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Evolution of DB white dwarfs in the Canuto and Mazzitelli theory of convection

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ABSTRACT

We compute the evolution of DB (helium-rich envelope) white dwarf models with masses between 0.4 and 1 $M_\odot$ and metallicities of $Z = 0.001$ and 0.004, taking into account Canuto & Mazzitelli's new theory of convection. This theory, which *has no free parameters* and includes the full spectrum of turbulent eddies, has been successfully tested in different stellar objects and represents a substantial improvement compared with the classical mixing length theory used in most white dwarf studies. Using thermal time-scales we find that, for the range of masses and metallicities assumed in this study, the Canuto & Mazzitelli theory yields theoretical blue edges between 24 200 and 25 600 K, which is in good agreement with observations of pulsating DB white dwarfs. Calculations are performed considering the mixing length theory as well. In this context, our results are consistent with previous computations.

Key words: convection – stars: evolution – white dwarfs.

1 INTRODUCTION

For a long time, the evolution of white dwarf (WD) stars has captured the attention of many researchers. In fact – crystallization, latent heat release, electrostatic and quantum corrections to the ideal equation of state, partial ionization, and convection, among other physical phenomena – have been explored in great detail over the last decade (see Koester & Chanmugam 1990 and D'Antona & Mazzitelli 1990 for reviews). In particular, the theory of convection is perhaps the most poorly understood of the cited physical phenomena. In recent years, efforts have been focused mainly on pulsational properties, which have made it possible to achieve a deeper knowledge of the outer layer structure of WDs (see e.g. Tassoul, Fontaine & Winget 1990, TFW). It is well known that after the temperature drops below the limit for recombination of the main outer layer constituents (H in DA or He in DB WDs, respectively), the radiative opacity increases substantially, leading to convective instabilities. The role played by convection in pulsating WDs as well as in their spectral evolution (through mixing episodes) has been repeatedly emphasized in a great number of investigations, such as Winget et al. (1982), Winget et al. (1983), TFW, D'Antona & Mazzitelli (1990), Fontaine & Wesemael (1991), Bradley & Winget (1994). In particular, as WDs move through instability strips, the location of the bottom (in Lagrangian coordinates) of the outer convective zone (OCZ) can differ by several orders of magnitude according to the assumed convective efficiency. Finally, the match to the observed blue edge of instability strips has allowed a calibration of the mixing length theory (MLT) in WD envelopes. Such a match requires a moderately high convective efficiency, as claimed by several authors (see TFW; Wesemael et al. 1991; Bradley & Winget 1994; Koester, Allard & Vauclair 1994). In the frame of the MLT (Böhm-Vitense 1958), the convective flux is assumed to be transported by single size, large eddies, which after travelling – on average – a distance l (the mixing length), break up, releasing their energy excess into the surrounding medium. This formulation involves three length parameters, which are usually reduced to a single one: the distance l, which is parameterized as a fraction of the local pressure scaleheight: $l = \alpha H_P$ where the free parameter α is commonly fixed from adjustments to the solar radius. The MLT is not a completely satisfactory approach to describe the actual process of convective transport in stellar environments. However, in view of the lack of an alternative formulation of convection that can be easily implemented in evolutionary codes, the MLT in its several versions has been employed so far in almost all WD calculations.

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Canuto & Mazzitelli (1991, 1992) (CM) have recently developed a new theory for turbulent convection (CMT) in stars, which constitutes a considerable improvement with respect to the MLT. The CMT takes the full spectrum of turbulent eddies into account by using modern theories of turbulence, avoiding the single large eddy approximation of the MLT. The much wider range of eddy sizes necessary in order to describe a nearly inviscid stellar fluid leads, at high convective efficiencies, to a turbulent flux up to ten times larger than the MLT one. CM fit their values for the new convective flux (proportional to Φ) as $\Phi = a_1 \Sigma^m [(1 + a_2 \Sigma)^n - 1]^p$, where Σ is a measure of the convective efficiency and $a_1 = 24.868$, $a_2 = 9.7666 \times 10^{-2}$, $m = 0.14972$, $n = 0.18931$, and $p = 1.8503$; whereas in the MLT the coefficients are $a_1 = 9/8$, $a_2 = 1$, $m = -1$, $n = 1/2$, and $p = 3$ (see CM for further details). Another improvement of the CMT is the absence of free parameters. In particular the mixing length is calculated as $l = z$, where z is the distance from the top of the convective zone to the point where the overadiabatic gradient is computed.

The CMT has been successfully tested in different stellar contexts, see e.g. D'Antona, Mazzitelli & Gratton (1992), D'Antona & Mazzitelli (1994), Stothers & Chin (1995), Paternò et al. (1993). Concerning WDs, in a preliminary work, Mazzitelli & D'Antona (1991) applied the CMT to study the OCZ of a typical 0.55 $M_\odot$ DB WD, finding a value of $T_{\rm eff} \approx 25\,000$ K for the blue edge of the DB instability strip.

In this paper, we present the first results of the complete evolution of carbon–oxygen DB WD models with different masses (0.40, 0.55, 0.80 and 1.0 $M_\odot$) in the CMT self-consistently. For comparison, the same sequences were computed using the MLT formalism as well. Specifically, we employed the three different versions of MLT discussed in TFW: ML1 corresponding to the standard version of Böhm-Vitense (1958) with $\alpha = 1$, ML2 with $\alpha = 1$ but more efficient than ML1, and ML3 being the same as ML2 but with $\alpha = 2$, which are all widely used in WD model calculations.

2 COMPUTATIONAL DETAILS AND INPUT PHYSICS

Our code has been written following Kippenhahn, Weigert & Hofmeister (1967). The atmosphere has been integrated in Eddington's grey approximation. In radiative equilibrium, it was integrated up to $\tau = 2/3$, otherwise we extended the integration up to the point where convection set in. From there on, we integrated the full stellar structure equations at constant luminosity (envelope integration) inwards, up to a fitting mass fraction $q_{\rm F}$ corresponding to the first Henyey shell. At the outset of each evolutionary sequence, we took $\log q_{\rm F} \approx -15$, which is small enough to yield a self-consistent description of the outer layers. Finally, the interior integration was treated according to the standard Henyey technique.

Our stellar models were divided into ~2000 mesh points, most of them distributed in the outer layers. Such thin zoning is necessary since in the CMT, l is measured from the top of the OCZ (which may be located in the atmosphere itself). In fact, as the overadiabatic gradient is very sensitive to z, it must be computed very accurately. In particular, a slight difference between r at the bottom of the envelope and that corresponding to $q_{\rm F}$ would lead to a change of z of several orders of magnitude (because $z \ll r$ in all the OCZ). This would abruptly modify the nature of the convective transport solution in those cases in which convection, at the bottom of the envelope, was far from being adiabatic. The size of the triangles in the HR diagram was taken to be small enough ($\Delta \log T_{\rm eff} = 2 \times 10^{-4}$, $\Delta \log L = 10^{-3}$) to overcome such a difficulty. Likewise, in each iteration, the location of the top of the OCZ is obtained through the usual interpolation in Kippenhahn et al.'s triangles. Of course such treatment is necessary because, in our code, $q_{\rm F}$ is not coincident with the photosphere ($q_{\tau=2/3}$).

Our starting models were obtained by the procedure employed in Benvenuto & Althaus (1995), and have the chemical composition profile given in fig. 1 of that work, i.e. with a helium-layer mass of $10^{-2}\,M_*$.

With regard to input physics, for densities $\rho \leq 2 \times 10^3$ g cm^{-3} we employed an updated version (Mazzitelli 1993, private communication) of the equation of state of Magni & Mazzitelli (1979). Higher density layers and neutrino emission were treated as in Benvenuto & Althaus (1995). We used new OPAL radiative opacities provided to us by Professor Forrest Rogers (1994, private communication) for $Z = 0.001$ and 0.004. Such tables cover widely the temperature–density regime that characterizes the outer layer structure of DB WD models at their pulsational stages. Conductive opacities were taken from Itoh et al. (1983) for the liquid phase and from Itoh et al. (1984) and Itoh & Kohyama (1993) for the solid phase. For the low-density regime, we employed the expressions given by Fontaine & Van Horn (1976). Also, we assumed the Schwarzschild criterion for the occurrence of convection.

3 ANALYSIS OF THE EVOLUTIONARY RESULTS

The main results of this work are presented in Figs 1, 2, 3 and 4. Since different values of the helium-layer thickness would barely modify the pulsation-related results presented below (Bradley & Winget 1994), we have not studied the effect of changing its quoted thickness. In each figure, we show the evolving structure of the OCZs and the evolution of the thermal time-scale $\tau_{\rm TH}$ for the various versions of convection and metallicities that we considered. The values of $\tau_{\rm TH}$ have been computed by means of the equation (see e.g. Kawaler 1993)

$$\tau_{\rm TH} = \int_0^{q_{\rm bc}} \frac{C_{\rm V} T}{L} M_* \, {\rm d}q, \qquad (1)$$

where $C_{\rm V}$ is the specific heat at constant volume, $q = 1 - M_r / M_*$, and M_* is the mass of the model. The subscript bc refers to the bottom of the OCZ.

As found in previous investigations (see TFW), at high $T_{\rm eff}$, the extent of the OCZ is strongly sensitive to the treatment of convection, but it is not as sensitive to the metallicity. In Table 1 we present the location of the blue edge of the DB instability strip according to our calculations. The temperature of the theoretical blue edge is estimated by calculating the $T_{\rm eff}$ at which $\tau_{\rm TH} = 100$ s (see e.g. TFW, Winget et al.

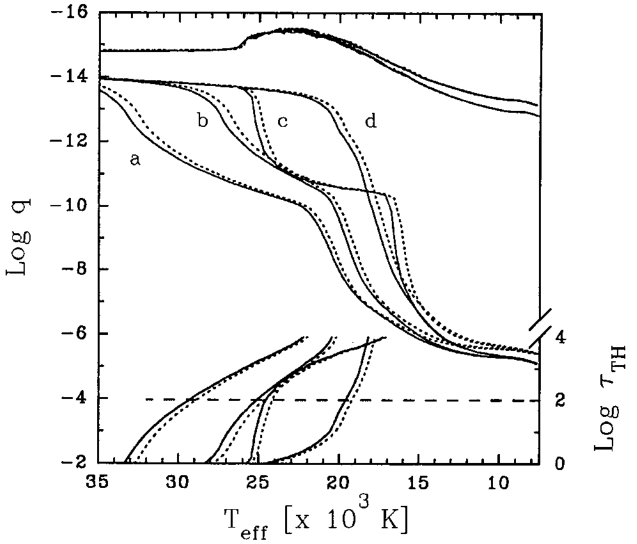

Figure 1. The size of the OCZ and the thermal time-scale (in s) for a 0.4-$M_\odot$ DB WD in different theories of convection as a function of the effective temperature is shown. q is the mass fraction (see text for definition). a, b, c and d stand for the cases of ML3, ML2, CMT and ML1 convection, respectively. The top of the convective zone almost coincides with the photosphere. Solid and short-dashed lines stand for the cases of $Z=0.001$ and 0.004, respectively. The scale for the thermal time-scale is given on the right-hand axis.

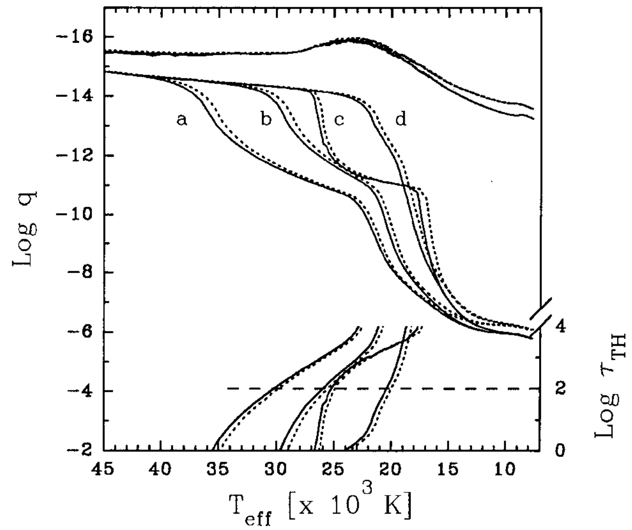

Figure 2. Same as in Fig. 1, but for a 0.55-$M_\odot$ DB WD.

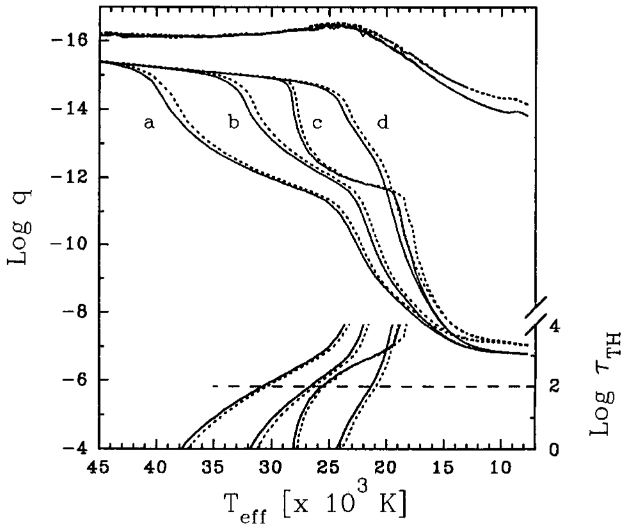

Figure 3. Same as in Fig. 1, but for a 0.80-$M_\odot$ DB WD.

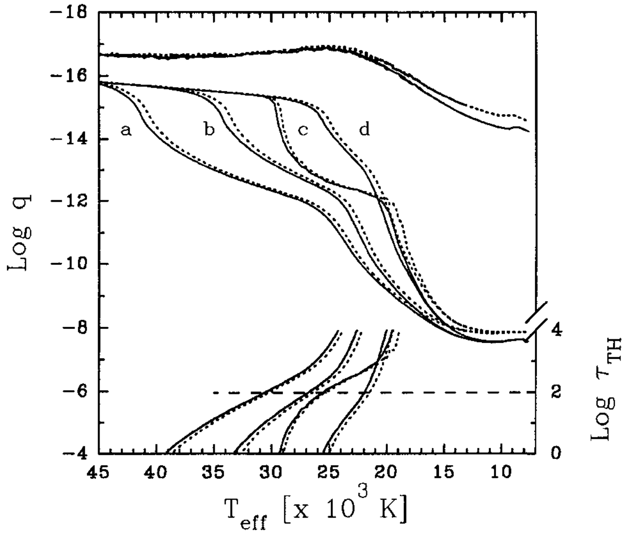

Figure 4. Same as in Fig. 1, but for a 1.0-$M_\odot$ DB WD.

1983). Particularly, with the CMT, we obtain DB blue edges in a narrow range between 24 200 and 25 600 K for the stellar masses and metallicities considered here. This is in good agreement with the observations of Thejll, Vennes & Shipman (1991) who placed an upper temperature limit on the observed blue edge at, or near, 25 000 K. Although the theoretical blue edges given by the CMT resemble those given by the ML2, it is obvious that the structure of the evolving OCZs is markedly different in both theories.

In Fig. 5, we show the dependence of the location of our blue edges on the stellar mass. We also added data available from TFW, Kawaler (1993), and Bradley & Winget (1994). In these works several versions of the MLT, both with and without performing the limiting procedure of Böhm & Stückl (1967) (BS and NBS respectively, see e.g. TFW) were employed. This procedure has not been taken into account in our computations. In the framework of the MLT (NBS), our blue edges are systematically hotter than those found by other researchers, particularly for efficient convection. These discrepancies are due essentially to our employment of the new OPAL radiative opacities. It is worth noting that, for the 0.4 $M_\odot$ CMT, we found a blue edge that was much hotter than the observed red edge located near 21 500 K (Thejll et al. 1991). Thus, in the CMT, it is no longer possible to assign a low mass for any non-variable DB object within the instability strip as suggested by Bradley & Winget (1994).

As mentioned previously, the CMT has been successfully tested in different stellar objects and evolutionary phases. The aim of this paper has been to perform a further test of

Table 1. Theoretical blue-edge temperatures for DB WDs.

Theory of Convection	Mass ($M/M_{\odot}$)	$T_{eff}(K)$ ($Z = 0.001$)	$T_{eff}(K)$ ($Z = 0.004$)
CM	0.40	24,560	24,160
ML1	0.40	19,570	19,190
ML2	0.40	25,130	24,690
ML3	0.40	29,330	28,890
CM	0.55	25,200	24,910
ML1	0.55	20,400	20,020
ML2	0.55	25,980	25,570
ML3	0.55	30,250	29,820
CM	0.80	25,620	25,400
ML1	0.80	21,330	20,920
ML2	0.80	26,670	26,280
ML3	0.80	30,740	30,370
CM	1.00	25,550	25,370
ML1	1.00	21,960	21,510
ML2	1.00	26,890	26,460
ML3	1.00	30,650	30,290

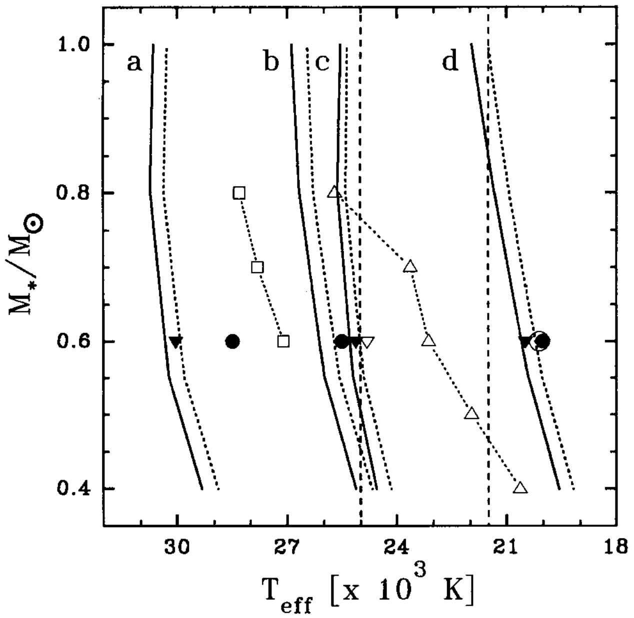

Figure 5. The stellar mass versus effective temperature relationship for the blue edge of DB WD models. The letters, solid- and short-dashed lines have the same meaning as in Fig. 1. The vertical dashed lines are the limits of the observed DB instability strip. From left to right, the filled circles represent data from Kawaler (1993) (BS) and filled triangles from TFW (NBS) for ML3, ML2, and ML1 respectively. Hollowed symbols are for data from Bradley & Winget (1994). Squares, 'down' triangles, 'up' triangles, and circles standard for ML3 (NBS), ML2 (NBS), ML2 (BS) and ML1 (BS) respectively.

this model in the context of WD stars. The agreement with observations of pulsating DB WDs as suggested by our results, indicate that the CMT can be regarded as being a very valuable tool in modelling convection also in the high-density, low-temperature regime that characterizes the outer layers of WD stars. We shall continue exploring the consequences of the CMT in WD objects in a more general context (mainly DA WD) in future works.

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