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Remarks Concerning the Renormalized Coupling Constants of the Gauge Theories.

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Summary. — Some basic properties of renormalized coupling constants are discussed in the framework of spontaneously broken gauge symmetries. It is pointed out that conventional definitions lead to some difficulties connected with infra-red divergences and two possible solutions are suggested. The ultraviolet divergent contributions to the lowest-order corrections are then studied and it is shown explicitly, in a number of particular examples, how these divergent terms cancel in the interrelations between renormalized coupling constants and masses. The specific calculations are carried out in the unitary gauge of the $SU_2 \times U_1$ gauge model, using the ν-dimensional regularization method.

1. – Introduction.

A very interesting recent development in particle physics has been the study of unified models of weak and electromagnetic interactions constructed on the basis of spontaneously broken gauge symmetries (1). In fact, recent

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(1) S. WEINBERG: *Phys. Rev. Lett.*, **19**, 1264 (1967); **27**, 1688 (1971); **29**, 388 (1972); A. SALAM: in *Elementary Particle Physics*, edited by N. SVARTHOLM (Stockholm, 1968); G. 'T HOOFT: *Nucl. Phys.*, **35** B, 167 (1971); B. W. LEE: *Phys. Rev. D*, **5**, 823 (1972).

investigations give strong support to the idea that such models are renormalizable, provided that Adler anomalies are cancelled internally [2].

In this paper we study some problems concerning the definition and interrelations of the renormalized constants of the theory. This subject is of course of general theoretical interest as it is connected with the renormalization program for the theories under consideration. It is also a matter of practical interest as can be readily ascertained by examining some recent papers and results concerning the higher-order weak and electromagnetic corrections to weak processes [3-6].

Our specific considerations are carried out in the framework of the $SU_2 \times U_1$ gauge model [1]. It shoulder be clear, however, that similar studies can be extended, in principle, to other spontaneously broken gauge symmetries.

The plan of the paper is as follows. In Sect. **2** we discuss a rather conventional definition of the renormalized coupling constants of the theory. We point out that this leads to difficulties connected with infra-red divergences, and offers two possible solutions to this problem. In Sect. **3** we study to lowest nontrivial order the ultraviolet divergent contributions to some relevant two- and three-point functions. We then show how these divergent contributions cancel out in the relations between renormalized coupling constants and masses. All the calculations are done in the unitary gauge using the ν-dimensional regularization method [7,8]. Some of the calculational procedures and, in particular, the field renormalization of the massive lepton in the presence of parity-violating interactions, are discussed in an Appendix.

2. – Definitions.

In field theory renormalized coupling constants associated with trilinear couplings are conventionally defined by considering the appropriate three-point functions with the three particles on mass shell. For example, in QED the renormalized charge e may be defined by means of the equation

$$e_0 Z_3^{\frac{1}{2}} Z_2 \bar{u}(p) \Gamma_\lambda(p, p) u(p) = e\bar{u}(p)\gamma_\lambda u(p) \,, \tag{1}$$

([2]) B. W. LEE and J. ZINN-JUSTIN: *Phys. Rev. D*, **5**, 3121, 3137, 3155 (1972). For an extensive review, see B. W. LEE: *Proceedings of the XVI International Conference on High-Energy Physics at Chicago-Batavia*, Vol. **4** (1972), p. 249.

([3]) G. RAJASEKARAN: *Divergences of the higher order corrections to μ decay in the Gauge theory*, preprint, Tata Institute of Fundamental Research TIFR/TH/72-11 (1972).

([4]) S. Y. LEE: *Phys. Rev. D*, **6**, 1701 (1972).

([5]) T. W. APPELQUIST, J. R. PRIMACK and H. R. QUINN: *Phys. Rev. D*, **6**, 2998 (1972).

([6]) For a recent summary, see A. SIRLIN: *Proceedings of the XVI International Conference on High-Energy Physics at Chicago-Batavia*, Vol. **2** (1972), p. 252.

([7]) G. 'T HOOFT and M. VELTMAN: *Nucl. Phys.*, **44** B, 189 (1972).

([8]) C. G. BOLLINI and J. J. GIAMBIAGI: *Phys. Lett.*, **40** B, 566 (1972); *Nuovo Cimento*, **12** B, 20 (1972).

where e_0 is the bare charge, Γ_λ is the proper unrenormalized vertex function and Z_2 and $\sqrt{Z_3}$ describe the field renormalizations associated with the external electron and photon lines. The left-hand member represents the sum of all the Feynman diagrams contributing to the three-point function with the two electrons on mass shell, evaluated at zero momentum transfer.

It seems natural to use a similar definition for the renormalized coupling constants in the gauge theories. For instance, in order to define a renormalized coupling constant $g_{e\nu_e W}$ for the trilinear coupling $e\nu_e W$ one may consider the sum of all the Feynman diagrams contributing to the three-point $e\nu_e W$ function with the three particles on mass shell. (Figure 1 illustrates the first- and

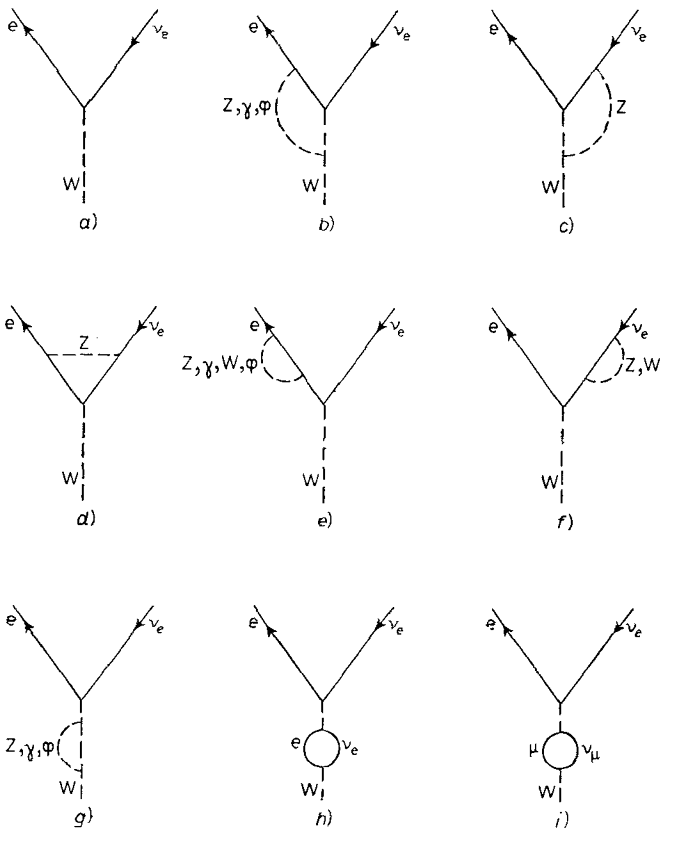

Fig. 1. – Diagrams contributing to $e\nu_e W$ coupling renormalization. Figure 1 *b*), for example, depicts three distinct diagrams, with the dashed line representing a Z, γ or φ.

third-order contributions to this function in the $SU_2 \times U_1$ gauge model.) An important theoretical advantage of this procedure is that the matrix elements involved are closely connected with physical amplitudes and are, therefore, independent of the electromagnetic gauge. In fact this definition, along with a similar one for $g_{\mu\nu_\mu W}$, has been used implicitly or explicitly in recent calculations concerning higher-order corrections to μ decay and lepton-ν scattering [3-5]. There is, however, the difficulty that the constants so defined are infra-red divergent [9]. This can be seen most easily if one observes, for example, that the diagrams of Fig. 1 describe the amplitude for $W \to e^- + \bar{\nu}_e$ and, therefore, the virtual-photon contributions to Fig. 1 must somehow cancel the infra-red divergences associated with soft-photon emission in $W^- \to e^- + \bar{\nu}_e + \gamma$. Similar difficulties occur if one wishes to define the pion decay constant f_π and the pion-nucleon coupling constant $g_{pn\pi}$ in the presence of electromagnetism. This situation is unlike what occurs in «pure QED». In fact, Z_3 is infra-red convergent and $Z_2 \bar{u}(p) \Gamma_\lambda(p, p) u(p) = (Z_2/Z_1) \bar{u}(p) \gamma_\lambda u(p)$; although Z_2 and Z_1 are infra-red divergent, the Ward identity tells us that $Z_2 = Z_1$ so that the renormalized charge is free from such divergences.

We briefly comment on two possible solutions to this difficulty:

a) Separate out the infra-red divergent contributions to the three-point function and define the renormalized coupling constant in terms of the remainder. A method for separating out the infra-red divergent contributions of order α in a manner which is independent of the gauge adopted for the covariant photon propagator has been already described in the literature [10]. The advantage of this method is that, although not unique, it is universal in the sense that it can be applied immediately to any three-point function and, furthermore, the separated terms are free from ultraviolet divergences.

b) An alternative procedure is to define the renormalized coupling constant $g_{e\nu_e W}$ by means of the relation

$$\Gamma(W \to e + \bar{\nu}_e) + \Gamma(W \to e + \bar{\nu}_e + \gamma) + \ldots = g^2_{e\nu_e W} m_W/(48\pi) , \tag{2}$$

where $\Gamma(W \to f)$ means the total decay probability for $W \to f$ and ... stands for the emission of two or more photons. Clearly the left-hand member is free from infra-red divergences. To lowest nontrivial order only the two first terms need be considered. Note that this «infra-red difficulty» does not arise in vertices involving three neutral particles, such as $\nu\nu Z$.

Having thus suggested two methods to define the renormalized coupling constants so that they do not involve infra-red divergences we now turn our attention to the problem of studying the ultraviolet divergences and their cancellations in the relations between renormalized quantities.

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(9) This point was already emphasized in ref. (5).

(10) A. SIRLIN: *Phys. Rev. D*, **5**, 436 (1972). See, in particular, Sect. **4** where the method is applied to the study of the pion decay constant. A similar procedure has been used to isolate the infra-red divergent contributions in the electromagnetic corrections to β decay: see A. SIRLIN: *Proceedings of the Topical Conference on Weak Interactions, CERN, 1969*, edited by J. PRENTKI and J. STEINBERGER (Geneva, 1969), p. 408.

3. – Calculations.

In this Section we discuss to lowest nontrivial order the divergent contributions to some of the renormalized coupling constants of the theory and show explicitly how they cancel in the relations between renormalized coupling constants and masses. Insofar as we are discussing only the ultraviolet divergent contributions, we can ignore the infra-red problem discussed in Sect. **2** (11). The calculations are done in the framework of the $SU_2 \times U_1$ leptonic gauge theory, with ν_e, e_L and ν_μ, μ_L constituting two independent doublets and e_R and μ_R two independent singlets. We have not included the effects of hadrons or quarks which can be incorporated into the theory in the manner discussed in the literature (12). The Feynman integrals are evaluated using the ν-dimensional regularization method (7,8) in the unitary gauge of the theory. In this method divergent contributions appear as terms proportional to $(\nu-4)^{-1}$ (ν is the number of dimensions in the regularization procedure and constitutes the regularizing parameter). To show that a certain quantity or combination of quantities is convergent, it is sufficient to show that the residue of the $\nu = 4$ poles cancel exactly. Proper vertex parts and two-point functions involving photons are gauge-dependent quantities; our results are given in the Feynman gauge for the photon propagator. However, as pointed out in Sect. **2**, with our definitions renormalized coupling constants and masses are independent of the gauge chosen for the covariant photon propagator; it follows that our final results enjoy the same property.

We first consider the renormalized coupling constant $g_{e\nu_e W}$. Figure 1 *a*) represents the first-order contribution while Fig. 1 *b*)-1 *i*) depict the lowest-order corrections. For brevity, a single Figure represents several Feynman graphs, for instance, Fig. 1 *b*) stands for three separate Feynman graphs with the virtual line representing either a Z-meson, a φ-meson or a photon. The sum of Fig. 1 *b*)-1 *d*) gives the proper vertex function with the three particles on mass shell. For this contribution we find

$$V = M_0 \frac{g_0^2}{16\pi^2}\left[-\frac{29}{6} + 2R + \frac{m_e^2}{2m_W^2}\left(R - \frac{1}{2}\right)\right]\frac{1}{\nu-4} + \mathrm{Pf}\,, \tag{3}$$

(11) Note, however, that for the evaluation of the finite corrections to the relations among renormalized quantities, it is important to give a precise definition of the coupling constants which takes into account the infra-red difficulty.

(12) Rigorously speaking, the inclusion of the multiplets such as quarks is necessary in order to eliminate the Adler anomalies (see, for example, B. W. LEE's review cited in footnote (2)). As the difficulties associated with the Adler anomalies do not manifest themselves to the order of our calculations, and as there are many different ways to introduce the hadrons, we have neglected them altogether.

where

$$R = m_W^2/m_Z^2\,, \tag{4a}$$

$$M_0 = -i\,\frac{g_0}{\sqrt{2}}\,\varepsilon^\lambda \bar{u}_e \gamma_\lambda a v_{\nu_e}\,, \tag{4b}$$

is the lowest-order matrix element, $a = (1-\gamma_5)/2$, g_0 is the unrenormalized coupling constant, ε^λ is the polarization vector of the W-meson and Pf stands for «finite part» of the corresponding Feynman diagrams.

Figures 1 e), 1 f) and 1 g)-1 i) depict the field renormalizations of the external e, $\bar{\nu}_e$ and W, respectively. These quantities can be obtained from the corresponding two-point functions which are studied in Appendix A. In particular, in that Appendix we discuss the determination of the field renormalization of the electron in the presence of parity-nonconserving interactions. Using the results of Appendix A, we obtain

$$1e) = M_0 \frac{g_0^2}{16\pi^2}\left[1 - R + \frac{m_e^2}{m_W^2}\left(1 - \frac{R}{2}\right)\right]\frac{1}{\nu - 4} + \text{Pf}\,, \tag{5}$$

$$1f) = -\,M_0 \frac{g_0^2}{16\pi^2}\,\frac{3}{4}\,\frac{m_e^2}{m_W^2}\,\frac{1}{\nu - 4} + \text{Pf}\,, \tag{6}$$

$$1g) + 1h) + 1i) = -\,M_0\,\frac{g_0^2}{16\pi^2}\left[\frac{8}{3} + R\right]\frac{1}{\nu - 4} + \text{Pf}\,. \tag{7}$$

The sum of eqs. (3), (5)-(7) gives the complete contribution of order g_0^3 to the three-point function:

$$V = +\,1e) + 1f) + 1g) + 1h) + 1i) = -M_0\,\frac{g_0^2}{16\pi^2}\,\frac{13}{2}\,\frac{1}{\nu - 4} + \text{Pf}\,. \tag{8}$$

Thus, we have

$$g_{e\nu_e W} = g_0\left[1 - \frac{g_0^2}{16\pi^2}\left(\frac{13}{2}\,\frac{1}{\nu - 4} + \text{Pf}\right) + O(g_0^4)\right]. \tag{9}$$

The corresponding calculation for $g_{\mu\nu_\mu W}$ is obtained by replacing $m_e \leftrightarrows m_\mu$. Note that although eqs. (3), (5) and (6) contain divergent contributions proportional to the squared lepton mass, such terms cancel in the sum of eq. (8) and, therefore, in the final answer of eq. (9). This is necessary for the success of the renormalization program as otherwise the ratio $g_{e\nu_e W}/g_{\mu\nu_\mu W}$ would have turned out to be divergent! This ratio is, in fact, finite and calculable in this theory in terms of such parameters as α, R, m_φ^2/m_W^2 and the lepton masses ([5,6]). The constants $g_{e\nu_e W}$ and $g_{\mu\nu_\mu W}$ appear naturally in the calculations of higher-order corrections to μ decay and to neutrino-lepton scattering ([3-5]). The actual calculation of the finite parts of $g_{e\nu_e W}/g_{\mu\nu_\mu W}$ is of interest in the detailed studies of such corrections. It may also be of interest in the distant future if it becomes feasible to test e-μ universality by measuring the ratio

$$\Gamma(\mathrm{W} \to \mathrm{e}^- + \bar{\nu}_\mathrm{e})/\Gamma(\mathrm{W} \to \mu^- + \bar{\nu}_\mu)$$

with a precision of order α.

Turning our attention to the renormalized charge, we note that only the renormalizations of the external photon line need be considered as all other contributions vanish for $q = 0$ by virtue of electromagnetic-current conservation. The relevant diagrams are depicted in Fig. 2. By gauge invariance the

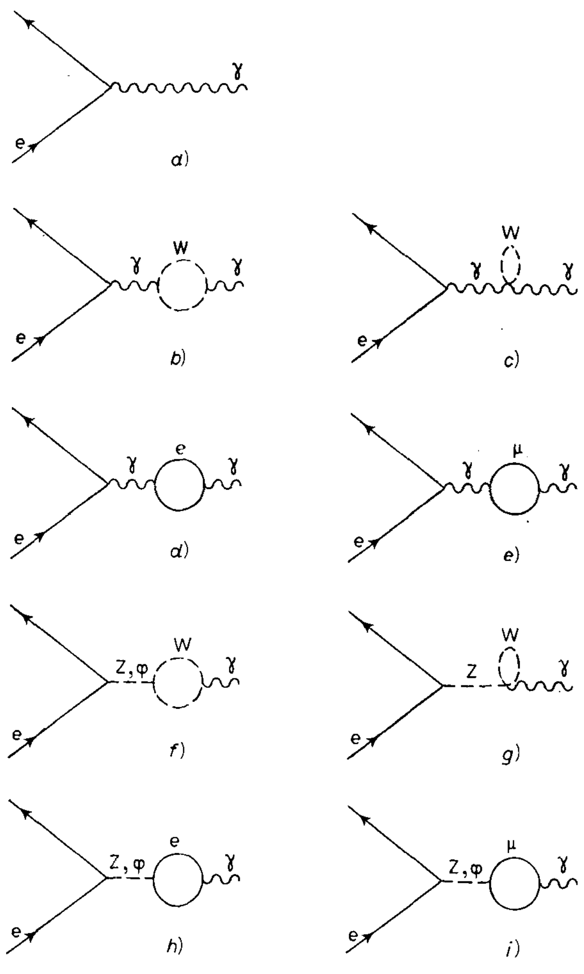

Fig. 2. – Diagrams contributing to charge renormalization.

sum of the diagrams involving the Z vector meson in Fig. 2 *f*) and 2 *g*) must be proportional to $D^{\mathrm{Z}}_{\mu\nu}(q)(q^2 g^{\nu\varrho} - q^\nu q^\varrho)\varepsilon_\varrho$, where $D^{\mathrm{Z}}_{\mu\nu}$ is the Z-meson propagator. As $m_\mathrm{Z}^2 \neq 0$, $D^2_{\mu\nu}$ is not singular as $q \to 0$ and therefore the sum of these two diagrams must vanish in that limit. The same is true for the diagram involving the φ-meson in Fig. 2 *f*) and for the matrix elements of Fig. 2 *h*) and 2 *i*). The other diagrams give a total contribution

$$(10)\qquad 2b) + 2c) + 2d) + 2e) = ie_0\,\bar{u}_e\gamma\; u_e\varepsilon_\lambda\left[-\frac{e_0^2}{16\pi^2}\left(\frac{13}{3(\nu-4)}+\mathrm{Pf}\right)\right],$$

where $e_0 > 0$ is the bare charge of the positron and $ie_0\bar{u}_e\gamma^\lambda u_e\varepsilon_\lambda$ is the lowest-order matrix element corresponding to Fig. 2 *a*). Thus on the basis of eq. (10) we obtain

$$(11)\qquad e = e_0\left[1-\frac{e_0^2}{16\pi^2}\left(\frac{13}{3(\nu-4)}+\mathrm{Pf}\right)+O(e_0^4)\right].$$

The ratio $g_{e\nu_e W}/e$ can be obtained by dividing eq. (9) by (11). Using the relation

$$(12)\qquad \frac{e_0^2}{g_0^2} = 1-\left(\frac{m_W^0}{m_Z^0}\right)^2 = 1-R\,\frac{(1-\delta m_W^2/m_W^2)}{(1-\delta m_Z^2/m_Z^2)}\,,$$

where $(m_W^0)^2$ and $(m_Z^0)^2$ are the squared bare masses of the W and Z mesons and δm_W^2 and δm_Z^2 the corresponding mass shifts, we obtain

$$(13)\qquad \frac{e^2}{g_{e\nu_e W}^2} = 1-R+R\left(\frac{\delta m_W^2}{m_W^2}-\frac{\delta m_Z^2}{m_Z^2}\right)+ \\ +\frac{g_0^2}{8\pi^2}(1-R)\left[\left(\frac{13}{2}-\frac{13}{3}\{1-R\}\right)\frac{1}{\nu-4}+\mathrm{Pf}\right]+O(g_0^4)\,.$$

The mass shifts δm_W^2 and δm_Z^2 can be obtained from the analysis of the two-point functions for the W and Z mesons. Using the results of Appendix A we have

$$(14)\qquad \frac{\delta m_W^2}{m_W^2}-\frac{\delta m_Z^2}{m_Z^2} = \frac{g_0^2}{8\pi^2}\,\frac{13}{6}\left(2R-1-\frac{1}{R}\right)\frac{1}{\nu-4}+\mathrm{Pf}\,.$$

Insertion of eq. (14) into eq. (13) shows that in fact the residue of the $\nu=4$ pole exactly cancels in the ratio of the renormalized constants. Note that evaluation of the finite parts in eqs. (13) and (14) would essentially give the leading correction to the lowest-order formula

$$e^2 = g_{e\nu_e W}^2(1-R)+O(g_{e\nu_e W}^4)\,.$$

As a further example, we consider the renormalized $\nu_e\nu_e Z$ coupling constant. The diagrams of order g_0 and g_0^3 are depicted in Fig. 3. Again we find that the proper vertex diagrams Fig. 3 *b*)-3 *d*) and the field renormalizations of the external particles Fig. 3 *e*)-3 *h*) contain divergent contributions proportional to m_e^2 but such terms cancel in the overall sum. The final answer can be writ-

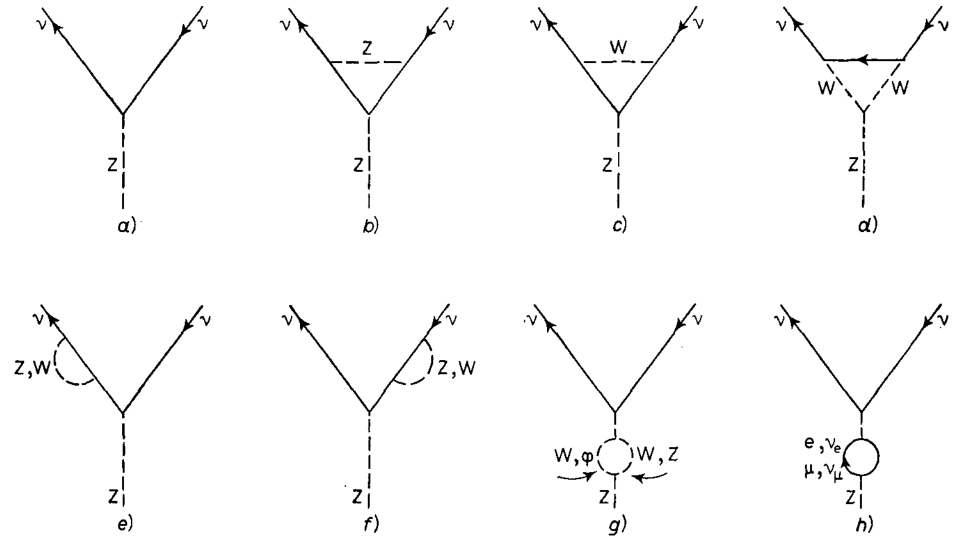

Fig. 3. – Diagrams contributing to ννZ coupling renormalization.

ten as

$$(15)\qquad g_{\nu_e\nu_e Z}=\frac{(g_0^2+g_0'^2)^{\frac{1}{2}}}{\sqrt{2}}\left\{1-\frac{g_0^2}{16\pi^2}\left[\frac{13}{3}\left(1-\frac{1}{2R}+R\right)\frac{1}{\nu-4}+\mathrm{Pf}\right]+O(g_0^4)\right\}.$$

The absence of divergent terms proportional to m_e^2 in eq. (15) implies that $g_{\nu_e\nu_e Z}/g_{\nu_\mu\nu_\mu Z}$ is finite, as expected. Dividing eqs. (11) and (15) and using the relation $e_0^2/(g_0^2+g_0'^2)=[1-(m_W^0/m_Z^0)^2][m_W^0/m_Z^0]^2$ we find

$$(16)\qquad \frac{1}{2}\left(\frac{e}{g_{\nu_e\nu_e Z}}\right)^2=R(1-R)+R(1-2R)\left(\frac{\delta m_Z^2}{m_Z^2}-\frac{\delta m_W^2}{m_W^2}\right)+\\ +\frac{g_0^2}{8\pi^2}R(1-R)\left[\frac{13}{3}\left(2R-\frac{1}{2R}\right)\frac{1}{\nu-4}+\mathrm{Pf}\right]+O(g_0^4)\,.$$

Inserting eq. (14) into eq. (16) we verify that the residues of the $\nu=4$ poles cancel in the expression for $(e/g_{\nu_e\nu_e Z})^2$.

Thus, we have shown in some simple examples how the lowest-order divergent corrections cancel against each other in the relations between renormalized coupling constants and masses. This is of course necessary for the success of the renormalization program (*). Furthermore, the present discussion

(*) *Note added in proofs.* – After having submitted this paper for publication we have received a preprint entitled *Renormalization of Gauge theories* W-*decay and* μ-*decay* by T. W. Appelquist, J. R. Primack and H. R. Quinn. Among other subjects, these authors have independently studied the renormalization of the $SU_2\times U_1$ model in the unitary gauge, and have also analysed in detail the role of the infrared divergences in the renormalization program.

establishes a convenient framework to study the leading finite corrections to lowest-order relationships between the renormalized constants of the theory.

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APPENDIX

In this Appendix we discuss some aspects of the evaluation of the regularized two-point functions (*i.e.* the self-energy parts) to lowest nontrivial order. From the corresponding expressions one obtains the field renormalizations and mass shifts used in the calculations of Sect. **3**.

The integrals are evaluated with the aid of the ν-regularization method [7,8]. In this scheme translation of variables is permissible and all integrals can be reduced to

$$\text{(A.1)}\qquad \int \frac{d^\nu k}{(2\pi)^\nu}\frac{(k^2)^l}{[k^2-L+i\varepsilon]^m}=\frac{i(-1)^{l-m}}{(2\sqrt{\pi})^\nu}L^{l+\nu/2-m}\frac{\Gamma(l+\nu/2)\Gamma(m-l-\nu/2)}{\Gamma(\nu/2)\Gamma(m)}\,.$$

This expression defines the regularized integral, with the number of dimensions ν playing the role of the regularization parameter. Divergent intervals exhibit simple poles at $\nu=4$.

A.1. – Electron two-point function.

After performing the mass renormalization, we find that the divergent part of the electron two-point function (Fig. 4 *a*)) can be written as

$$\text{(A.2)}\qquad \Sigma_e(\not p)-\Sigma_e(m)=B(\not p-m)+C(\not p-m)^2+D(\not p-m)^3-$$
$$-A\not p\gamma_5-E\not p\gamma_5(p^2-m^2)+\mathrm{Pf}\,,$$

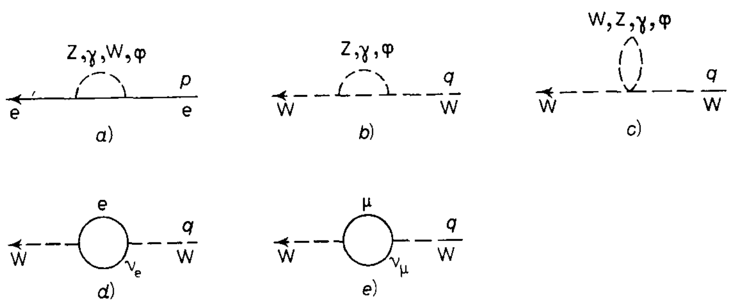

Fig. 4. – Diagrams contributing to electron and W-meson two-point functions.

where m stands now for the lepton mass and

$$B = \frac{g_0^2}{16\pi^2}\left[2(1-R)+\frac{m^2}{m_W^2}\right]\frac{1}{\nu-4}, \tag{A.3}$$

$$A = \frac{g_0^2}{16\pi^2}\left[\frac{m^2}{m_W^2}(1-R)\right]\frac{1}{\nu-4}, \tag{A.4}$$

and C, D and E are known constants proportional to $(\nu-4)^{-1}$. Note, however, that the terms involving C, D and E do not contribute to the field renormalization of the electron. This is obvious for the terms involving C and D. On the other hand the term proportional to E can be written as $E(\not p - m)\not p\gamma_5(\not p - m)$, thus it gives no contribution when inserted into diagram 1e).

We must then take into account the contributions of $B(\not p - m)$ and $A\not p\gamma_5$ to the field renormalization of the electron. The appearance of the last term is, of course, due to the fact that the interactions are parity violating. We discuss two methods of taking into account the effect of the last term.

i) Following Hiida's suggestion ([13]), one replaces the free Lagrangian of the electron by

$$\mathscr{L}_0 = \bar\psi[i\gamma^\mu\partial_\mu(1+A\gamma_5)-m]\psi \tag{A.5}$$

and adds a compensating counterterm $-\bar\psi i\gamma^\mu\partial_\mu A\gamma_5\psi$ which cancels the $-A\not p\gamma_5$ term in Σ.

In this formulation the « free spinor » u' is not a solution of Dirac's equation but rather it satisfies

$$\not p(1+A\gamma_5)u' = mu'\,. \tag{A.6}$$

Calling u the solution of Dirac's equation $\not p u = mu$, writing $u' = (1-c\gamma_5)u$ and neglecting terms of $O(g_0^4)$, one finds $c = A/2$. Therefore, the « zeroth order » term (Fig. 1 a)) gives now a contribution

$$-i\frac{g_0}{\sqrt2}\varepsilon_\lambda\bar u'_e\gamma^\lambda a v_{\nu_e} = -i\frac{g_0}{\sqrt2}\varepsilon_\lambda\bar u_e\left(1+\frac{A}{2}\gamma_5\right)\gamma^\lambda a v_{\nu_e} = \left(1+\frac{A}{2}\right)M_0\,. \tag{A.7}$$

Thus the overall effect of the term $A\not p\gamma_5$ in the corrections to $W^- \to e^- + \bar\nu_e$ is to give a contribution $(A/2)M_0$, which we have included in eq. (5). The contribution of $B(\not p - m)$ can be calculated in the usual manner and gives $(B/2)M_0$, after taking into account the factor $(1/\sqrt{Z_2}) \approx 1 - B/2$ in the reduction formula.

ii) An alternative although somewhat more involved method is to rescale independently the fields $((1-\gamma_5)/2)e$ and $((1+\gamma_5)/2)e$. If we write

$$\frac{(1-\gamma_5)}{2}e = \sqrt{Z_{2L}^{(e)}}\,\frac{(1-\gamma_5)}{2}e', \qquad \frac{(1+\gamma_5)}{2}e = \sqrt{Z_{2R}^{(e)}}\,\frac{(1+\gamma_5)}{2}e',$$

([13]) K. Hiida: *Phys. Rev.*, **132**, 1239 (1963).

the free Lagrangian generates the counterterms

$$\mathscr{L}_{\text{c.t.}} = (Z_{2L}^{(e)} - 1)\, \bar{e}' i\gamma^\mu \partial_\mu a e' + (Z_{2R}^{(e)} - 1)\, \bar{e}' i\gamma^\mu \partial_\mu \bar{a} e' - \\ - \left[m\left(\sqrt{Z_{2L}^{(e)} Z_{2R}^{(e)}} - 1\right) - \delta m \sqrt{Z_{2L}^{(e)} Z_{2R}^{(e)}}\right] \bar{e}' e' \,. \tag{A.8}$$

Adjusting $Z_{2L}^{(e)} - 1$ and $Z_{2R}^{(e)} - 1$ to cancel the terms $B(\not p - m) - A\not p\gamma_5$ in $\Sigma_e(\not p) - \Sigma_e(m)$ gives

$$Z_{2L}^{(e)} - 1 = B + A + \text{Pf}\,, \tag{A.9}$$

$$Z_{2R}^{(e)} - 1 = B - A + \text{Pf}\,. \tag{A.10}$$

If we now express the interaction term $\bar{e}\gamma^\lambda a \nu_e W_\lambda$ in terms of the renormalized field e' we see that a factor $\sqrt{Z_{2L}^{(e)}} \approx 1 + (Z_{2L} - 1)/2$ is introduced. This contributes a second-order correction $\big((Z_{2L} - 1)/2\big) M_0 = \big((A + B)/2\big) M_0 + \text{Pf}$, in agreement with the previous method.

A.2. – Vector-meson two-point functions.

We illustrate the procedure with the W-meson two-point functions. The polarization tensor $\pi_{\sigma\lambda}(q)$ is defined as $(-i)$ times the Feynman diagrams depicted in Fig. 4 b)-4 e) with the external legs extracted. We find

$$\pi_{\sigma\lambda}^{(\text{W})}(q) = \\ -\frac{g_0^2}{16\pi^2}\mathscr{P}_{\sigma\lambda}(q)\left[\frac{26}{3} + R + \frac{7}{3}R^2 + \frac{q^2}{m_\text{W}^2}\left(-\frac{5}{3} + \frac{1}{2}R - \frac{7}{6}R^2\right) - \frac{q^4 R^2}{6m_\text{W}^4}\right]\frac{1}{\nu - 4} - \\ - \frac{g_0^2}{16\pi^2} m_\text{W}^2 g_{\sigma\lambda}\left[3\left(1 - \frac{1}{2}R - \frac{1}{2R} - \frac{R^2}{2}\right) + \frac{q^2 R^2}{2m_\text{W}^2} + \frac{m_e^2 + m_\mu^2}{m_\text{W}^2}\right]\frac{1}{\nu - 4} + \text{Pf}\,, \tag{A.11}$$

where

$$\mathscr{P}_{\sigma\lambda}(q) = q^2 g_{\sigma\lambda} - q_\sigma q_\lambda\,. \tag{A.12}$$

If we write:

$$\pi_{\sigma\lambda}(q) = B(q^2) g_{\sigma\lambda} + C(q^2)\, q_\sigma q_\lambda\,, \tag{A.13}$$

the squared-mass shift δm_W^2 and the conventionally defined renormalization constant $Z_3^{(\text{W})}$ are given by

$$\delta m_\text{W}^2 = \text{Re}\, B(m_\text{W}^2)\,, \tag{A.14}$$

$$\frac{1}{Z_3^{(\text{W})}} = 1 - \frac{\text{d}}{\text{d}q^2}\text{Re}\, B(q^2)\bigg|_{q^2 = m_\text{W}^2}\,. \tag{A.15}$$

Using eq. (A.11) we find

$$\frac{\delta m_W^2}{m_W^2} = \frac{g_0^2}{16\pi^2}\left[-10 + \frac{3}{2R} - \frac{m_e^2 + m_\mu^2}{m_W^2}\right]\frac{1}{\nu - 4} + \mathrm{Pf} \tag{A.16}$$

and

$$Z_3^{(W)} - 1 = -\frac{g_0^2}{16\pi^2}\left[\frac{16}{3} + 2R\right]\frac{1}{\nu - 4} + \mathrm{Pf}\,. \tag{A.17}$$

With this conventional definition of $Z_3^{(W)}$, eq. (7) is given by $(Z_3^{(W)} - 1)(M_0/2)$.

The field renormalizations of the Z, γ and ν and the mass shift of the Z-meson are obtained in the same manner. We list the results:

$$Z_2^{(\nu_e)} - 1 = -\frac{g_0^2}{16\pi^2}\frac{3}{2}\frac{m_e^2}{m_W^2}\frac{1}{\nu - 4} + \mathrm{Pf}\,, \tag{A.18}$$

$$Z_3^{(\gamma)} - 1 = -\frac{g_0^2}{8\pi^2}\frac{13}{3}(1 - R)\frac{1}{\nu - 4} + \mathrm{Pf}\,, \tag{A.19}$$

$$Z_3^{(Z)} - 1 = \frac{g_0^2}{8\pi^2}\left[-\frac{5}{3} + \frac{7}{3R} - \frac{13}{3}R\right]\frac{1}{\nu - 4} + \mathrm{Pf}\,, \tag{A.20}$$

$$\frac{\delta m_Z^2}{m_Z^2} = \frac{g_0^2}{16\pi^2}\left[-\frac{17}{3} + \frac{35}{6R} - \frac{26}{3}R - \frac{m_e^2 + m_\mu^2}{m_W^2}\right]\frac{1}{\nu - 4} + \mathrm{Pf}\,. \tag{A.21}$$

We make two further observations: i) as there is no mass counterterm for the photon available in the theory, the polarization tensor $\pi_{\sigma\lambda}^{(\gamma)}$ for the photon must be proportional to $P_{\sigma\lambda}(q)$ in order to satisfy gauge invariance; ii) to order g_0^2 the Z and γ mesons can transform into each other. Because the Z and γ are not degenerate in zeroth order, this mixing can only contribute to our results through the diagrams of Fig. 2 *f*)-2 *i*). As pointed out in the text, however, at $q = 0$ such matrix elements vanish by virtue of gauge invariance.

● RIASSUNTO (*)

Si discutono alcune proprietà fondamentali delle costanti di accoppiamento rinormalizzate nel contesto delle simmetrie di gauge spontaneamente infrante. Si mette in rilievo che le definizioni convenzionali portano ad alcune difficoltà connesse con le divergenze infrarosse e si suggeriscono due possibili soluzioni. Si studiano poi i contributi alle correzioni di ordine inferiore divergenti nell'ultravioletto e si mostra esplicitamente, con numerosi esempi particolari, come questi termini divergenti si eliminano nelle interrelazioni fra costanti di accoppiamento rinormalizzate e masse. Si eseguono i calcoli specifici nella gauge unitaria del modello di gauge $SU_2 \times U_1$, facendo uso del metodo di regolarizzazione ν-dimensionale.

(*) *Traduzione a cura della Redazione.*

Замечания, касающиеся перенормированных констант связи калибровочных теорий.

Резюме (*). — В рамках спонтанно нарушенных калибровочных симметрий обсуждаются некоторые основные свойства перенормированных констант связи. Отмечается, что общепринятые определения приводят к некоторым трудностям, связанным с инфракрасными расходимостями. Предлагаются два возможных решения. Затем исследуются ультрафиолетовые расходящиеся вклады в поправки низшего порядка и явно показывается на ряде частных пределов, как эти расходящиеся члены взаимно уничтожаются в соотношениях между перенормированными константами связи и массами. Проводятся специальные вычисления в унитарной калибровке для $SU_2 \times U_1$ калибровочной модели, используя ν-мерный метод регуляризации.

(*) *Переведено редакцией.*