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Radial pulsations of strange stars and the internal composition of pulsars

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SUMMARY

We present calculations of radial oscillations of homogeneous strange stars, showing that the particular form of the equation of state allows some simple and general scaling relations which may prove to be very useful for the search of these objects.

1 INTRODUCTION

Presently, the very nature of matter at supranuclear densities is a subject of controversy. While the dominant opinion is that the lowest energy state of hadronic matter is a collection of nucleons (see for example, Lattimer 1981 and references therein), it may well be that we have overlooked the existence of a still lower energy state currently known as strange matter (Witten 1984; Farhi & Jaffe 1984). Astrophysical implications of this hypothesis have been recently addressed by a number of authors (see the review articles by Alcock & Olinto 1989 and Benvenuto, Horvath & Vucetich 1990 for extensive references). Particularly, the structure of compact objects composed of this matter (strange stars) has been shown to be remarkably similar to the structure of neutron star models in the range $M \geq 1\ M_{\odot}$. Thus, even if the strange matter hypothesis implies that neutron stars should convert to this state in a characteristic (and unknown) time-scale τ_0, the actual way of distinguishing both types of stars by performing suitable observations is not obvious.

If $\tau_0 < H^{-1}$, which would be a physically interesting case, then the actual value of τ_0 could be inferred by counting the relative populations of normal (neutron) and strange stars in a given sample (Alpar 1987). However, it may be that $\tau_0 \ll \tau_p$ (where τ_p is the typical age of a pulsar) is a feature of models where the conversion occurs inside the Kelvin–Helmholtz period (~ seconds). As a consequence of the latter, strange stars should be the debris of supernova explosions and therefore should be identified with pulsars, which would contain ordinary hadronic matter in tiny amounts, if at all. Thus, the radical nature of the strange matter hypothesis calls for extensive studies of strange star properties in order to extract observational signatures which can distinguish both types of objects. This point is of great importance to elucidate the question of the actual composition of pulsars.

It is generally believed that, among other potentially useful properties, the knowledge of the oscillation spectra could provide a powerful insight to the equation of state, and therefore to the nature of the constituent matter. This kind of study has been presented for neutron star radial oscillations in Lindblom & Detweiler (1983) where a wide set (collected by Arnett & Bowers 1979) of equations of state were employed to extract the oscillatory features of those stars. We shall present in this work a similar analysis corresponding to the homogeneous strange star models. The general features and numerical results for radial pulsations are addressed in Section 2. Section 3 is devoted to a discussion of the physical conditions for excitation and damping of these modes, as well as their possible observational signatures.

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2 THE STRANGE MATTER EQUATION OF STATE AND THE OSCILLATION SPECTRUM OF STRANGE STARS

The strange matter equation of state (hereafter EOS) in the MIT bag model approach has been discussed elsewhere (see e.g. Freedman & McLerran 1978). This model contains three parameters, the energy density of the perturbative QCD vacuum represented by the bag constant B, the strength of the perturbative strong interactions measured by the 'strong fine structure constant' α_c, and the mass of the strange quark M_s. For our purposes it will be convenient to employ the parametric form of the EOS given in Benvenuto & Horvath (1989)

$$P = \frac{1}{3+a}[\rho - (4+b)B], \tag{1}$$

where a and b are complicated functions of M_s and α_c. We shall consider a fixed at some constant value a_0 in the remainder of this work because it is known (Benvenuto & Horvath 1989) to be a small correction to the ultrarelativistic behaviour (introducing negligible error in our calculations) of the EOS throughout the window for strange matter stability. Thus we will only focus on the important effects of the second term in brackets which determines the degree of selfbinding of the state (Fahri & Jaffe 1984). Even if a is in fact also weakly dependent on the density ρ, a constant value a_0 will be very useful to illustrate the behaviour of the strange matter EOS in relation to the oscillation problem.

It has been previously noted (Witten 1984) that the equilibrium stellar structure variables admit a scaling transformation:

$$P_c' = a^2 P_c; \qquad \rho_c' = a^2 \rho_c; \qquad R' = R/a;$$
$$M' = M/a; \qquad Z' = Z, \tag{2}$$

where P_c and ρ_c are the central pressure and density respectively, R the stellar radius, M the stellar mass and Z the surface redshift. Inserting the EOS equation (1) in the relativistic stellar structure equations it can be shown that the scaling factor a relating two models with different values b, B and b', B' is given by

$$a = \left[\frac{(4+b')B'}{(4+b)B}\right]^{1/2}. \tag{3}$$

If no parametrization for the EOS is used, it can be shown that the scaling law still holds, although no simple expression for a can be found in this general case.

The equations of motion of linear, adiabatic radial perturbations have been presented in Bardeen, Thorne & Meltzer (1966) and Meltzer & Thorne (1966). Using their equations and the scaling relationships equation (2) it can be further checked that the coefficients of the Schwarzschild metric are invariants under the same transformation, and therefore the radial oscillation eigenfrequencies scale as

$$\omega_n' = a\omega_n. \tag{4}$$

While, generally speaking, the features of a strange star model depend on the strength of the confinement forces through the parameter B (bag constant), the above relation equation (4) allows us to assert that the quotient of two radial eigenfrequencies is independent of B. Thus, following Glass & Lindblom (1983), we can construct an universal plot $\omega_n/\omega_{n=0}$ versus Z with n the number of nodes of the corresponding eigenfunction. There is no analogue of this plot for conventional neutron models due to the form of the nuclear matter EOS's, although their plots $\omega_n/\omega_{n=0}$ versus Z also show a fairly regular behaviour. Thus, even if no scaling can be found for EOS's which are not linear in ρ, this feature is not different enough so as to envisage a clear observational signature for the near future. This situation is illustrated in Fig. 1. Figs 2 and 3 display the behaviour of the periods T_0 and T_1 (corresponding to the eigenfunctions having no nodes and one node respectively) depicted as functions of the redshift Z for three representative cases of the strange matter EOS that cover the extremes of the parameter space (Fahri & Jaffe 1984; Benvenuto & Horvath 1989). These correspond to the EOS's labelled as A, C and D in Benvenuto & Horvath (1989) and represent different choices of the parameter a_0. Any other model can be calculated by simply applying the scaling relationship equation (4).

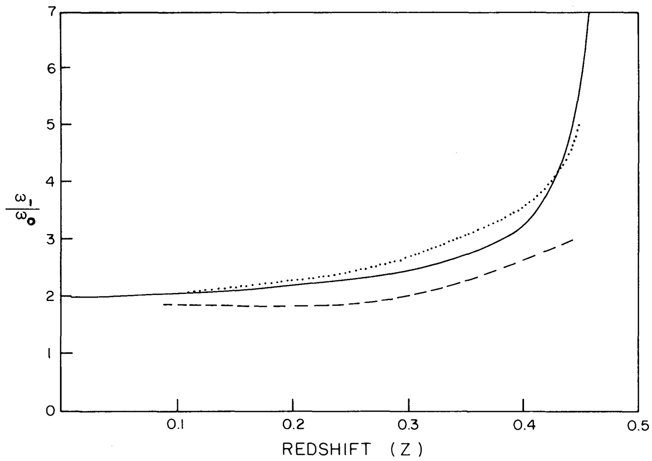

Figure 1. The quotient of the first two radial eigenfrequencies ω_1/ω_0 versus surface redshift Z for strange stars (solid curve). Two curves corresponding to the Arnett & Bowers (1977) collection, EOS B (dashed) and EOS C (dotted) are shown for comparison. For the sake of clarity the curves for $\omega_{n\geq 1}/\omega_0$ are not shown, although they are found to repeat the ω_1/ω_0 pattern moving upwards with n.

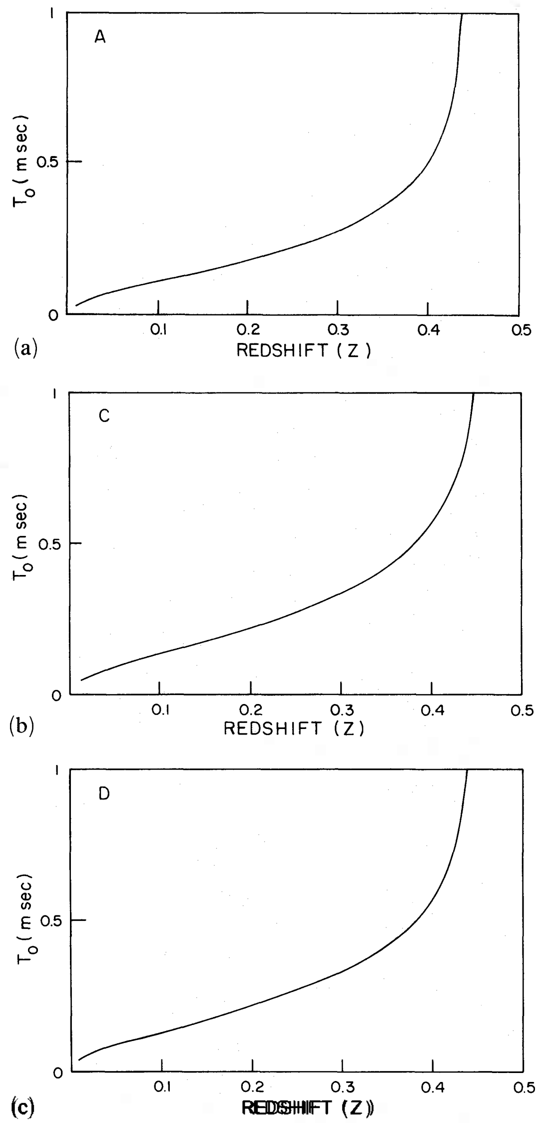

Figure 2. (a) The fundamental ($n=0$) mode period T_0 versus surface redshift Z corresponding to the sequence A in Benvenuto & Horvath (1989); (b) the same as (a) for the sequence C; (c) the same as (a) for the sequence D.

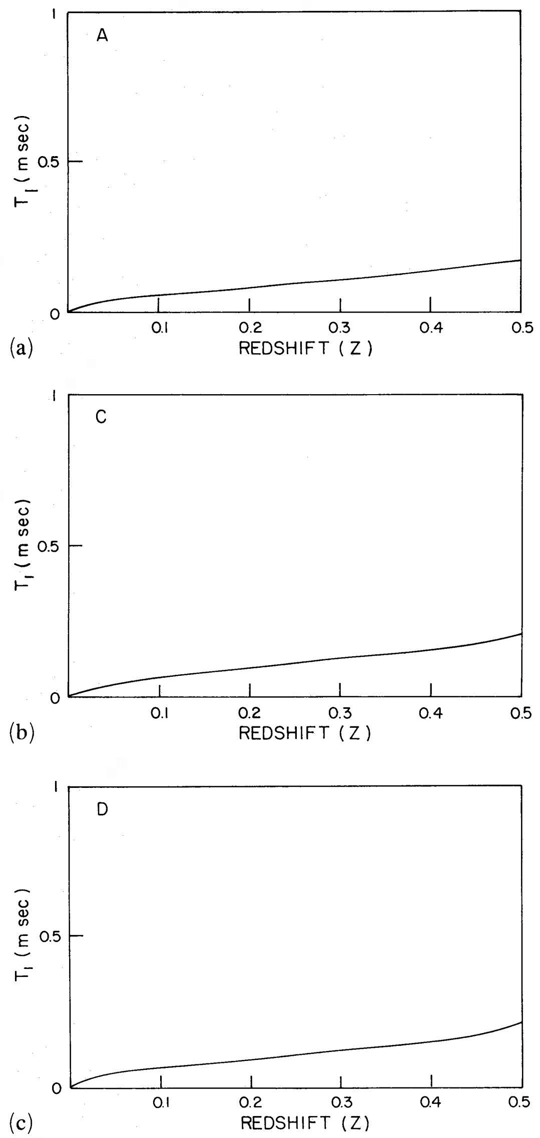

Figure 3. (a) The first overtone ($n=1$) period T_1 versus surface redshift Z corresponding to the sequence A in Benvenuto & Horvath (1989); (b) the same as (a) for the sequence C; (c) the same as (a) for the sequence D.

3 EXCITATION AND DAMPING OF THE RADIAL MODES

It is presently quite clear that any effort to detect a signal from a radially oscillating compact star left over from gravitational collapse would be hopeless for the case of quark interiors. It has been shown that radial pulsations of quark stars are quickly damped on time-scales $\tau_D < 1$ s due to the enormous bulk viscosity of the hot quark matter (Wang & Lu 1984; Sawyer 1989) which is many orders of magnitude larger than that of neutronic matter. Even if this damping mechanism were not effective, there is another important sink of pulsational energy arising from the rotation of the pulsar (Chau 1967) which drives gravitational radiation: the former destroys the spherical symmetry and thus causes a non-zero varying quadrupole moment. The damping time-scale associated with this effect is $\approx 10^3 P^4$ yr (with P the rotation period in s) and is generally much shorter than a typical pulsar lifetime τ_p. Thus, the persistence of 'primordial' radial oscillations are not to be expected.

The above considerations lead us to expect the possible relevance of radial oscillations only in the aftermath of transient events in the pulsars life. It is nowadays widely believed (although by no means proved) that transients like gamma-ray bursts originate at or near the surface of compact stars (Liang 1990). Then, if enough energy from that powering the burst could be transferred to vibrational modes we should be able to identify them in a high time-resolution gamma-ray burst. To be quantitative we have calculated the energies necessary to excite the fundamental and first harmonic radial modes for a 1.4 $M_\odot$ model belonging to the sequence A. They turned out to be $2.48 \times (\xi/R)^2 \times 10^{54}$ erg and $1.7 \times (\xi/R)^2 \times 10^{55}$ erg respectively, where ξ is the actual amplitude of the oscillation. As very small ξ's (< 1 mm) should be expected for such events (see for example the ξ values needed to fit the glitch phenomena into the starquake theory in Shapiro & Teukolsky 1983), these modes might be reasonably expected to be excited after a glitch or some other catastrophic event. However, it is worth emphasizing that the important question of how to relate the amplitude of such oscillations to an observable electromagnetic variability is still unanswered. Furthermore, we have to check that damping effects at the given epoch (mainly the bulk viscosity ones) are not so strong that they suppress the oscillatory behaviour before it can be unambiguously detected. Using the results of Sawyer (1989) we find that the requirement of a reasonably long damping time-scale, say $\tau_D \geq 1$ d, is only possible if the interior temperature satisfies $T_1 \leq 10^7$ K, which in turn implies that relatively old pulsars (e.g. PSR 0355+54) should be better candidates than the young ones (e.g. the Crab) for the proposed detection.

In the present work we have centred our attention on the radial modes because we believe they should be more easily excited than $l=1$ dipole modes. The latter share the same scaling law of the radial modes, which opens the possibility of giving a generalized universal plot like that presented in Fig. 1. Thus *all* potentially important oscillation modes could be used to distinguish strange from neutron stars. We finally stress that plots like the one in Fig. 1 are *not dependent* on the assumed strength of confinement forces (a point which is subject to important uncertainties), and only depend on the asymptotic freedom of the partons reflected in the strange matter EOS. This feature may make oscillation studies extremely useful in the search for strange stars.

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