

## **THEORETICAL APPROACH FOR THE DETERMINATION OF THE ODEN CURVES**

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### **ABSTRACT**

A mathematical equation is presented for the Oden Curve which is employed to determine the grain size distribution of fine-grained suspended sediments. This equation has proved to give a satisfactory fit for more than 115 sample data collected from Bahía Blanca Estuary and offshore Monte Hermoso Beach. It has been possible to find parameters which represent intrinsic characteristics of the suspended sediment.

### **RESUMEN**

Una expresión matemática es presentada para la Curva de Oden, la cual es empleada para determinar la granulometría de los sedimentos finos en suspensión. Esta ecuación ha ajustado satisfactoriamente más de 115 muestras extraídas en el Estuario de Bahía Blanca y la zona de externa de la playa de Monte Hermoso. Fueron encontrados parámetros que representan características intrínsecas del sedimento en suspensión.

### **1. INTRODUCTION**

The determination of the grain size distributions of the suspended load (very fine sand, silt and clay) is not trivial. Many instruments and analytical techniques have been developed to obtain it. Some methodologies are based on settling velocity distributions like the Oden Method (Krumbein and Pettijohn, 1938), Owen Method (Owen, 1976) developed for in situ sampling, Small Settling Tube (Anderson and Kurtz, 1979) and others. There are also methods based on different properties of the sediments such as the SediGraph, based on X-ray absorption (Jones, 1988), the Hydrophotometer, which uses a photo-extinction device (Jordan *et al.*, 1971), and the Coulter Counter (Kranck, 1979), considering the electrical resistance of the particles.

Depending on the method employed, the same sample may yield different size distributions (Singer *et al.*, 1988). For example in the Owen Method, flocculation is allowed and the time measured at every bottom extraction is the time that a floc needs to deposit. So the results obtained are in relation to the floc sedimentation, so an individual particle-size distribution becomes meaningless. On the other hand Oden Method deals with the ultimate grain size analysis, independently from the quality of the water or chemical reactions which may take place during the experiment.

Normally the size distribution is found by estimating the particle settling velocity. Although there are many factors that modify the settling velocity, such as particle shape, mineralogy, salinity, flocculation, organic material, electrochemical forces,

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sediment concentration, etc.. Methods based on this variable are widely employed, less complex, provide reliable and repeatable results and they are economically feasible.

The Oden's Tube Method, as described by Krumbein and Pettijohn (1938), is a simple laboratory procedure but time consuming. However, the procedure has measurement periods of the order of the standard pipette method for grain size analysis of fine sediments. A drawback of the procedure occurs during data processing since it is based on drawing the Oden's curves and manually defining tangents to it. Obviously, the problem of this technique is that it relays on the ability and expertise of the operator to make the graphical fit. The same set of values may produce different results according to the experience of the operator.

The objective of this paper is to introduce a physical model of the Oden Method leading to a mathematical equation representing the Oden Curve, which can be fitted to any data using regression procedures. This will provide an objective method, independent of the operator's experience or ability. The parameters obtained from the regression will be a function of the size distribution. The analytical approach to the curve is preferable to the graphical one and also it will give robust results that can be used to compare with other samples. This approach may be used to relate the intrinsic characteristics of the suspended load which describe the distribution of solid suspended particles.

## **2. STANDARD METHOD**

The principle for the determination of the Oden Curve in the laboratory is based on the free fall law of a particle in a viscous fluid. A water column with solid particles homogeneously distributed and previously sieved through the ASTM 230 sieve, is allowed to settle in a 100 to 120 cm long, 3.25 cm diameter tube. Then portions of the column with sediment are extracted from the bottom at different predefined times. The extracted portions contain a certain amount of deposited load which is dried out and weighed. An example of the laboratory results is given in Table 1.

The results are plotted on a percentage of solids decanted versus time (figure 1). Once the points are plotted, a curve is drawn on the paper fitting the data obtained. Next step is to draw tangents to the curve manually fitted at specific times. Then, estimate the y-axis intersection of each tangent, which represents the percentage in weight of a decanted load, for equal and larger diameters having the corresponding deposition time on the x-axis.

## **3. ANALYTICAL MODEL**

### **3.1 Falling velocity of a sphere in a viscous fluid**

As the method is based on the free fall sphere principle, for the sake of completeness we will show the full derivation of the equations. Using figure 2 as a reference, we can apply Newton's second law to obtain

TABLE 1. Indicates the set of values obtained for an inner Bahía Blanca estuary following the procedure described by Krumbein and Petijhon.

Time (min)	Suspended load (%)
0.00	100
1.00	98.0
4.44	96.1
25.00	91.6
133.60	50.5
280.00	73.0
616.42	63.7
1391.90	49.6
32.90	34.7

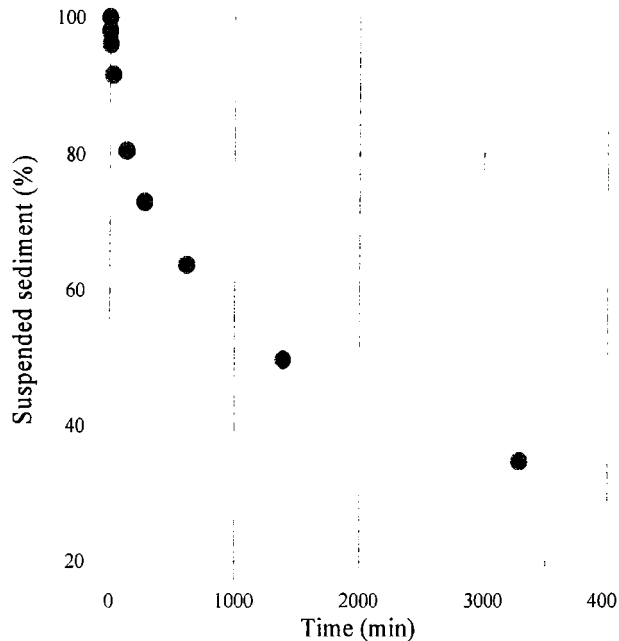
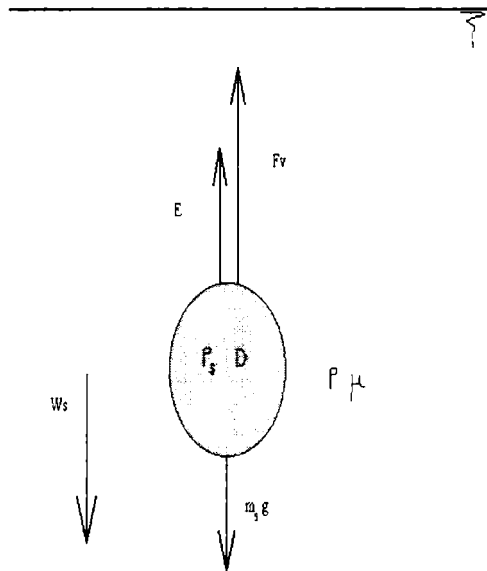


Figure 1. Set of data corresponding to the values of table 1. The points represent the values of the suspended load versus time obtained in laboratory following the methodology described in Krumbein and Petijohn (1938)

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$$m_s \frac{\partial w_s}{\partial t} = m_s g - \rho g V_s - C \mu w_s \quad (1)$$

where  $m_s$  is the mass of the particle,  $w_s$  is the particle settling velocity,  $t$  is the time,  $g$  is the acceleration of gravity,  $V_s$  is the volume of the sphere,  $\rho$  is the fluid density,  $C$  is the Stokes coefficient and  $\mu$  is the dynamic viscosity of the fluid.



**Figure 2. Illustration of a sphere falling in a viscous fluid.  $w_s$  is the velocity,  $m_s g$  is the weight of the sphere,  $E$  and  $F_v$  represent the Archimedes force and the viscous force,  $\rho_s$  is the specific mass of the solid particle,  $\rho$  is the specific mass of the fluid,  $\mu$  is dynamic viscosity of the fluid and  $D$  is the diameter of the particle**

Dividing (1) by the mass of the particle we have,

$$\frac{\partial w_s}{\partial t} = \frac{(\rho_s - \rho)}{\rho_s} g - \frac{C \mu w_s}{V_s \rho_s} \quad (2)$$

where  $\rho_s$  is the particle density. Through the experiments of Stokes we know that,

$$C = 3\pi D_i \quad (3)$$

where  $D_i$  is the diameter of the sphere. Equation (2) is a differential equation, whose solution is as follows (Boyce and Di Prima, 1976)

$$w_{(s,t)} = \frac{(\rho_s - \rho)gD_i^2}{18\mu} \left(1 - e^{-\frac{18\mu t}{\rho_s D_i^2}}\right) \quad (4)$$

Expression (4) gives the variation of the velocity of the particle as a function of time assuming its initial velocity as zero. The equation shows that when time is long enough, the influence of the exponential term can be neglected and so, the velocity becomes constant and equal to the settling velocity  $w_s$ ,

$$w_s = \frac{(\rho_s - \rho)gD_i^2}{18\mu} \quad (5)$$

Integrating expression (4) in the time, we obtain the distance covered at every time interval by a particle. That integration leads to,

$$y - y_0 = \frac{18\mu}{(\rho_s - \rho)gD_i^2} t + \frac{\rho}{(\rho_s - \rho)g} \left(1 - e^{-\frac{18\mu t}{\rho_s D_i^2}}\right) \quad (6)$$

where  $y$  is the distance traveled by a particle of diameter  $D_i$  in a time  $t$ ,  $y_0$  is the initial position of the particle from a fixed coordinate system. For our case the values of the largest particle diameters are 0,00625 cm, an ASTM 230 sieve nominal diameter. The time needed for such particle to reach a velocity 99% of the settling one (5) is only,  $t_{(0.99)} = 5 \cdot 10^{-4}$  s considering  $\rho_s = 2.65 \text{ g cm}^{-3}$ ,  $\rho = 1 \text{ g cm}^{-3}$ ,  $\mu = 0.01 \text{ g cm}^{-1} \text{ s}$ ,  $g = 981 \text{ cm s}^{-2}$ . The rest of the particles that are smaller, will need even less time to reach the 99% of their settling velocity. Due to this numeric result given by expression (3) using typical values, it will be assumed that particles reach the settling velocity instantaneously. Then the exponential term in (3) can be neglected. Using the same set of values, the distance covered by the particle is also very small (0.01 cm) in such a short time and does not compare to the total tube length. Therefore, based on these calculations, it is safe to assume that the settling velocity of a particle is constant and acquired almost instantaneously as it is free to fall.

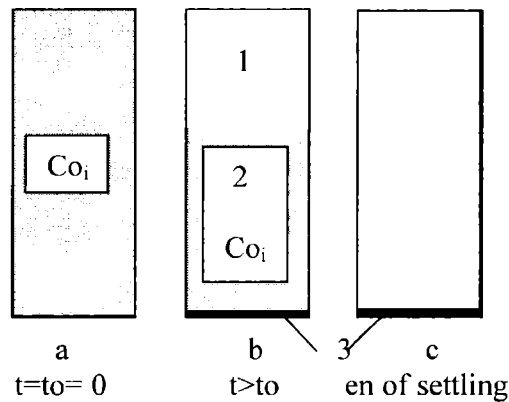
Also the Reynolds number (Re) for this kind of conditions is very low. For Reynolds numbers less than one the sphere moves in a laminar regime. If so, the result given by equation (5) is mathematically correct. Between Reynolds numbers of one and two the relative error when calculating the settling velocity with equation (5), is

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below 3%. Assuming the largest value in the samples,  $D=0.00625$  cm, the corresponding Reynolds number for the settling velocity is 0.22. Therefore, the procedure is valid even for particles with  $D = 0.014$  cm (fine sand).

### 3.2 Oden Curve model

To obtain the Oden's Curve equation it is assumed: 1) all the particles are uniformly distributed in the fluid mass at the initiation of the experiment ( $t = 0$ ) (figure 3a); 2) the particles settle down in a laminar regime without disturbing each other (figure 3b); 3) the settling velocity is constant (linear process); 4) the diameters of the particles have a log-normal distribution.



**Figure 3. Illustrates the assumptions made for the model.  $C_{0i}$  is the initial mass concentration of the particles for the diameter  $D_i$ , (1) has no particles of  $D_i$ , (2) has the initial concentration and (3) represents the accumulated mass for a single diameter  $D_i$ .**

To explain the process, we assume some solid particles with the same diameter ( $D_i$ ) suspended in a fluid. At the initiation of the experiment, these particles produce a volumetric concentration of  $C_{0i}$ . Then all particles are uniformly distributed in the fluid mass within the tube (figure 3a). Then each particle begins to settle independently with a velocity  $w_{si}$  and, as we assumed only one particle size after a certain time  $t$ , there would be two layers. An upper one (layer 1, figure 3b) with no solid particles, an intermediate layer (layer 2, figure 3b) with the original mixture concentration, and a lower layer (layer 3, figure 3b) at the bottom of the tube, where certain amount of mass  $M_{si}$  is deposited. figure 3c shows the end of the experiment when all particles have settled to the bottom of the tube.

At any given time,  $t > t_0$  the mass accumulated at the bottom for the particles  $D_i$  is,

$$\frac{M_{ac,i}}{M_{D_i}} = \frac{Y_{D_i}}{L} \quad (7)$$

where  $M_{ac,i}$  is the accumulated mass at the bottom of the tube for the diameter  $D_i$ ,  $M_{D_i}$  is the total initial mass corresponding to the diameter  $D_i$ ,  $y_{D_i}$  is the distance traveled by a particle, with constant settling velocity, in a period  $\Delta t (= t - t_0)$  and  $L$  is the total length of the tube. The grain size distribution can be expressed,

$$\Phi_{(D_i)} = \frac{1}{2\pi} \frac{1}{\beta} \frac{1}{D_i} e^{-\frac{(\ln D_i - \alpha)^2}{2\beta^2}} \quad (8)$$

where  $\beta$  is the variance of the population and  $\alpha$  is a constant. Then the initial concentration is given by

$$C_0 = \frac{M}{V} = \frac{\int_0^{D_*} \rho_s V_p \Phi_{(D)} \delta D}{LA} \quad (9)$$

where  $A$  is the cross-section of the tube,  $M$  is the total solid mass in the system,  $V$  is the total volume concerned in the process and  $D_*$  is the largest diameter in the sample. The solution for (9) is,

$$C_0 = \frac{1}{LA} \frac{\pi}{12} \rho_s D_0^3 e^{\frac{9}{2}\beta^2} \left\{ \left[ \operatorname{erf} \left( \frac{\ln(D_*)}{\sqrt{2}\beta} - \frac{3}{2\sqrt{2}\beta} \right) \right] - 1 \right\} \quad (10)$$

where  $\operatorname{erf}$  is the error function,  $D_0$  is the mean diameter of the Log-normal distribution ( $\alpha = \ln D_0$ ). Considering all the hypotheses, the suspended mass for any diameter can be calculated solving the following integration,

$$M_{ac} = \int \rho_s \frac{\pi}{6} D^3 \Phi_{(D)} \left( \frac{Y_{(D)}}{L} \right) \delta D \quad (11)$$

where  $M_{ac}$  is the accumulated mass at the bottom of the tube. The percentage is calculated by

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$$\frac{M_{susp}}{M_{tot}} 100 = 100 - \frac{M_{ac}}{C_0 LA} 100 \quad (12)$$

where  $M_{susp}$  is the suspended mass in the fluid. The value  $y_D$  of (7) can be estimated from (5) as

$$Y_D = w_s t = \frac{(\rho_s - \rho)gD^2}{18\mu} t \quad (13)$$

Replacing (11) and (12) in (10) and solving for  $M_{ac}$

$$\frac{M_{ac}}{M_{tot}} = 100 - 100A_0 t \left\{ \text{erf} \left[ \frac{1}{2\beta^2} \ln \left( \frac{D}{D_0} \right) - \frac{5}{4} [2\beta^2] + 1 \right] \right\} \quad (14)$$

where  $A_0$  is a constant. Now if we write  $D$  in time ( $t$ ) according to (13), we arrive at the expression of the Oden Curve,

$$M_{ac\%} = 100 - 100A_0 t \left\{ \text{erf} \left[ \frac{1}{A_2} \ln \left( \frac{1}{A_1 t} \right) - \frac{5}{4} A_2 \right] + 1 \right\} \quad (15)$$

where

$$A_0 = \frac{1}{\text{erf} \left[ A_2 \ln \left( \frac{D_*}{D_0} \right) - \frac{3}{2} \frac{1}{A_1} \right] - 1} \quad (16)$$

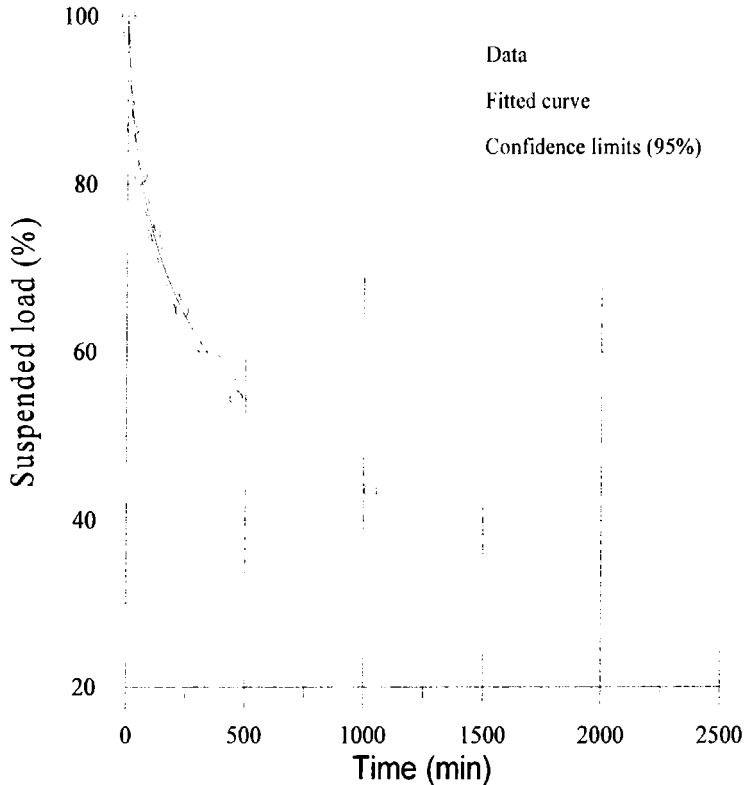
$$A_2 = 2 \sqrt{2\beta^2} \quad (17)$$

$$A_1 = \frac{(\rho_s - \rho)gD_0^2}{18L\mu} \quad (18)$$

The values of these three constants are calculated through a regression technique that fits the data obtained in the laboratory. figures 4a and 4b show examples of two different cases fitted with (13) and their residuals (figure 4c). All the statistical results provide a good indication of the adequate fit of the curve to the data, resulting in highly significant constants when compared with the value  $F_{(1,8) 99\%}$ . based on ANOVA tests.



The standard error is also remarkably low suggesting a good approximation of the curve to the data. figures 4a, 4b and 4c show only representative examples of the more than 115 cases studied. Similar results have been obtained with all the samples processed with this method.



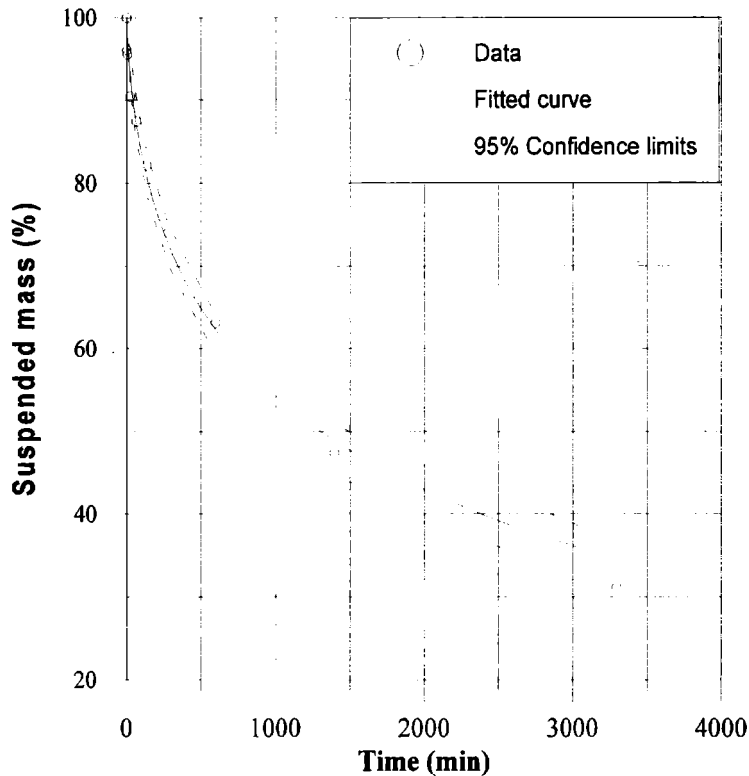
**Figure 4a. Representation of a set of data belonging to th inner Bahía Blanca Estuary with the corresponding fitted Oden Curve. The 95% confidence limits are also shown. The  $r^2 = 0.998$ , the fit standard error is 1.07and the F-statistic value is 1894.6. The values of the three constants are  $A_0 = 0.0122$ ,  $A_1 = 6.77267e-12$  and  $A_2 = 4.4843$ .**

#### 4. DETERMINATION OF THE SIZE DISTRIBUTION CURVE

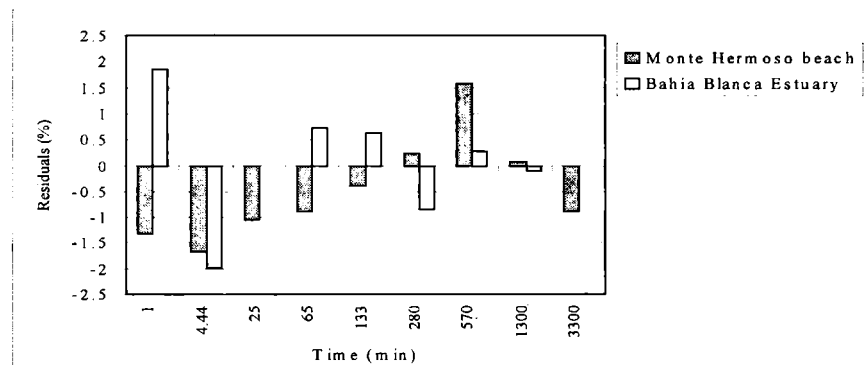
The vertical axis intersection with a straight line tangent to the curve at a certain value of time  $t$ , gives the sediment percentage in weight (or mass) for equal and smaller diameters to the corresponding instant  $t$  evaluated. The general equation for a straight line is,

$$M_{ac\%} = pt + q \quad (19)$$

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**Figure 4b.** Representation of a set of data belonging to offshore Monte Hermoso beach with the corresponding fitted Oden Curve. The 95% confidence limits are also shown. The  $r^2 = 0.995$ , the fit standard error is 2.17 and the F-statistic value is 839. The values of the three constants are  $A_0 = 0.003329$ ,  $A_1 = 1.72892 \text{ e-}11$  and  $A_2 = 4.2215$ .

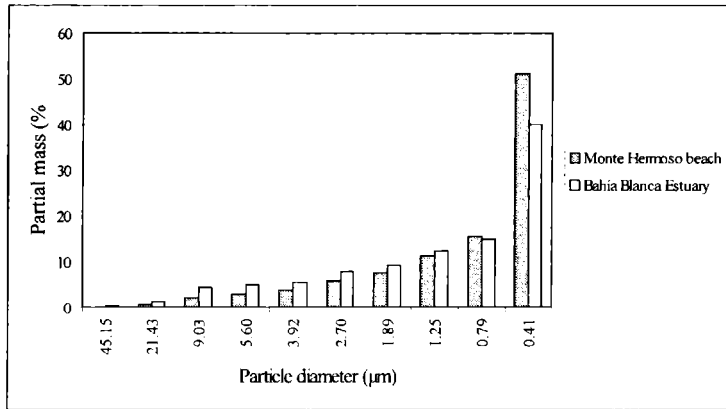


**Figure 4c.** This figure represents the residuals, difference between the predicted values and the data values versus time. In every case the error is bellow 2%.

Where  $p$  is the slope,  $t$  is the time and  $q$  is the Y-axis intersection. For our particular case the value of  $p$  will be the time derivative of (15), and  $q$  is the percentage of accumulated mass for diameters equal to or larger than the corresponding diameter for any  $t$ . So deriving and replacing in (19) it can be concluded that,

$$q = 100 - 200 \frac{A_0}{A_2 \cdot \pi} \left\{ e^{\left( \frac{5}{4} A_2 - \frac{1}{A_2} \ln \left( \frac{1}{A_1 t} \right) \right)^2} \right\} \quad (20)$$

Thus equation (20) represents the accumulated size curve for the experiment in time. An example is shown on figure 5, which shows the histogram for both samples employed throughout this paper. As they belong to different environments, differences in the relative percentage of mass at each range can be seen. The offshore Monte Hermoso beach sample has more fine grained material (more than 50% of particles equal to or lower than  $0.41 \mu$ ), than the Inner Bahía Blanca Estuary sample, which has a lower relative clay percentage (just 40% for the same diameter range).



**Figure 5. The bars in the diagram represent the mass in % discriminated by particle diameter. The last column indicates the mass percentage for particles smaller than 0.41 microns, corresponding to clay. The values for the columns were calculated using eq. (19).**

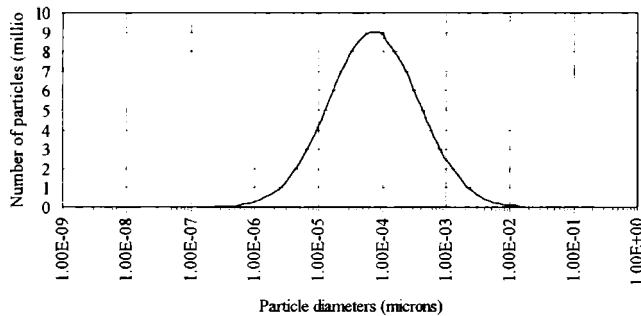
Once the regression is applied to a set of data the mean diameter and the variance of the grain size distribution can be calculated using equations (17) and (18) as follows,

$$D_0 = \frac{18L\mu A_1}{(\rho_s - \rho)g} \quad (21)$$

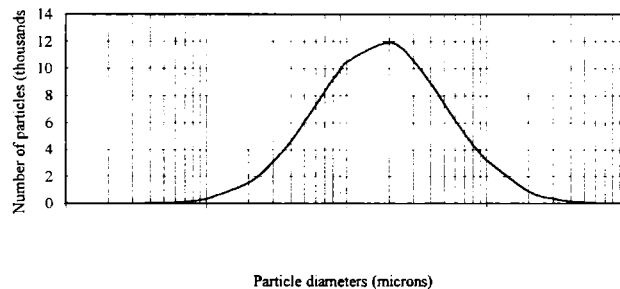
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$$\beta = \frac{A_2}{2.42} \quad (22)$$

Equations (21) and (22) were deduced from equations (17) and (18) respectively. These values allow us to know the size distribution in numbers of particles present against the diameter. Using the equation of the log normal distribution, a typical chart is shown in figure 6. The two size distributions (figure 6a and figure 6b), represent the number of particles on the y-axis and the corresponding diameter on the logarithmic x-axis. Clearly, there exists smaller particles in the Offshore Monte Hermoso Beach sample than in the Inner Bahía Blanca Estuary. Figure 6a shows a Monte Hermoso sample with a log-normal mean diameter of  $1.07 \cdot 10^4 \mu\text{m}$ , figure 6b shows a Bahía Blanca Inner Estuary log-normal mean diameter of  $5.49 \cdot 10^1 \mu\text{m}$ . These samples represent the largest difference between the most frequent grain size diameters in both areas.



**Figure 6a.** Represents the log-normal distribution for the offshore Monte Hermoso beach sample. Numbers of particles is plotted versus particles diameters. The two values the determine the log-normal curve are  $D_0 = 1.0752 \cdot 10^{-3} \text{ :m}$  and  $\beta = 1.655$ . Both values were calculated using equations (20) and (21) respectively.



**Figure 6b.** Represents the log-normal distribution for the inner Bahía Blanca Estuary sample. Numbers of particles is plotted versus particles diameters. The two values the determine the log-normal curve are  $D_0 = 2.01 \cdot 10^{-1} \text{ :m}$  and  $\beta = 1.0699$ . Both values were calculated using eqs. (20) and (21) respectively.

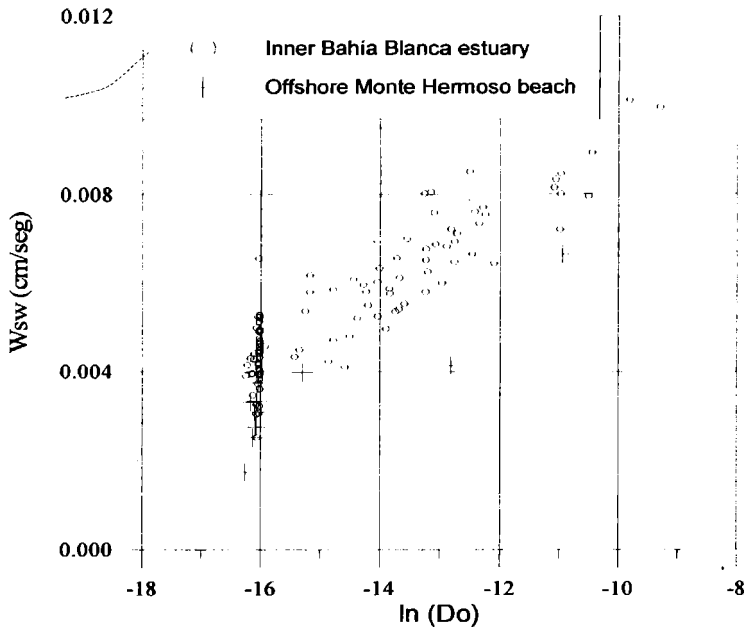
Calculation of the weighted averaged settling velocity ( $W_{sw}$ ) of the suspended sediment is possible according to the following equation,

$$W_{sw} = \frac{\int_0^D W_{sD} \Phi_D \delta D}{\int_0^D W_{sD} \delta D} \quad (23)$$

This integration leads to,

$$W_{sw} = W_0 e^{A_2} \frac{\{erf[\frac{2}{A_2} \ln(\frac{D}{D_0}) - \frac{A_2}{2}] + 1\}}{erf[\frac{2}{A_2} \ln(\frac{D}{D_0})] + 1} \quad (24)$$

where  $W_0$  is the mean diameter,  $D_0$ , settling velocity. Equation (23) gives the variation of the weighted averaged settling velocity with  $D$ . figure (7) shows the value of the equation.



**Figure 7. Representation of the effective settling velocity values versus  $D_0$ . It can be seen the self grouping of the values corresponding to the inner Bahía Blanca Estuary and offshore Monte Hermoso beach.  $W_{sw}$  has been calculated with equation 23.**

## 5. RESULTS AND DISCUSSION

The analytical equation developed allows an objective fitting of the Oden Curve through any sample, whereas, with the manual method is necessary to trust the ability of the person that draws the curve. Also two statistical parameters were calculated through the model described.

We tested the method with more than 115 sets of data from inner Bahía Blanca Estuary and offshore Monte Hermoso beach (Argentina) fitted with equation (15). Both environments are quite different although having variable concentrations of suspended sediments ranging from 100 to 400 mg l<sup>-1</sup>. Descriptions of both areas may be found elsewhere (Perillo and Cuadrado, 1990; Perillo and Piccolo, 1998) and will not be repeated here. The suspended sediments at Monte Hermoso beach are derived from both the outflow of the estuary and the plume of the Colorado River (Cuadrado *et al.*, 1998).

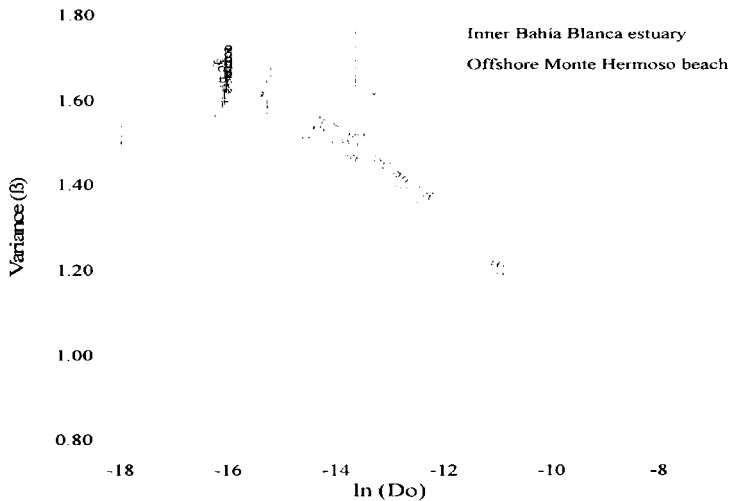
Examples of the level in which the equation provides an adequate fit to all the samples and the typical confidence intervals are shown in figure 4a and figure 4b. Further to this fit the grain size curves corresponding to the samples are readily calculated using equation (20) as are plotted in figure 6. The model also allows to establish physical parameters of the grain size distributions of the samples studied. Once the Oden Curve is fitted with equation (15), the values of the coefficients in (16), (17), (18) are known. The coefficient  $A_0$  gives information about the largest particle present in the sample.  $A_1$  provides a value related to the most frequent diameter in the sample ( $D_0$ ). In fact  $A_1$  is the settling velocity for  $D_0$  divided for the total length of the tube.  $A_2$  is related to the variance of the log-normal distribution. Mean diameters ( $D_0$ ) and the variances ( $\beta$ ) of the log normal distributions can be calculated through the values obtained by regression method using equations (21) and (22). Also the value for the effective settling velocity is given in the diameter in equation (24).

Parameter  $W_{sw}$  gives the skewness of the size distribution curve. The higher the value of  $W_{sw}$ , the more skewed to the left the distribution curves become. An inverse correlation independent from the environment exists when  $\beta$  is plotted against the  $\ln D_0$  (figure 8). The more skewed the curve, the smaller the mean grain sizes (mostly clay). Both environments can be differentiated in the graph, the beach samples showing consistently lower values than the estuarine ones.

Because we assumed that the data corresponding to  $W_{sw}$ , and  $D_0$  or its equivalent the effective settling velocity, may be used to differentiate environments, we tested them statistically to find out whether they show a difference between them. A t-test for independent samples was applied to the effective settling velocity from inner Bahía Blanca Estuary (group 1) and offshore Monte Hermoso beach (group 2) samples. Before that, the series were tested through normal probability plots and showed that the effective settling velocities series were normally distributed.

For group 1 the effective settling velocity mean is 0.0057 cm s<sup>-1</sup>, its standard deviation is 0.002002 and the number of cases is 109. For group 2 the mean results in 0.0036 cm s<sup>-1</sup>, its standard deviation is 0.001349 and the number of cases is 10. The t-value for the test is  $t = 3.47$  to which corresponds a probability of 0.7% for a level of

95% of confidence. Therefore, the effective settling velocities from offshore Monte Hermoso Beach are different from those of the Inner Bahía Blanca Estuary with a level of significance of less than 1% for a confidence level of 95% using a  $t_{(10)}$ .



**Figure 8. The plot shows the correlation between  $\beta$  and  $D_0$  for the whole set of data studied.**

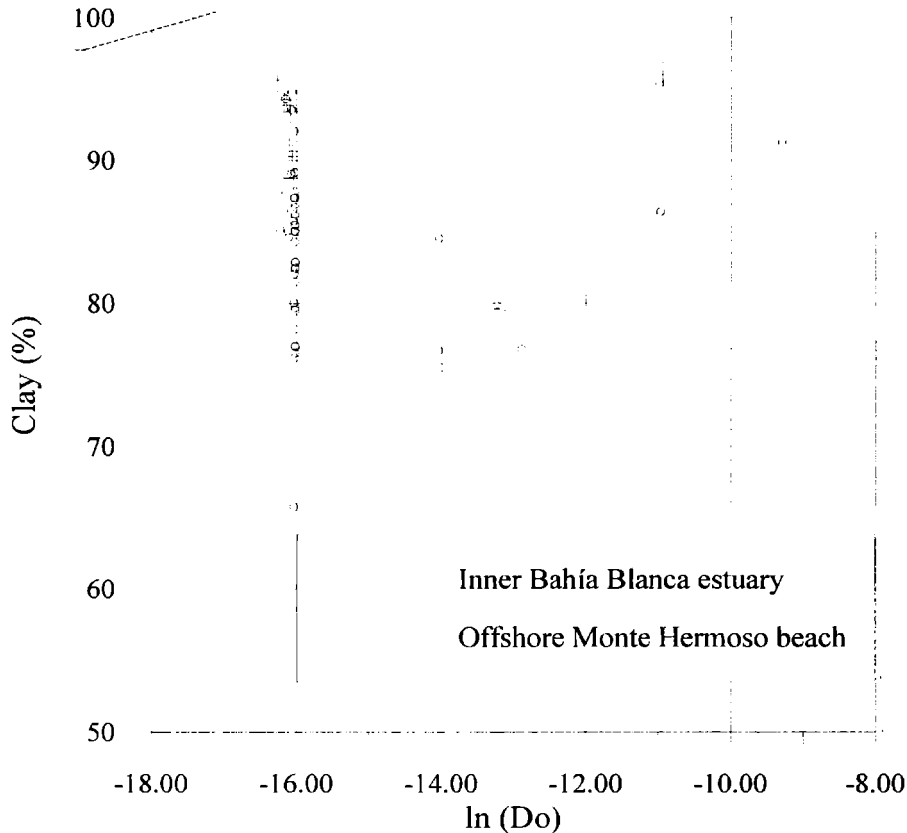
An intrinsic characteristic of the suspended sediment is its mean diameter  $D_0$ . The series from both places were studied statistically to find whether this parameter can be used to distinguish two environments. The series of mean diameters were not normally distributed so the Kolmogorov-Smirnoff non-parametric test was done. The mentioned test has the same interpretation of the t-test and is one of the most powerful tools among the non-parametric tests. The  $D_0$  data had a mean of  $3.55 \cdot 10^{-2} \mu\text{m}$ , its standard deviation was  $1.12 \cdot 10^{-1} \mu\text{m}$  and the cases studied were 109. For group 2, we had  $2.1503 \cdot 10^{-1} \mu\text{m}$ ,  $5.55 \cdot 10^{-2} \mu\text{m}$  and 10 respectively. The probability for this series to be the same is less than 1%. This says that the mean diameter of the log-normal distributions has a statistical difference between the groups studied. As the data suggest, the mean diameter average is smaller at offshore Monte Hermoso beach. This result also supports the idea that finer suspended load is found in offshore Monte Hermoso beach. Figure 9 represents the percentage of clay versus  $D_0$ , showing the highest percentages of clay in offshore Monte Hermoso beach samples.

## **6. CONCLUSIONS**

An analytical model has been developed to provide a fit for the Oden Curve. The method eliminates manual-operator dependant normal procedures used to decide the suspended load grain-size distributions. As a basic result, the model makes the process of determination of the grain size distribution much more accurate, faster and

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reproducible. The inaccuracy introduced by the operator is avoided as the data is processed by computer and not manually.



**Figure 9. Percentage of clay versus  $D_0$ , showing the highest percentages of clay in offshore Monte Hermoso beach samples.**

Two basic parameters that represent the intrinsic characteristics of the grain size distribution were calculated, the mean diameter ( $D_0$ ) and the variance of a log-normal distribution ( $\beta$ ). The advantage of these parameters is that they represent physical characteristics of the suspended load size distribution.  $D_0$  is the most frequent particle diameter size of the sample.  $\beta$  represents the skewness of the size distribution curve. The effective velocity ( $w_s$ ) is related to the average settling velocity of the whole mixture and is also given in terms of the diameter.

The parameters described here hold much more information than a simple graph can give, and may be used to compare different results from different places. However larger sets of data have to be processed in order to prove the effectiveness in discriminating environments.



**Acknowledgements:** Partial funding for this study was provided by CONICET and Universidad Nacional del Sur grants for the project "Estudio Integrado de Estuarios Argentinos (EIDEA) and the European Economic Community grant Joint Project 9350448AR The authors wish to thank Dr Angel Marcos for providing the Inner Bahía Blanca Estuary sample data.

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