

## E2-Transition Probabilities in the Davydov Model (\*).

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**Summary.** — Ratios of  $E2$ -transition probabilities for the de-excitation of levels belonging to the ground-state, beta and gamma rotational bands in the deformed region  $150 < A < 190$  are calculated with the Davydov model. A general good agreement with experiment is obtained.

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### 1. — Introduction.

Recent experimental information of high-spin levels in even-even deformed nuclei as well as of the corresponding electromagnetic transitions between levels of different rotational bands have increased the interest for making comparison with the theoretical prediction of current nuclear models. Since the microscopic models at least in their present stage, do not permit such a comparison, the use of the most refined phenomenological models is imposed <sup>(1)</sup>.

The different approaches of the Davydov-Chaban model <sup>(2)</sup> have allowed

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(1) C. A. HERAS, S. M. ABECASIS and H. E. BOSCH: *Phenomenological and microscopic analysis of properties of even-even nuclei*, Part A, Serie de Comunicaciones LR7 (1965); Part C, Serie de Comunicaciones LR9 (1966) (Dinfia, IIAE, Laboratorio de Radiaciones, Buenos Aires).

(2) A. S. DAVYDOV and A. A. CHABAN: *Nucl. Phys.*, **20**, 499 (1960).

a successful comparison of its theoretical predictions with experimental results. However, this model cannot be applied to some nuclides (cf. ref. (1)). Consequently, the assumption emerges naturally that the agreement between theory and experiment would be improved if the Davydov model (3)—which implies a refinement of the former—is used.

Up to now, the theoretical predictions of this model concerning the  $E2$ -transition probabilities have not been applied to fit any experimental data. The aim of this work is to compute the ratios of the  $E2$ -transition probabilities for the de-excitation of levels belonging to ground-state, beta, and gamma rotational bands in the deformed region  $150 < A < 190$  with the algorithm proposed by DAVYDOV and ROSTOVSKY (4).

## 2. — Results.

The formulas proposed by DAVYDOV and ROSTOVSKY (4) have been applied to particular cases of some rare-earth nuclides for which there is available a meaningful group of experimental results. The ratios of the second  $2^+$  level and the first  $0^+$  excited level with respect to the first  $2^+$  level are taken as the parameters of the present calculations. When the energies of the former were not experimentally known, their values were taken respectively, from the theoretical works of BES *et al.* (5,6).

In Table I the theoretical predictions of the reduced  $E2$ -transition probabilities from the first  $2^+$  level to ground-state are compared with the experimental data. Inspection of this Table shows that on the average the calculated values agree with the experimental data to within 20 %.

The results obtained for rotational and vibrational transition ratios are presented in Table II. From these data the conclusion can be drawn that this model gives a general good agreement with respect to the quoted ratios.

However, some points deserve special attention. Firstly, in some cases—labeled with a question mark—unphysical situations are reached. Secondly, for the ratio  $B(E2; 2_\gamma \rightarrow 4_g)/B(E2; 2_\gamma \rightarrow 2_g)$  the same calculated value is obtained for all the nucleides under consideration, which in turn is in complete disagreement with experiment. Finally, for high-spin levels the model seems to be inadequate for fitting the corresponding data.

(3) A. S. DAVYDOV: *Nucl. Phys.*, **24**, 682 (1961).

(4) A. S. DAVYDOV and V. S. ROSTOVSKY: *Nucl. Phys.*, **60**, 529 (1964).

(5) D. R. BES, P. FEDERMAN, E. MAQUEDA and A. ZUKER: *Nucl. Phys.*, **65**, 1 (1965).

(6) D. R. BES: *Nucl. Phys.*, **49**, 544 (1963).

TABLE I. - *E2 transitions from the first excited 2<sup>+</sup> level to the ground state compared with the predictions of Davydov model.*

Nuclide	<i>R</i> ( <sup>22</sup> )	<i>R</i> ( <sup>10</sup> )	<i>B</i> (E2; 20 → 0)/e <sup>2</sup> (10 <sup>-48</sup> cm <sup>4</sup> )	
			Experiment	Theory
<sup>152</sup> <sub>62</sub> Sm	8.93	5.57	3.51 ± 0.10	1.98
<sup>154</sup> Sm	17.6	13.4	4.53 ± 0.40	3.82
<sup>154</sup> <sub>64</sub> Gd	8.13	5.54	3.83 ± 0.10	2.04
<sup>156</sup> Gd	13.0	11.98	4.75 ± 0.13	3.74
<sup>158</sup> Gd	14.97	18.97 ( <sup>b</sup> )	5.10 ± 0.21	4.82
<sup>160</sup> Gd	13.6	20.3 ( <sup>b</sup> )	5.43 ± 0.40	5.16
<sup>158</sup> <sub>66</sub> Dy	9.57	10.0	4.65 ± 0.23	3.66
<sup>160</sup> Dy	11.14	17.83 ( <sup>b</sup> )	4.99 ± 0.22	3.90
<sup>162</sup> Dy	11.00	19.11 ( <sup>b</sup> )	4.68 ± 0.35	4.49
<sup>164</sup> Dy	10.38	20.36 ( <sup>b</sup> )	5.64 ± 0.25	4.95
<sup>164</sup> <sub>68</sub> Er	9.40	13.56	5.00 ± 0.18	4.23
<sup>166</sup> Er	9.72	18.02	5.79 ± 0.19	4.90
<sup>168</sup> Er	10.29	18.18 ( <sup>b</sup> )	5.60 ± 0.18	4.97
<sup>170</sup> Er	11.77	15.70	6.13 ± 0.45	4.70
<sup>168</sup> <sub>70</sub> Yb	11.22	13.15	5.43 ± 0.25	4.53
<sup>170</sup> Yb	14.63	15.96 ( <sup>b</sup> )	5.76 ± 0.30	4.80
<sup>172</sup> Yb	18.647	15.083 ( <sup>b</sup> )	6.66 ± 0.44	5.01
<sup>174</sup> Yb	15.00 ( <sup>a</sup> )	17.37	5.91 ± 0.30	5.17
<sup>176</sup> Yb	15.49	10.66	5.28 ± 0.40	4.44
<sup>178</sup> <sub>72</sub> Hf	15.86	12.84	4.51 ± 0.20	3.83
<sup>180</sup> <sub>74</sub> W	12.19	11.78	4.48 ± 0.29	3.39
<sup>182</sup> W	12.198	11.788	4.19 ± 0.21	3.25
<sup>184</sup> W	8.13	14.01 ( <sup>b</sup> )	4.18 ± 0.30	3.01
<sup>186</sup> W	5.959	12.441	3.56 ± 0.15	2.82
<sup>188</sup> <sub>76</sub> Os	5.61	11.23 ( <sup>b</sup> )	3.35 ± 0.34	2.59
<sup>188</sup> Os	4.08	11.39	2.66 ± 0.06	1.92
<sup>190</sup> Os	2.98	7.87 ( <sup>b</sup> )	2.50 ± 0.25	1.55

(a) Value of the <sup>22</sup><sup>+</sup> level taken from ref. (<sup>7</sup>).(b) Value of the <sup>10</sup><sup>+</sup> level taken from ref. (<sup>6</sup>).

TABLE II. - *Vibrational and rotational ratios of  $E2$* 

Ratios		$^{152}\text{Sm}$	$^{154}\text{Gd}$	$^{156}\text{Gd}$
$B(E2; 2_\beta \rightarrow 0) / B(E2; 2_g \rightarrow 0)$	exp theor	$0.013 \pm 0.003$ 0.14	$0.035 \pm 0.024$ 0.13	$0.015 \pm 0.007$ 0.047
$B(E2; 2_\beta \rightarrow 2_g) / B(E2; 2_\beta \rightarrow 0)$	exp theor	$2.36 \pm 0.70$ 1.43	$3.8 \pm 2.9$ 1.43	
$B(E2; 2_\beta \rightarrow 2_g) / B(E2; 2_g \rightarrow 0)$	exp theor	$0.024 \pm 0.007$ 0.20	$0.084 \pm 0.040$ 0.19	
$B(E2; 2_\beta \rightarrow 4_g) / B(E2; 2_\beta \rightarrow 2_g)$	exp theor	$3.79 \pm 0.67$ 1.00	$5.4 \pm 2.3$ 1.0	
$B(E2; 2_\beta \rightarrow 0) / B(E2; 2_\beta \rightarrow 4_g)$	exp theor	$0.048 \pm 0.015$ 0.39	$(4 \pm 3) \cdot 10^{-2}$ 0.39	
$B(E2; 4_\beta \rightarrow 2_g) / B(E2; 4_\beta \rightarrow 4_g)$	exp theor		$< 0.12$ 1.10	
$B(E2; 4_\beta \rightarrow 2_g) / B(E2; 4_\beta \rightarrow 6_g)$	exp theor		$(8 \pm 5) \cdot 10^{-3}$ 0.63	
$B(E2; 6_\beta \rightarrow 4_g) / B(E2; 6_\beta \rightarrow 8_g)$	exp theor		$\leq 0.054$ 0.73	
$B(E2; 2_\gamma \rightarrow 0) / B(E2; 2_\gamma \rightarrow 2_g)$	exp theor	$0.535 \pm 0.183$ 0.46	$0.53 \pm 0.14$ 0.44	$0.64 \pm 0.06$ 0.52
$B(E2; 2_\gamma \rightarrow 4_g) / B(E2; 2_\gamma \rightarrow 2_g)$	exp theor		$\leq 0.6$ 1.00	$0.13 \pm 0.04$ 1.00
$B(E2; 3_\gamma \rightarrow 2_g) / B(E2; 3_\gamma \rightarrow 4_g)$	exp theor	$0.90 \pm 0.40$ 1.02	$1.7 \pm 1.0$ 0.94	$1.61$ $1.32$
$B(E2; 3_\gamma \rightarrow 2_\gamma) / B(E2; 3_\gamma \rightarrow 2_g)$	exp theor			
$B(E2; 4_\gamma \rightarrow 2_g) / B(E2; 4_\gamma \rightarrow 4_g)$	exp theor	$0.25 \pm 0.15$ 0.035	$(8.2 \pm 2.5) \cdot 10^{-2}$ 0.01	

*transitions compared with the predictions of Davyдов model.*

$^{166}\text{Er}$	$^{182}\text{W}$	$^{168}\text{Er}$	$^{186}\text{Os}$	$^{188}\text{Os}$	$^{190}\text{Os}$
	$0.023 \pm 0.008$ 0.048				
	$1.03 \pm 0.08$ 1.43				
	$0.029 \pm 0.015$ 0.07				
	$0.45 \pm 0.02$ 0.39				
	$1.4 \pm 1.0$ 1.10				
0.55 0.47	$0.63 \pm 0.04$ 0.51	$0.563 \pm 0.020$ 0.48	0.38 0.35	0.33 0.27	0.23 0.19
0.087 1.00	$0.005 \pm 0.002$ 1.00	$0.08 \pm 0.03$ 1.00			
$1.31 \pm 0.10$ 1.09	$1.14 \pm 0.11$ 1.27	$1.56 \pm 0.04$ 1.14		0.50 0.36	$\sim 0.30$ 0.12
			22 5.01	8 3.93	7 3.88
$0.171 \pm 0.011$ 0.057	$0.14 \pm 0.04$ 0.11	$0.185 \pm 0.005$ 0.07	0.11 ?	0.07 ?	0.026 ?

TABLE II (*continued*)

Ratios		$^{152}\text{Sm}$	$^{154}\text{Gd}$	$^{156}\text{Gd}$
$B(E2; 4_\gamma \rightarrow 6_g)/B(E2; 4_\gamma \rightarrow 4_g)$	exp theor		$< 0.4$ 0.22	
$B(E2; 4_\gamma \rightarrow 2_\gamma)/B(E2; 4_\gamma \rightarrow 2_g)$	exp theor			
$B(E2; 4_\gamma \rightarrow 2_\gamma)/B(E2; 4_\gamma \rightarrow 4_g)$	exp theor			
$B(E2; 5_\gamma \rightarrow 3_\gamma)/B(E2; 5_\gamma \rightarrow 4_g)$	exp theor			
$B(E2; 5_\gamma \rightarrow 6_g)/B(E2; 5_\gamma \rightarrow 4_g)$	exp theor			
$B(E2; 6_\gamma \rightarrow 4_g)/B(E2; 6_\gamma \rightarrow 6_g)$	exp theor			
$B(E2; 6_\gamma \rightarrow 8_g)/B(E2; 6_\gamma \rightarrow 6_g)$	exp theor			
$B(E2; 6_\gamma \rightarrow 4_\gamma)/B(E2; 6_\gamma \rightarrow 4_g)$	exp theor			
$B(E2; 6_\gamma \rightarrow 4_\gamma)/B(E2; 6_\gamma \rightarrow 6_g)$	exp theor			
$B(E2; 7_\gamma \rightarrow 5_\gamma)/B(E2; 7_\gamma \rightarrow 6_g)$	exp theor			
$B(E2; 7_\gamma \rightarrow 8_g)/B(E2; 7_\gamma \rightarrow 6_g)$	exp theor			
$B(E2; 8_\gamma \rightarrow 6_g)/B(E2; 8_\gamma \rightarrow 8_g)$	exp theor			
$B(E2; 8_\gamma \rightarrow 6_\gamma)/B(E2; 8_\gamma \rightarrow 6_g)$	exp theor			

$^{166}\text{Er}$	$^{182}\text{W}$	$^{168}\text{Er}$	$^{186}\text{Os}$	$^{188}\text{Os}$	$^{190}\text{Os}$
0.20		0.08			
0.20		0.19			
			40 ?	44 ?	48 ?
11.9					
34					
34					
8.5					
$1.43 \pm 0.05$		1.0			
2.41		2.21			
$0.0748 \pm 0.0042$		0.096 ?			
		0.19 0.28			
			88 ?	?	?
19					
3.70					
59					
30					
$2.35 \pm 0.13$		1.6			
7.69		6.1			
$0.037 \pm 0.012$					
?					
$\sim 730$					
?					

Comparison of these results with those provided by the RV model (cf. ref. (7)) indicates an approximate equivalence of the two theories in explaining the transition ratios, although both of them exhibit discrepancies with respect to experiment in some cases. However, the RV model appears to be more suitable in those cases in which the Davydov model predicts unphysical situations.

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(7) A FAESSLER, W. GREINER and R. K. SHELINE: *Nucl. Phys.*, **70**, 33 (1965); S. M. ABECASIS and H. E. BOSCH: *Nuovo Cimento* (to be published).

### RIASSUNTO (\*)

Col modello di Davydov si calcolano i rapporti delle probabilità delle transizioni  $E2$  per la diseccitazione dei livelli appartenenti alle bande rotazionali beta, gamma e dello stato fondamentale nella regione deformata  $150 < A < 190$ . Si ottiene un buon accordo generale con i risultati sperimentali.

(\*) Traduzione a cura della Redazione.

### Вероятности $E2$ переходов в модели Давыдова.

**Резюме** (\*). — В модели Давыдова вычисляются отношения вероятностей  $E2$  переходов для девозбужденных уровней, принадлежащих основному состоянию,  $\beta$ - и  $\gamma$ -вращательным полосам в деформированной области  $150 < A < 190$ . Наблюдаются хорошее согласие с экспериментом.

(\*) Переведено редакцией.