27]). *A graph H is a clique graph if and only if there is a collection $\mathcal{F}$ of complete sets of H such that every edge of H is contained in some complete set of $\mathcal{F}$, and $\mathcal{F}$ satisfies the Helly property.*

A similar characterization for 2-weighted clique graphs was presented in [21, 27]. We can extend this characterization to weighted graphs.

Theorem 4. *Let H be a graph, provided with weightings w of sizes $m_1, \ldots, m_\ell$. Then there exists a graph G such that $H = K^w_{m_1,\ldots,m_\ell}(G)$ if and only if there is a collection $\mathcal{F}$ of complete sets of H, not necessarily pairwise distinct, such that:*
(a) every edge of H is contained in some complete set of $\mathcal{F}$,
(b) $\mathcal{F}$ satisfies the Helly property,
(c) $\mathcal{F}$ is separating,
(d) for every $1 \leq j \leq \ell$, each complete set $M_{i_1}, \ldots, M_{i_{m_j}}$ of H is contained in exactly $w(M_{i_1}, \ldots, M_{i_{m_j}})$ complete sets of $\mathcal{F}$.

It would be interesting to analyze the computational complexity of deciding if a weighted graph is a weighted clique graph. For 1-weightings, the result is negative.

Theorem 5. *The problem of deciding if a* 1*-weighted graph is a* 1*-weighted clique graph is NP-complete.*

It remains as an open question to analyze the problem for other weighting sizes.

4 Characterization of classical graph classes by means of the weighted clique operator

Some graph classes can be naturally defined in terms of their weighted clique graphs. This is the case of clique-Helly graphs and their generalizations. A family of subsets of a set satisfies the (p, q, r)*-Helly property* when every subfamily of it in which every collection of p members have q elements in common, has a total intersection of at least r elements. A graph is (p, q, r)*-clique-Helly* when its cliques satisfy the (p, q, r)-Helly property [6].

Proposition 1. *Let* G *be a graph. Then* G *is clique-Helly if and only if* $K^w_{3,\ldots,\omega(K(G))}(G)$ *satisfies* $w(M_{i_1}, \ldots, M_{i_\ell}) > 0$ *for every complete set* $M_{i_1}, \ldots, M_{i_\ell}$ *of* $K(G)$.

Proposition 2. *Let* G *be a graph. Then* G *is* (p, q, r)*-clique-Helly if and only if* $K^w_{3,\ldots,\omega(K(G))}(G)$ *satisfies that every complete set in which all its subsets of size* p *have weight at least* q*, has weight at least* r*.*

By the results in [7] shown in Table 1, we have the following corollary.

Corollary 1. *Let if* H *be graph and* w *a full weighting of* H *that is strictly positive over every complete set of* H*. If there is a graph* G *such that* $H = K^w_{3,\ldots,\omega(H)}(G)$*, then* H *is clique-Helly.*

Diamond-free graphs have also a natural characterization in terms of their weighted clique graph. It is proved in [5] that a graph is diamond-free if and only each edge belongs to exactly one clique. This property can be restated as follows.

Proposition 3. *Let* G *be a graph. Then* G *is diamond-free if and only if* $K^w_2(G)$ *satisfies* $w(M_i, M_j) = 1$ *for every edge* M_iM_j *of* $K(G)$.

In particular, by the results in [5] shown in Table 1, we have the following corollary, that was also pointed out in [21].

Corollary 2. *Let* H *be a graph and* w *a* 2*-weighting of* H*. If* $w(v_i, v_j) = 1$ *for every* v_iv_j *in* $E(H)$*, then there exists some graph* G *such that* $H = K^w_2(G)$ *if and only if* H *is diamond-free.*

Moreover, since diamond-free graphs are clique-Helly, we have that in a fully weighted clique graph of a diamond-free graph, the weight of each complete set of size at least two is exactly one. In [2], the authors establish when a 1-weighted graph H is $K^w_1(G)$ for some diamond-free graph G, thus completing the characterization of weighted clique graphs of diamond-free graphs.

Theorem 6 (Barrionuevo and Calvo, 2004 [2]). *Let H be a graph and w a 1-weighting of H. Then there exists some diamond-free graph G such that $H = K_1^w(G)$ if and only if H is diamond-free and $w(M) \geq \max\{2, |\mathcal{C}_H(M)|\}$ for each M in $V(H)$.*

The result above can be obtained also as a corollary of Theorem 4. Joining it with Proposition 3, we have the following Corollary.

Corollary 3. *Let H be a graph and w be weightings of H of sizes 1 and 2, such that $w(M_i, M_j) = 1$ for each edge M_iM_j of H. Then there exists a graph G such that $H = K_{1,2}^w(G)$ if and only if H is diamond-free and $w(M) \geq \max\{2, |\mathcal{C}_H(M)|\}$ for each M in $V(H)$.*

It is clear that diamond-free graphs cannot be characterized by their 1-weighted clique graph, since the diamond and two triangles sharing a vertex have the same 1-weighted clique graph.

A connected graph G with at least two vertices is triangle-free if and only if $w(M) = 2$ for each vertex M of $K_1^w(G)$. Indeed, the results in [25] showed in Table 1 imply the following proposition.

Proposition 4. *Let H be a graph and w a 1-weighting of H such that $w(M) = 2$ for each vertex M of H. Then there exists a graph G such that $H = K_1^w(G)$ if and only if H is linear domino.*

Also linear domino graphs can be naturally defined in terms of their weighted clique graph.

Proposition 5. *Let G be a graph. Then G is linear domino if and only if $K_2^w(G)$ is triangle-free and satisfies $w(M_i, M_j) = 1$ for every edge M_iM_j of $K(G)$.*

In the remaining of this section, we will show characterizations of some classical and widely studied graph classes in terms of their weighted clique graphs. Many of them are subclasses of chordal and/or clique-Helly graphs.

4.1 Hereditary clique-Helly graphs

A first characterization of hereditary clique-Helly graphs is the following.

Theorem 7. *Let G be a graph. Then G is hereditary clique-Helly if and only if $K_{2,3}^w(G)$ satisfies $w(M_i, M_j, M_k) = \min\{w(M_i, M_j), w(M_j, M_k), w(M_i, M_k)\}$, for every $1 \leq i < j < k \leq |K(G)|$.*

Moreover, this property holds also for m-weightings, with $m \geq 3$.

Theorem 8. *[26, 32] Let G be an hereditary clique-Helly graph, and let $m \geq 3$. Then $K_{2,m}^w(G)$ satisfies $w(M_{i_1}, \ldots, M_{i_m}) = \min\{w(M_i, M_j) : i, j \in \{i_1, \ldots, i_m\}, i < j\}$, for every $1 \leq i_1 < \ldots < i_m \leq |K(G)|$.*

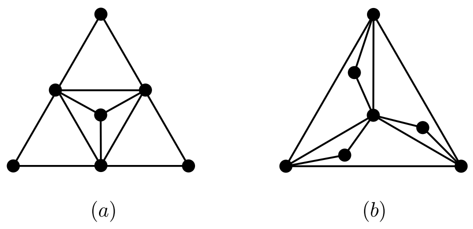

Fig. 1. Two graphs G, G' such that $K^w_{1,2}(G) = K^w_{1,2}(G')$. The rightmost one is hereditary clique-Helly, the leftmost one is not even clique-Helly. The leftmost one is UV, the rightmost is not.

The examples in Figure 1 show that $K^w_{1,2}$ is not sufficient to characterize neither hereditary clique-Helly graphs nor clique-Helly graphs. But we can obtain a characterization of hereditary clique-Helly graphs in terms of K^w_3.

Theorem 9. *Let G be a graph. Then G is hereditary clique-Helly if and only if $K^w_3(G)$ satisfies $w(M_i, M_j, M_k) \geq \min\{w(M_i, M_j, M_\ell), w(M_j, M_k, M_\ell), w(M_i, M_k, M_\ell)\}$, for every complete set M_i, M_j, M_k, M_ℓ of size four in $K(G)$.*

4.2 Trees and block graphs

The characterization of trees and block graphs are as follows.

Theorem 10. *Let G be a graph, $|V(G)| > 1$. Then G is a tree if and only if $K^w_1(G)$ is a connected block graph such that $w(M_i) = 2$, $1 \leq i \leq |K(G)|$.*

Theorem 11. *Let G be a connected graph. Then G is a block graph if and only if $K^w_2(G)$ is a connected block graph such that $w(M_i, M_j) = 1$, for every edge M_iM_j of $K(G)$.*

The same example used in the case of diamond-free graphs shows that block graphs cannot be characterized by their 1-weighted clique graph.

4.3 Split graphs

A characterization of split graphs in terms of $K^w_{1,2}$ is the following.

Theorem 12. *Let G be a graph. Then G is split and connected if and only if $K^w_{1,2}(G)$ is a star with center M_1 and $w(M_1, M_j) = w(M_j) - 1$, $2 \leq j \leq |K(G)|$.*

The examples in Figure 2 show that K^w_1 and K^w_2 are not sufficient to characterize split graphs.

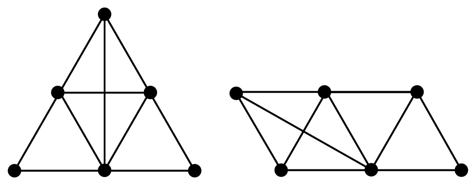

Fig. 2. Two graphs G, G' such that $K_1^w(G) = K_1^w(G')$ and $K_2^w(G) = K_2^w(G')$. The leftmost one is split, the rightmost one is not. The rightmost one is proper interval, the leftmost one is not.

4.4 Interval graphs

For interval and proper interval graphs, we have the following characterizations.

Theorem 13. *Let G be a graph. Then G is an interval graph if and only if $K_{2,3}^w(G)$ admits a linear ordering $M_1, \ldots, M_p$ of its vertices such that for every $1 \leq i < j < k \leq p$, $w(M_i, M_j, M_k) = w(M_i, M_k)$.*

Theorem 14. *Let G be a graph. Then G is a proper interval graph if and only if $K_{1,2}^w(G)$ admits a linear ordering $M_1, \ldots, M_p$ of its vertices such that for every triangle M_i, M_j, M_k, $1 \leq i < j < k \leq p$, $w(M_j) = w(M_i, M_j) + w(M_j, M_k) - w(M_i, M_k)$.*

The examples in Figure 2 show that K_1^w and K_2^w are not sufficient to characterize proper interval graphs.

4.5 Chordal and UV graphs

It is a known result that clique graphs of chordal graphs are dually chordal graphs. Moreover, it holds that, for a chordal graph G, there is some canonical tree T of $K(G)$ such that, for every vertex v of G, the subgraph of T induced by $\mathcal{C}_G(v)$ is a subtree. Such a tree is called a *clique tree* of G. McKee proved [24] that those trees are exactly the maximum weight spanning trees of $K_2^w(G)$. Also in the context of chordal graphs, 2-weighted clique graphs where considered in [10, 12–14, 19, 23, 29].

Theorem 15. *Let G be a connected graph. Then G is chordal if and only if $K_{2,3}^w(G)$ admits a spanning tree T such that for every three different vertices M_i, M_j, M_k of T, if M_j belongs to the path M_i—M_k in T, then $w(M_i, M_j, M_k) = w(M_i, M_k)$.*

Let G be a connected UV graph, and let $(T, \mathcal{F})$ be a representation of G as the intersection graph of a family of paths of a tree T, $\mathcal{F}$ being the family of paths. By taking a tree T that minimizes the number of vertices preserving the intersection relationship in the family of paths, we obtain that $V(T) = \mathcal{C}(G)$ and each path in $\mathcal{F}$ representing vertex v corresponds to $\mathcal{C}_G(v)$ [12]. That will be called a *clique tree* of the UV graph G.

Theorem 16. *Let G be a connected graph. Then G is UV if and only if $K^w_{2,3}(G)$ admits a spanning tree T such that for every three different vertices M_i, M_j, M_k of T, if M_j belongs to the path M_i—M_k in T, then $w(M_i, M_j, M_k) = w(M_i, M_k)$, and for every M in T and M_i, M_j, M_k in $N_T(M)$, it holds $w(M_i, M_j, M_k) = 0$.*

The examples in Figure 1 show that $K^w_{1,2}$ is not sufficient to characterize UV graphs.

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