THE INVERSE COMPTON EFFECT IN THE ELECTRONS OF THE VAN ALLEN RADIATION BELT*

JORGE RAUL ALBANO

Observatorio Astronómico, Universidad Nacional de La Plata, Argentina.

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Abstract. A first estimate of the energy that reaches the Earth's surface and is produced by the inverse Compton effect between the electrons in the Van Allen belt and the solar flux was made. Since in the belt there are electrons with energies between 0.5 and 7 MeV, it was possible to use the Klein-Nishina formula in an approximate form and estimate the energy that is Compton scattered by all the electrons in the Van Allen belt by using Vette, Lucero and Bright's model. The result was compared (a) with the measurements of the continuum in the regions of soft X-rays, and (b) with the energy that is produced by the trapped electrons through the synchrotron mechanism.

1. Introduction

Measurements of the energy spectrum of trapped electrons by the Earth magnetic field have shown that it extends into the regions of high energy from 0.5 up to 7 MeV and that the flux is extremely variable. These electrons are able to undergo Compton scattering with the solar photon flux of average photon energy 1.32 eV, and for electrons with energy larger than 0.5 MeV (relativistic velocity) we will have inverse Compton scattering.

By using different satellite measurements Vette *et al.* (1966) have constructed a model of the trapped radiation environment for the above mentioned energy range. Vesecky (1969) has utilized this model for the calculation of the radio-frequency synchrotron radiation from trapped electrons in the auroral zones.

In the present we plan to use Vette, Lucero and Wright's model for to estimate the intensity of the radiation produced by inverse Compton effect between the solar flux and the trapped high energy electrons.

This work refers only to the integrated intensity and gives no information about the spectral distribution of the radiation.

2. Generalities on the Compton Effect

Let be an electron of momentum $\mathbf{P}_1 = \gamma m \mathbf{v}_1$, in a system of coordinates fixed to the Earth or to some other astronomical reference systems, and be E_1 its energy. Let also be a photon of initial momentum $\mathbf{K}_1$ and energy ε_1 (Figure 1). During the process of scattering the electron and photon exchange energy and momentum, so that their final values are E_2 and ε_2, and $\mathbf{P}_2$ and $\mathbf{K}_2$, respectively.

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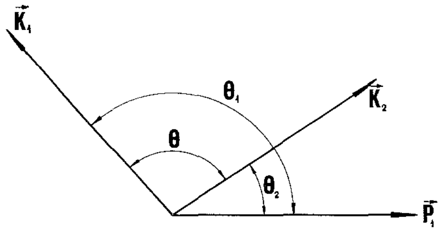

Fig. 1. Scattering of a photon (momentum $\mathbf{K}_1$, energy ε_1) by an electron (momentum $\mathbf{P}_1$, energy E_1); the scattered photon has momentum $\mathbf{K}_2$ energy ε_2. The vectors $\mathbf{K}_1$, $\mathbf{K}_2$, and $\mathbf{P}_1$ generally are not in the same plane.

The energy of the scattered photon because of the laws of conservation of energy and momentum is given by

$$\varepsilon_2 = \varepsilon_1 \frac{(1-\beta\cos\theta_1)}{1-\beta\cos\theta_2 + \frac{\varepsilon_1}{E_1}(1-\cos\theta)}, \tag{1}$$

where $\beta = v_1/c$ and the invariant cross section for the process by the well known Klein-Nishina formula (Ginzburg, 1967):

$$\sigma_c = 2\left(\frac{e^2}{mc^2}\right)^2 \frac{\varepsilon_2^2}{m^2c^4x_1^2}\left\{4\left(\frac{1}{x_1}+\frac{1}{x_2}\right)^2 - 4\left(\frac{1}{x_1}+\frac{1}{x_2}\right) - \left(\frac{x_1}{x_2}+\frac{x_2}{x_1}\right)\right\}, \tag{2}$$

where

$$\begin{aligned} x_1 &= -\frac{2}{m^2c^4}\,\varepsilon_1 E_1 (1-\beta\cos\theta_1), \\ x_2 &= \frac{2}{m^2c^4}\,\varepsilon_2 E_1 (1-\beta\cos\theta_2). \end{aligned} \tag{3}$$

Since the electron energy spectrum in the Van Allen radiation belt goes up to 7 MeV we will work within the energy interval that corresponds to relativistic velocities, namely $0.5 \leqslant E_1 \leqslant 7$ MeV. Moreover, we shall consider the solar flux characterized by a photon mean density $\varrho = 2.10^7$ photons/cm^3 and a photon mean energy $\varepsilon_1 = 1.35$ eV (Feemberg and Primakoff, 1948). This implies

$$\frac{\varepsilon_1}{E_1} \ll 1, \quad \frac{\varepsilon_1}{E_1} \ll 1-\beta. \tag{4}$$

Then we can approximate (1) and (2) as follows

$$\varepsilon_2 \approx \varepsilon_1 \frac{(1-\beta\cos\theta_1)}{(1-\beta\cos\theta_2)}, \tag{5}$$

$$\sigma_c \approx \frac{1}{\gamma^2}\left(\frac{e^2}{mc^2}\right)^2 \frac{1}{(1-\beta\cos\theta_2)^2}, \tag{6}$$

we shall use expressions (5) and (6) throughout the present paper.

By definition of effective cross section, the number of scattered photons per unit time into the solid angle $\mathrm{d}\Omega_2$ will be

$$\mathrm{d}N = \sigma_c \, \mathrm{d}\Omega_2 I \,, \tag{7}$$

where $I = \varrho c(1 - \beta \cos\theta_1)$ is the flux intensity of the photons scattered by the electrons (ϱ is the photons density, $\mathbf{v} \equiv \mathbf{v}_1$ is the scattering electrons velocity, θ_1 is the angle between $\mathbf{K}_1$ and $\mathbf{v}_1$). The cross section $\sigma_c \mathrm{d}\Omega_2$ is a relativistic invariant.

In (6) we see that the energy scattered is symmetrical with respect to the direction of the electron velocity. A good approach is to consider that the electrons scatter all the energy inside a cone with its axis in the direction of the electron velocity and with the half angle α defined by

$$\frac{E_{\Omega_\alpha}}{E_{4\pi}} = a \,, \tag{8}$$

where E_{Ω_α} is the energy scattered inside the cone of half angle α and $E_{4\pi}$ the energy scattered in all directions. In our work we have assumed $a = 0.99$

$$E_{\Omega_\alpha} = \int_0^{\Omega_\alpha} \sigma_c I \varepsilon_2 \, \mathrm{d}\Omega_2 = \int_0^{\Omega_\alpha} \frac{\sigma_0}{\gamma^2} I \varepsilon_1 \frac{(1 - \beta \cos\theta_1)}{(1 - \beta\cos\theta_2)^3} \, \mathrm{d}\Omega_2 =$$

$$\sigma_0 = \left(\frac{e^2}{mc^2}\right)^2$$

$$= \frac{\sigma_0}{\gamma^2} I\varepsilon_1 (1 - \beta\cos\theta_1) \frac{\pi}{\beta} \left[\frac{1}{(1-\beta)^2} - \frac{1}{(1 - \beta\cos\alpha)^2}\right], \tag{9}$$

$$E_{4\pi} = \int_0^{4\pi} \sigma_c I \varepsilon_2 \, \mathrm{d}\Omega_2 \,,$$

and with this equation we arrive at the following expression for $\cos\alpha$:

$$\cos\alpha = \frac{1}{\beta} - \frac{(1-\beta)}{\beta\left(1 - \dfrac{4a\beta}{(1+\beta)^2}\right)^{1/2}} . \tag{10}$$

3. Determination of Compton Scattering in the Van Allen Belt

The problem is sketched in Figure 2 where T is the magnetic equator; N the magnetic pole; e an electron in the Van Allen radiation belt; for simplicity the direction of zenith is taken at 90° with the direction of the solar flux.

As we want to find out only the integrated intensity and not the monochromatic intensity, we can assume that the scattered photons are isotropically distributed inside the cone and that all of them have the same energy

$$\bar{\varepsilon}_2 = \frac{\int_0^{\Omega_\alpha} \varepsilon_2 \sigma_c \, \mathrm{d}\Omega_2}{\int_0^{\Omega_\alpha} \sigma_c \, \mathrm{d}\Omega_2} = \frac{\varepsilon_1}{2} (1 - \beta \cos \theta_1) \frac{(2 - \beta(\cos\alpha + 1))}{(1-\beta)(1 - \beta\cos\alpha)}. \tag{11}$$

Therefore, the energy scattered by unit of solid angle is given by

$$j_{\Omega_\alpha} = \frac{\varepsilon_1}{2} \varrho c \frac{\sigma_0}{\gamma^2} \frac{(1 - \beta\cos\theta_1)^2 (2 - \beta(\cos\alpha + 1))}{(1-\beta)^2 (1 - \beta\cos\alpha)^2}. \tag{12}$$

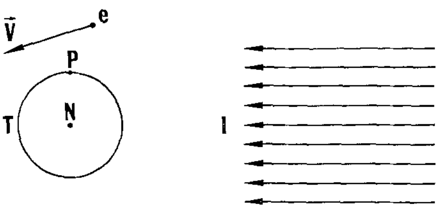

Fig. 2. Schematic illustration of Compton scattering process in the Van Allen belt.

For this problem in particular we can consider that the electron velocities are uncorrelated, then we can add together the energy scattered by the different electrons.

Let us consider that the electron flux is isotropic inside the cone. In (10) we see that for a fixed a, $\cos\alpha$ depends only on the electrons energy then, only the electrons with velocities inside the cone of half-angle α can scatter photons capable to arrive at P (Figure 3).

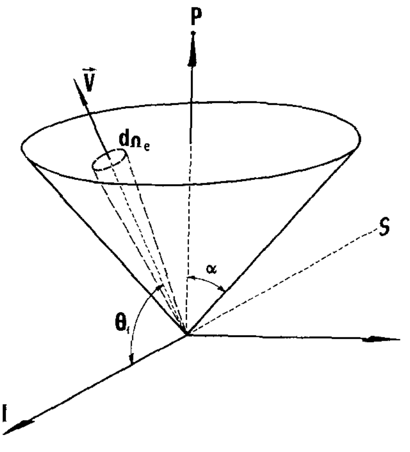

Fig. 3. Scattering of photon by electrons with velocities inside the cone of half angle α. Only the photons scattered by these electrons can arrive at P.

If we consider the electrons that occupy a volumen dv located at O, with velocities inside the solid angle $d\Omega_e$ and his energy between E_1 and $E_1 + dE_1$, then the energy scattered by unit of solid angle will be

$$dj^* = j_{\Omega_\alpha}\left[\frac{N}{4\pi}\,d\Omega_e\right] dv\,dE_1\,, \tag{13}$$

where $N(\mathbf{r}, E)$ is the electron density in $\mathbf{r}$ in an interval E_1, $E_1 + dE_1$.

Now, we can compute the intensity of the radiation that will arrive to a detector placed at P (since we do not consider atmospheric absorption this point P is at the top of Earth atmosphere). This intensity can be derived from the following expression.

$$I = \int_{E_1}^{E_2}\int_{R_1}^{R_2}\int_{0}^{\Omega_\alpha} j_{\Omega_\alpha}\frac{N}{4\pi}\,d\Omega_e\,dr\,dE_1$$

$$I = \int_{E_1}^{E_2}\int_{R_1}^{R_2} \frac{\varepsilon_1}{2}\frac{\varrho c}{\gamma^2}\frac{\sigma_0}{4\pi}\frac{(2-\beta(\cos\alpha+1))}{(1-\beta)^2(1-\beta\cos\alpha)^2} \tag{14}$$

$$\times\left(2\pi(1-\cos\alpha)+\beta^2\pi\left(\frac{\cos^3\alpha}{3}-\cos\alpha+\tfrac{2}{3}\right)\right)N(rE)\,dr\,dE\,.$$

This double integration had been computed numerically by using the values of $N(\mathbf{r}, E)$ that correspond to Vette, Lucero and Wright model. The integration limits have been $E_1 = 0.5$ and $E_2 = 7$ MeV; $R_1 = 1.2\ R_T$ and $R_2 = 5.8\ R_T$.

The value obtained for I is

$$I = 2.8\ \text{eV/cm}^2\ \text{sec ster}\,.$$

Most of this energy is placed in the spectral range limited by $\varepsilon_2 = 0.01$ keV and $\varepsilon_2 = 0.5$ keV.

4. Concluding Remarks

It is interesting to compare this last value with the intensity of the galactic X-radiation and with the synchrotron radiation emitted by the same trapped electrons, in spite of the fact that they lie in different spectral regions.

The galactic background X-radiation in the interval $\varepsilon_2 = 0.3$ keV and 0.5 keV (Baxter *et al.*, 1969) has an intensity $I \approx 46$ keV/cm^2 ster, a value which is 10000 times the calculated intensity for trapped electrons. The radiations that is produced by the Compton scattering will be undistinguishable from the galactic background X-radiation. The only possibility to measure the radiation we have computed, irrespective of the availability of a detetcor for such a weak radiation, will be with the use of a detector placed beyond the Van Allen belt and oriented towards the Earth.

In the region of 1 MeV and for spectral interval $\Delta\varepsilon_2 = 0.5$ keV the intensity of galactic background X-radiation is still of the order of a keV (Felten and Morrison, 1966), that is to say, stronger than the radiation we have calculated.

The trapped electrons radiate also through the mechanism of synchrotron radiation: Vesecky (1969) has calculated this radiation intensity by also using Vette, Lucero and Wright trapped electrons model. For the integrated intensity in the interval 5–20 MHz Vesecky obtains $I \approx 0.4$ eV/cm^2 sec ster which is of the order of 10% of the intensity produced by Compton effect.

With this intensity estimate we see that the energy produced by Compton scattering is very weak relative to the galactic background X-radiation but of the same order as the synchrotron radiation energy produced by the same electrons. Therefore we must take into account Compton scattering only when we make energy balance inside the magnetosphere and we consider synchrotron electron loss.

Our aim has been to estimate the integrated intensity produced by inverse Compton scattering in order to know how important the process is. Next we will try to calculate the spectral intensity distribution by using the most recent pitch angle distribution measurements.

If we consider one star in place of the Earth we would have a possible X-ray source model. We also plan to compute such a X-ray source model in the case of different magnetic field values and differents trapped electrons densities, and to compare the spectral distribution of the compute radiation with the different observational values of X-ray sources.

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