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PRONY'S METHOD APPLIED TO ANOMALIES' SEPARATION ON AN ANDEAN CORDILLERA SECTION

Beatriz Introcaso
Grupo de Geofísica - Instituto de Física de Rosario - CONICET
Fac. de Cs. Ex., Ing. y Agrim.- Universidad Nac. de Rosario, e-mail:beatriz@unrctu.edu.ar

ABSTRACT

Prony's method is applied in this paper to separate gravity anomalies on an E-W section near 22° S in Western South America. The method allows us to fit the coefficients in a linear combination of exponentials representing the gravity field power spectrum, to obtain an estimate of the causative masses and depths. For the profile under consideration the results agree with geological models of the area. Some possible difficulties of the method are mention.

RESUMEN

En este trabajo se aplica el método de Prony para la separación de anomalías gravimétricas en una sección E-W ubicada en las proximidades del paralelo 22° S en el oeste sudamericano. El método consiste en ajustar los coeficientes de una combinación lineal de exponenciales que representa el espectro de potencia de un gravimétrico para obtener las magnitudes de las masas y profundidades de fuentes equivalentes causantes. Para el perfil considerado se obtienen resultados adecuados a los modelos geológicos disponibles. Se advierte sobre las posibles dificultades del método.

1. INTRODUCTION

The aim of this work is to find the depths of shallow and deep masses causing gravity effects observed on an Andean Cordillera section. Separation of regional and residual fields is carried out using power spectrum techniques. Power spectrum $P(\alpha)$ is defined as the square of the gravity field Fourier transform module. Its value for a field caused by masses m_0 and m_1 located at depths z_0 and z_1 respectively, can be calculated by means of (see Introcaso and Guspí, 1995):

$$P(\alpha) = 4\pi^2 G^2 (m_0^2 e^{-4\pi|\alpha|z_0} + m_1^2 e^{-4\pi|\alpha|z_1}) \tag{1}$$

where G is the Universal Gravity Constant, and is the frequency variable.

Prony's method is used to fit multiplicative constants and exponents in a linear combination of exponentials, to obtain - here - the values of the masses and the depths of the sources causing the observed gravity.

2. THEORETICAL CONSIDERATIONS

Prony´s method (see, for example, Kay and Marple, 1981) is a technique for modelling data of equally spaced samples by a linear combination of exponentials; it is not a spectral estimation technique in the usual sense, but a spectral interpretation can be provided.

Prony (1795) proposed a method for providing interpolated data points in the measurements by fitting an exponential model to the measured points and computing the interpolated values by evaluation of the exponential model at these points. For the case where only an approximate fit with a combination of exponentials to the data set is desired, a least squares estimation procedure is used. The solution involves an iterative process in which an arbitrary initial approximation of the unknown parameters is successively improved.

Prony´s method solves two sequential sets of linear equations with an intermediate polynomial rooting step that concentrates the nonlinearity of the problem.

The method is based on the following property:

Property

Let $f(x)$ be:

$$f(x) = \sum_{k=1}^{m} C_k \; e^{\beta_k x} \tag{2}$$

and $s \, \varepsilon R^{+}$. Then there exist constants $A_1, A_2, \ldots, A_m$ satisfying

$$f(x) = \sum_{k=1}^{m} A_k \; f(x+ks) \qquad \forall x \in R \tag{3}$$

Demonstration:

Because of the way in which $f(x)$ was defined, we have:

$$f(x+ks) = \sum_{i=1}^{m} C_i \, e^{\beta_i(x+ks)} = \sum_{i=1}^{m} C_i \, e^{\beta_i x} e^{\beta_i ks} \tag{4}$$

Calling:

$$B_j = e^{\beta_j s} \qquad \forall j=1,\dots,m \tag{5}$$

it turns out:

$$f(x+ks) = \sum_{i=1}^{m} C_i \, B_i^k \, e^{\beta_i x} \tag{6}$$

The constants A_i we are looking for must satisfy:

$$\sum_{k=1}^{m} A_k \left(\sum_{i=1}^{m} C_i \, B_i^k e^{\beta_i x} \right) = \sum_{k=1}^{m} C_k \, e^{\beta_k x} \tag{7}$$

$$\sum_{k=1}^{m} A_k \, B_i^k = 1 \qquad \forall i=1,\dots,m \tag{8}$$

which leads to the resolution of an *m* per *m* linear system on A_i :

$$A_1B_1 + A_2B_1^2 + \ldots + A_mB_1^m = 1$$

$$A_1B_2 + A_2B_2^2 + \ldots + A_mB_2^m = 1$$

$$A_1B_m + A_2B_m^2 + \ldots + A_mB_m^m = 1$$

The coefficients make a Van der Monde matrix:

$$\begin{matrix} B_1 & B_1^2 & B_1^m \\ B_2 & B_2^2 & B_2^m \\ & & \\ B_m & B_m^2 & B_m^m \end{matrix}$$

in which $B_i \neq B_j \, \forall i \neq j$, guaranteeing the existence and uniqueness of the A_i we were looking for.

Corollary

B_j are the roots of the polynomial equation:

$$\sum_{k=1}^{m} A_k Z^k = 1 \qquad \textbf{(9)}$$

Demonstration:

It follows immediately from (8). Let us return to the particular case we are dealing with. We can assume the power spectrum values of the observed gravity field are given. They can be calculated from the gravity data using, for example, maximum entropy methods (Burg, 1967). Let these values be

$$P(0), P(1), \ldots, P(n)$$

If we consider $s = 1$, the former property and (1) allow us to affirm that there exist A_1 y A_2 satisfying

$$P(0) = A_1 P(1) + A_2 P(2)$$

$$P(1) = A_1 P(2) + A_2 P(3)$$

$$P(n-2) = A_1 P(n-1) + A_2 P(n)$$

overdetermined system (n>3) that can be solved by means of least squares techniques:

$$A = (Q^T Q)^{-1} Q^T P$$

where

$$A = \begin{pmatrix} A_1 \\ A_2 \end{pmatrix}, \quad Q = \begin{pmatrix} P(1) & P(2) \\ P(2) & P(3) \\ \vdots & \vdots \\ P(n-1) & P(n) \end{pmatrix} \quad and \quad P = \begin{pmatrix} P(0) \\ P(1) \\ \vdots \\ P(n-2) \end{pmatrix}$$

3. DETERMINING THE VALUES

Following the corollary of the property, we can find B_1 and B_2 as the roots of the equation

$$A_1 Z + A_2 Z^2 = 1$$

obtaining then

$$z_0 = \frac{\log B_1}{-4\pi} \quad and \quad z_1 = \frac{\log B_2}{-4\pi}$$

Another way of obtaining these depths can be based on the fact that a linear combination of exponentials is the homogeneous solution to a constant coefficient linear difference equation (see Introcaso and Guspí, 1995).

Once these values are obtained, the masses magnitudes can be achieved by solving the linear system:

$$\boldsymbol{EC = P}$$

where

$$E = \begin{matrix} a_{01} & a_{02} \\ a_{11} & a_{12} \\ \\ a_{n1} & a_{n2} \end{matrix} \quad and \quad C = \begin{matrix} C_1 \\ C_2 \end{matrix}$$

being $a_{j} = e^{-4\pi\alpha z j}$ and $C_i = 4\pi^2 G^2 m_i^2$.

Finally:

$$m_0 = \frac{\sqrt{C_1}}{2\pi G} \quad and \quad m_1 = \frac{\sqrt{C_2}}{2\pi G}$$

4. APPLYING THE METHOD

Let us consider an East-West section located near 22° South latitude on the Argentine western (Fig. 1). From the gravity profile (Fig. 2) the power spectrum was calculated (Andersen, 1974). Prony´s method applied to these values gave the following results:

Mass 1 of 18 unities at a depth of 6 km
Mass 2 of 2800 unities at a depth of 72 km

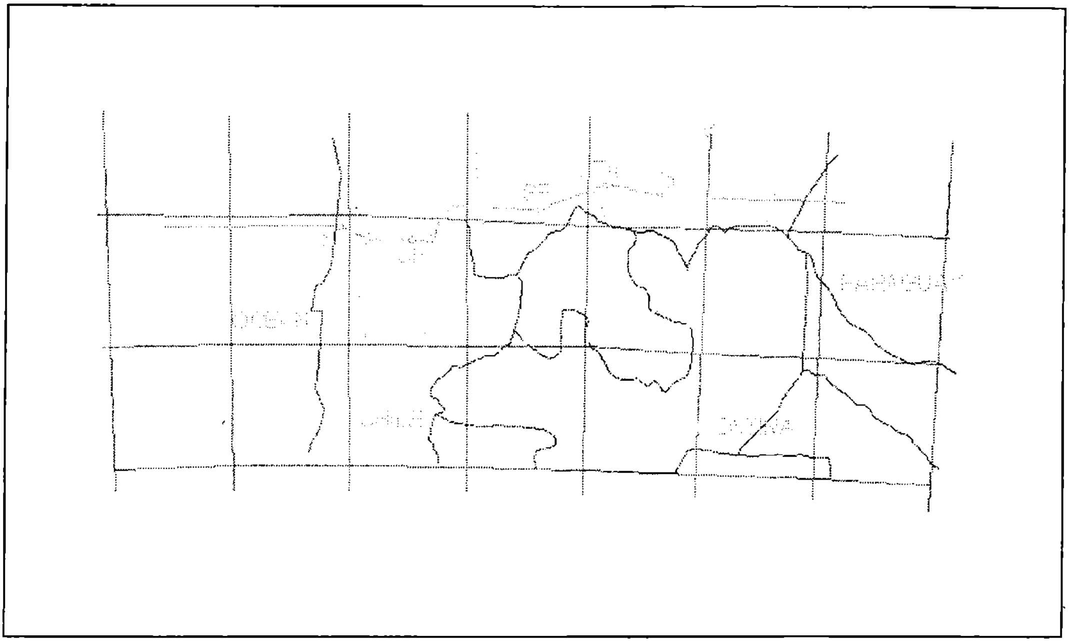

Figure 1. location of the studied section (Source: Abriata and Introcaso, 1990).

From these values and using (1) we have compared the logarithmic graphs of the original spectrum and the calculate one (Fig. 3) obtaining a reasonable agreement. The gravity model existing for the zone (Abriata and Introcaso, 1990) shows a maximum crustal depth of 66 km below the Cordillera Principal (Fig. 4). We can see that the maximum depth we have obtained using Prony´s method is fairly consistent with the maximum depth of the section. The shallow mass located at 6 km deep is so small comparing it with the deep one that can be neglected for modelling purposes.

Therefore, in this case, the method yields to acceptable results. However, it must be taken into account that the method cannot elude the potential field´s ambiguities, and so it is advisable to compare the obtained results with other data existing for the zone. Besides, spectral filtering techniques are often limited because of the Discrete Fourier Transform approximations. At this respect, some work is being carried out to improve it (i.e., Sacchi and Ulrych, 1996).

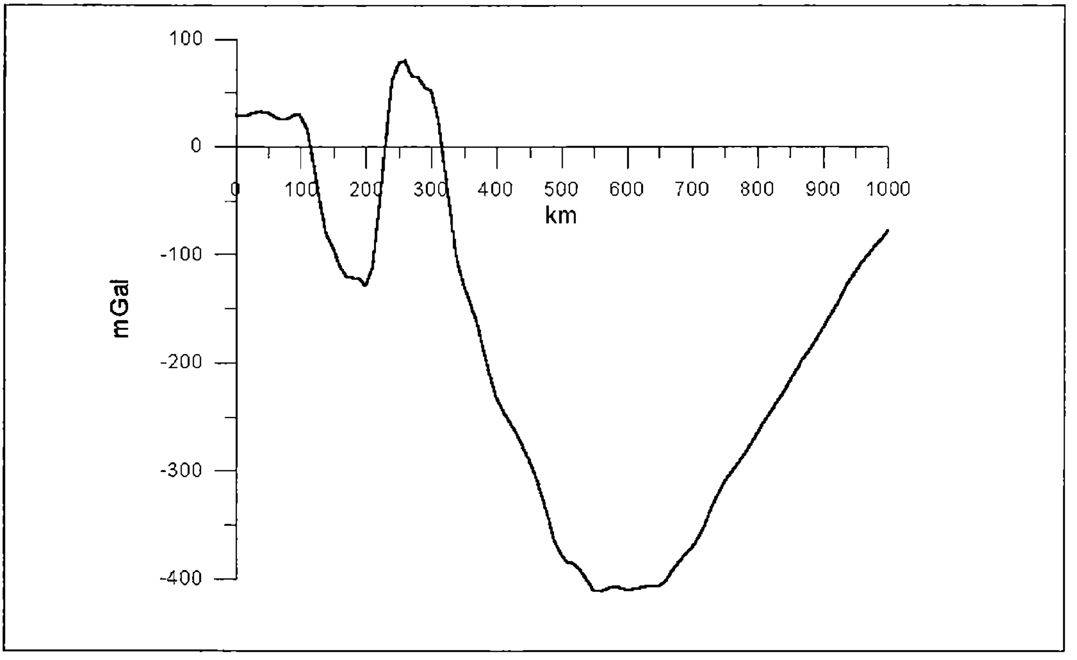

Figure 2. Gravity profile

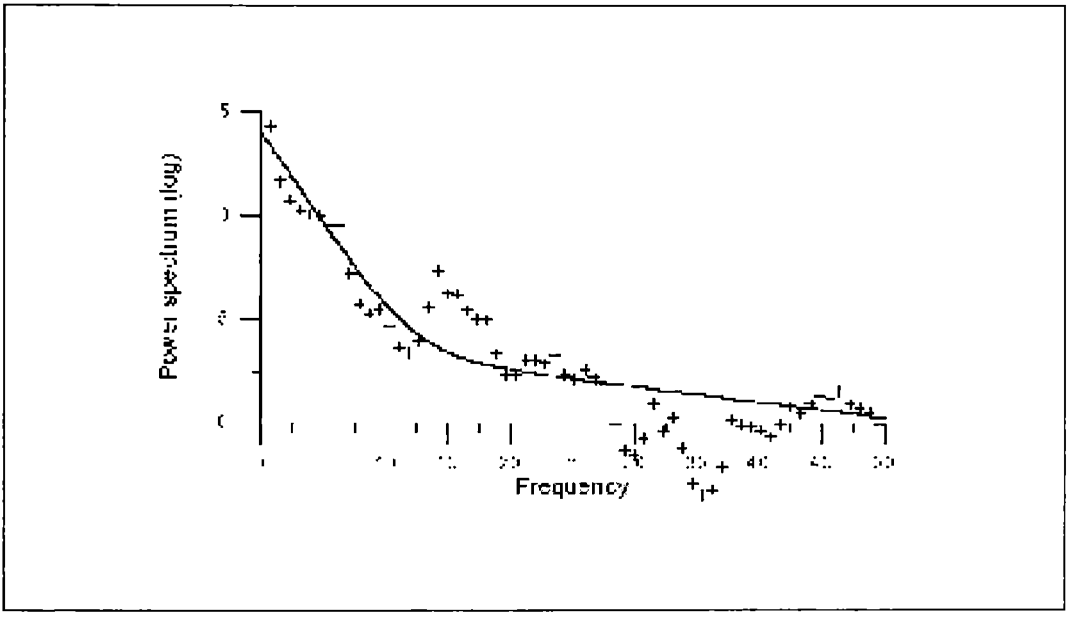

Figure 3. Logaritmic graph of the power spectrum: (a) calculated with maximum entropy method from the observed gravity data (symbols) and (b) calculated using (1) from the obtained results (solid line).

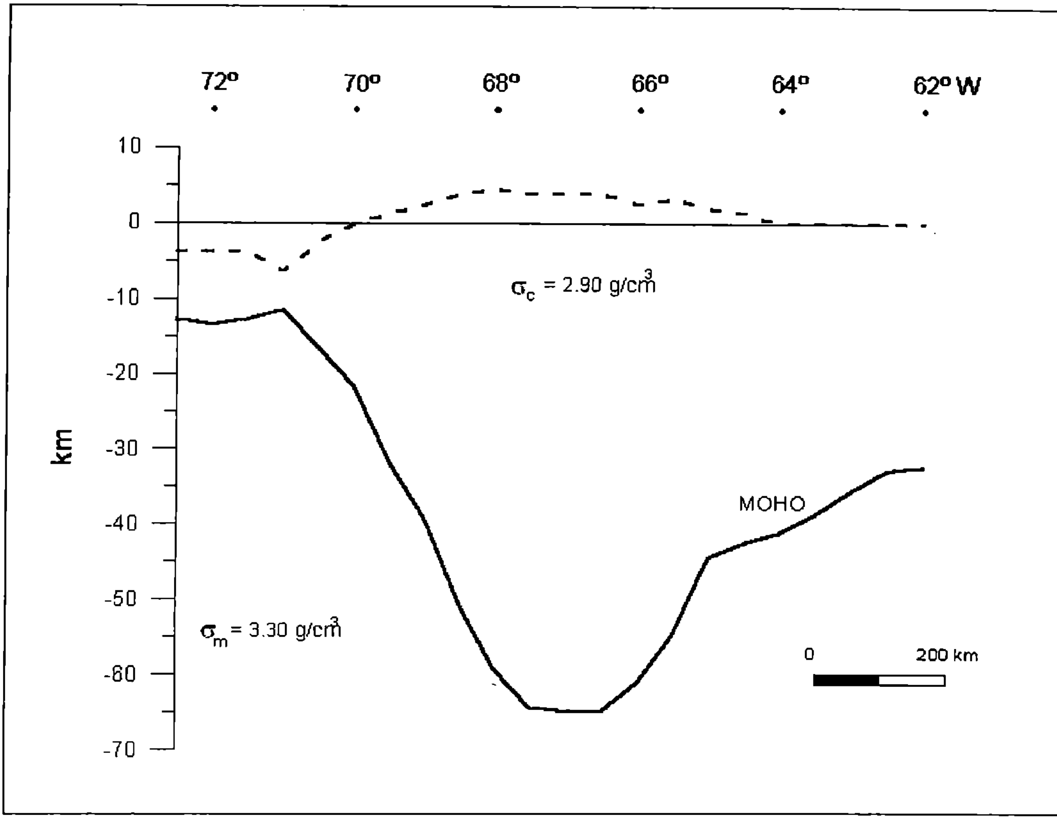

Figure 4. Crustal model obtained by inversion (Source: Abriata and Introcaso, 1990)

5. CONCLUSIONS

We have applied a method to fit the power spectrum of a gravity field on an Andean Cordillera section, in order to obtain the depths of the probable causative masses. The results obtained for this case are satisfactory. The depth of the bigger mass is consistent with the maximum crustal depth found from the gravimetrical model existing on the zone. However, it must be taken into account that the model assumed in the Prony´s method is a set of exponentials of arbitrary amplitude, phase and frequency. Besides, noise affects the accuracy of the Prony estimations greatly in some situations (Van Blaricum and Mittra, 1978). The principal advantage of the method lies in its easy automatization, that makes possible to interact with gravity application programs and allows the rapid visualizing of the results.

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