COMPARATIVE STUDIES OF THE $2\nu\beta\beta$ DECAY *)

O. CIVITARESE

Dto. de Fisica, UNLP, c.c. 67, 1900 La Plata, Argentina

P. O. HESS

Inst. de Ciencias Nucleares, UNAM, A. P. 70-543 México 04510 D.F., México

J. G. HIRSCH

Dto. de Fisica, CINVESTAV del IPN, A. P. 14-740 México 07000 D.F., México

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The microscopic description of nuclear double beta decay transitions is presented in the framework of a schematic model. The comparison between these results and the ones corresponding to realistic calculations shows that the physics of the double beta decay may be dominated by non-perturbative effects.

1 Introduction

Since the introduction of renormalized particle-particle interactions in the treatment of proton-neutron correlations in open shell systems [1, 2] the physical interpretation of double-beta decay transitions becomes heavily dependent upon the adopted values of the model parameters. The sensitivity of the two-neutrino double-beta decay mode upon g_{pp} was found to be present both in schematic and in realistic models. Several methods have been proposed to cure for this severe dependence since it become obvious that the renormalization was needed and that physical values of the coupling constants were hard to obtain from first principles. The possibility of renormalizations, originated in the ground state correlations of the initial and final nuclei induced by charge-conserving modes of excitations and acting upon single-quasiparticle occupancies, was suggested in [3]. However, these approaches suffer from a common disease, e.g. the violation of the Ikeda Sum Rule [4, 5]. Both the collapse of the QRPA [6] and the violation of the Ikeda Sum Rule can be interpreted in terms of a "phase-transition" rather similar to the pairing one [7] with the number of pn-pairs playing the role of an order parameter. In the present note we shall extent the similarities between the "critical" behaviour of the QRPA against renormalized particle-particle interactions and the more familiar concept of symmetry breaking. Details are given in [8].

2 The Hamiltonian

The starting Hamiltonian [9,10] is written

$$H = H_p + H_n + H_{\rm res}\,, \tag{1}$$

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where

$$H_p = \sum_p e_p a_p^\dagger a_p - G_p S_p^\dagger S_p , \qquad H_n = \sum_n e_n a_n^\dagger a_n - G_n S_n^\dagger S_n ,$$
$$H_{\rm res} = 2\chi \beta_J^- \cdot \beta_J^+ - 2\kappa P_J^- \cdot P_J^+ , \tag{2}$$

where the operators $S_{p(n)}$ are monopole pair operators, $G_{p(n)}$ are the pairing coupling constants, $\beta_J^\pm$ and $P_J^\pm$ are particle-hole and particle-particle proton-neutron operators, χ and κ are the coupling constants of the separable proton-neutron two body interaction.

This Hamiltonian was shown to produce results, for pn-excitations and double beta decay transition probabilities, which are similar to the ones obtained by using realistic interactions [10]. We shall consider the one-shell limit of this Hamiltonian [4].

After performing BCS transformations, for protons and neutrons separately, the single-particle and pairing terms are written as

$$H_p = \epsilon_p \sum_{m_p} \alpha_{pm_p}^\dagger \alpha_{pm_p}, \qquad H_n = \epsilon_n \sum_{m_n} \alpha_{nm_n}^\dagger \alpha_{nm_n}, \tag{3}$$

where ϵ_p, ϵ_n are the quasiparticle energies and $\alpha_p^\dagger, \alpha_n^\dagger$ are the quasiparticle creation operators [6]. The linearized Hamiltonian, neglecting the scattering terms $(\alpha_p^\dagger \alpha_n, \alpha_n^\dagger \alpha_p)$ which do not contribute at the QRPA order, reads

$$\begin{aligned} H =& H_p + H_n \\ &+ 2\chi(2j+1)\left[(u_p^2 v_n^2 + v_p^2 u_n^2) A^\dagger \cdot A \right. \\ &\left. + u_p v_n v_p u_n A^\dagger \cdot A^\dagger + v_p u_n u_p v_n A \cdot A\right] \\ &- 2\kappa(2j+1)\left[(u_p^2 u_n^2 + v_p^2 v_n^2) A^\dagger \cdot A \right. \\ &\left. - u_p u_n v_p v_n A^\dagger \cdot A^\dagger - v_p v_n u_p u_n A \cdot A\right] . \end{aligned} \tag{4}$$

Fermi excitations ($J = 0$) have been studied in [4] while the case of Gamow-Teller ($J = 1$) excitations have been presented in [5]. For the sake of simplicity and without loss of generality, we shall proceed with the case of $J = 0$. In this limit, the quasiparticle-pair operators have the form

$$A^\dagger = \left[\alpha_p^\dagger \otimes \alpha_n^\dagger\right]_{M=0}^{J=0} . \tag{5}$$

Thus, at the QRPA order of approximation, e.g. by keeping bilinear products of $A^\dagger$ and A, we arrive at the expression

$$H = \epsilon\, C + \lambda_1 A^\dagger A + \lambda_2 (A^\dagger A^\dagger + AA), \tag{6}$$

where the proton and neutron quasiparticle energies have been replaced by a common value ϵ. The operator C and the coupling constants λ_1 and λ_2 are given by

$$\begin{aligned} C &= \sum_{m_p} \alpha_{pm_p}^\dagger \alpha_{pm_p} + \sum_{m_n} \alpha_{nm_n}^\dagger \alpha_{nm_n} , \\ \lambda_1 &= 4\Omega\left[\chi(u_p^2 v_n^2 + v_p^2 u_n^2) - \kappa(u_p^2 u_n^2 + v_p^2 v_n^2)\right] , \\ \lambda_2 &= 4\Omega(\chi + \kappa) u_p v_p u_n v_n , \end{aligned} \tag{7}$$

where 2Ω is the degeneracy of the shell.

3 Collectivity of the modes and the QRPA

The currently adopted QRPA treatment of this Hamiltonian [4] has shown that the collapse of the QRPA [11] is controlled by the ratio between κ and natural scale of the model (e.g. G or the quasiparticle energies). Of course, one can always refer the values of κ to the value of χ but, as shown in [4] χ is fixed by the position of the resonance associated to the decay mode while κ is mostly responsible for the fragmentation of the low-lying intensities. This result, which is also found in the exact solution of the model, does not show-up in the renormalized QRPA treatment of refs. [3]. The collapse of the QRPA excitation energy has also been found in the extension of the present model to a larger group representation [5]. Since, as we have said before, it remains valid in more realistic shell model treatments, we shall attempt to understand the underlying physical mechanism from a more direct and simple picture where it can be featured as the signature of a phase transition. Consequently, we shall introduce a boson representation which transforms the combination of fermionic degrees of freedom into bosonic ones and which preserves Pauli's Principle. The link with the phase-transition mechanism is established by introducing, in this boson basis, coherent states and an order parameter.

4 Boson-mapping techniques and the QRPA and RQRPA

The Dyson's mapping of the Hamiltonian is performed by replacing the quasiparticle-pair operators by

$$A^{\dagger} \to b^{\dagger}\left(1 - \frac{b^{\dagger}b}{2\Omega}\right), \qquad A \to b\,, \qquad C \to 2b^{\dagger}b\,. \tag{8}$$

The operators $b^{\dagger}$ and b are boson creation and annihilation operators, which obey exact boson-commutation relations. The number of bosons n_b is restricted by the condition $n_b \leq 2\Omega$. This restriction guarantees that spuriousities due to non-physical states with a larger number of bosons will not be present in the basis.

The transformed Hamiltonian is written

$$\begin{aligned} H = {} & (2\epsilon + \lambda_1)b^{\dagger}b - \frac{\lambda_1}{2\Omega}b^{\dagger 2}b^2 + \lambda_2\left(1 - \frac{1}{2\Omega}\right)b^{\dagger 2} \\ & - \frac{\lambda_2}{\Omega}\left(1 - \frac{1}{2\Omega}\right)b^{\dagger 3}b + \frac{\lambda_2}{4\Omega^2}b^{\dagger 4}b^2 + \lambda_2 b^2\,. \end{aligned} \tag{9}$$

5 Understanding the results as a critical phenomena

The meaning of the QRPA collapse as a phase transition is better illustrated with the help of coherent states [12]

$$|\alpha\rangle = N_0 \sum_{l=0}^{2\Omega} \frac{\alpha^l}{l!} b^{\dagger l}|0\rangle\,, \tag{10}$$

where α is a complex parameter and N_0 is a normalization factor. Expectation values of products of boson creation and annihilation operators in the coherent state $|\alpha>$ are given by the general expression

$$\langle\alpha|b^{\dagger n_1}b^{n_2}|\alpha\rangle = {\alpha_0}^{n_1+n_2}e^{-i\phi(n_1-n_2)} \tag{11}$$

and with them the expectation value of the transformed Hamiltonian, Eq. (9), gives the potential energy surface in terms of the order parameter α. Different regimes of the solution will therefore be determined by non-trivial values of the order parameter.

The trial-function (10), when only even values of l are considered, reduces to the more familiar (quadratic) set of states used, for instance, to minimize the QRPA equations of motion. Concerning the present discussion the removal of one-boson components of (10) does not alter significantly the results, as shown next.

6 Potential energy surfaces

The potential-energy surface $E(\alpha)$, i.e. the expectation value of H on the coherent state, was minimized as a function of the order parameter α. Then the dependence of α with the coupling constant κ, at the minimum, was determined. The results are shown in Figure 1, for the case of $N_p = N_n = 4$. This behaviour demostrates that a sudden change of correlations occurs around some critical value of the coupling constant κ (κ_c). The onset of the phase transition is observed at values of κ just before the point where $2\epsilon + \lambda_1 - 2\lambda_2$ vanishes.

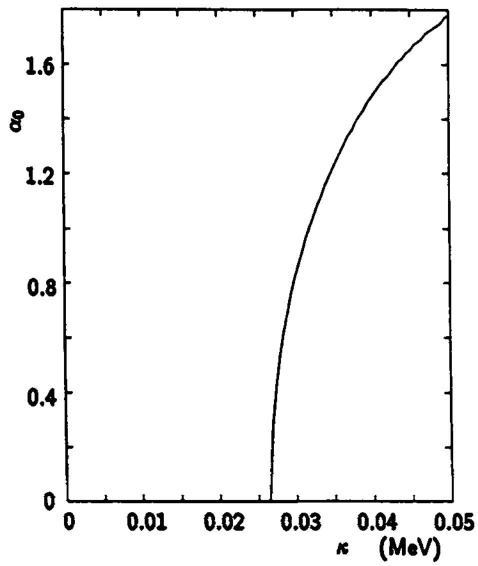

Fig. 1. The real part of the order parameter, α, as a function of the coupling constant κ, for a fixed value of $\chi = 0.04$ and for $N_n = N_p = 4$.

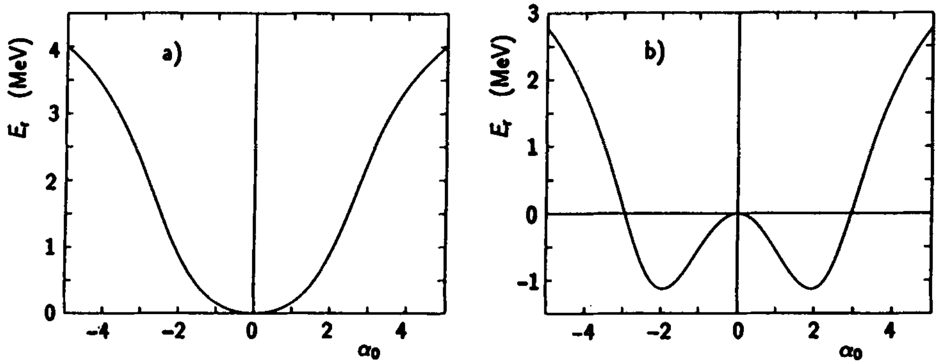

Fig. 2. Real part (E_r) of the energy, as a function of the order parameter α, for $\kappa < \kappa_c$ (a) and for $\kappa > \kappa_c$ (b). The results correspond to the case $N_n = N_p = 4$ and $\chi = 0.04\,\mathrm{MeV}$.

The critical behaviour of the potential-energy surfaces is well demostrated by the results shown in Figure 2. The upper box shows the harmonic dependence of the energy, as a function of α_0. This is the domain of validity of the QRPA harmonic expansion around the minimum corresponding to $\alpha_0 = 0$. For the example shown Fig. 2a we have used the value $\kappa = 0.02\,\mathrm{MeV}$. Figure 2b where the value $\kappa = 0.06\,\mathrm{MeV}$ is used, shows the characteristic shape of a double-well potential with symmetric minima located at non-zero values of α_0. This is a clear evidence of the symmetry breaking induced by the renormalized particle-particle interactions passed the critical value of κ.

7 Conclusions

In the above examples the interesting analogy existing between symmetry breaking mechanisms, either spontaneous like the breaking of the number of particle symmetry by the BCS vacuum or dynamical like the breaking of the isospin by the residual particle-particle interactions, can be established. Own to this analogy, the non-perturbative nature of the expansions around the critical point in the parametric space where the QRPA collapses, indicates that efforts to correct it based on perturbative methods can yield to non-physical solutions. By the other side, the double well shape of the potential energy surfaces for coupling constants passing by the critical point indicates that other methods, like the BRST treatment [13] may be in order to calculate perturbatively the wave functions and matrix elements involved in nuclear double beta decay transitions.

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