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Current State of the Investigation of Superallowed Fermi β Decays

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Nuclear Structure: Superallowed Fermi β transitions; derivation of the effective vector (G'_V) and induced scalar (f_S) coupling constants from experimental data.

At the present time, the crucial point in a systematic study of superallowed 0^+–0^+ β transitions is the evaluation of the isospin impurity correction δ_c. In the literature, δ_c is decomposed into two parts, δ_{c1} and δ_{c2}. Several estimates of δ_{c1} have been published, while only one is available for δ_{c2}. We analyze the compatibility of the different estimates of δ_{c1} with the most recent surveys of experimental data. The simplest evaluation of δ_{c1} reported some years ago by Damgaard is found to yield the most satisfactory $\mathscr{F}t$ values; these provide reliable values of the effective vector coupling constant G'_V [e.g., $G'_V = (1.41242 \pm 0.00023) \times 10^{-49}$ erg cm^3]. These values are in excellent agreement with a recent value $G'_V = (1.41248 \pm 0.00044) \times 10^{-49}$ erg cm^3 obtained by Wilkinson on the basis of a phenomenologic approach to δ_c. Conversely, the most recent and detailed parentage-expansion approaches to δ_{c1} lead to $\mathscr{F}t$ values which increase with Z, showing pronounced slopes. This fact might be due to a relative overestimation of δ_{c1} for the lighter nuclei. Using the $\mathscr{F}t$ values calculated with δ_{c1} as reported by Damgaard, we evaluate the coupling constant for the induced scalar interaction following a procedure described in a previous paper. The mean of such values is $f_S/f_V = (-0.17 \pm 0.80) \times 10^{-3}$. In addition, we develop an alternative way of determining a limit for f_S/f_V using the phenomenological approach to δ_c suggested by Wilkinson. This new procedure yields $f_S/f_V = (-0.16 \pm 0.87) \times 10^{-3}$, a result which is in excellent agreement with that obtained using the former method; both values are consistent with a value of zero, supporting the conserved vector current theory. The better accuracy of the experimental data makes it possible to reduce by a factor of two the limit established in a previous work.

1. Introduction

The main effort in a standard study of superallowed 0^+–0^+ β transitions is focused on properties which can lead to a test of the conserved vector current (CVC) theory and the Cabbibo universality hypothesis. In relation to 0^+–0^+ Fermi decays, the CVC theory predicts that [1–6]: (a) the vector coupling constant of the weak interaction, G_V ($= f_V G_\beta$, where f_V is the vector coupling constant and G_β is the weak coupling constant for β decay), should not be renormalized, and thus should be constant from nucleus to nucleus; and (b) the coupling constant of the induced scalar (IS) interaction, f_S, should be zero.

Since the three independent comprehensive papers [2–4] on 0^+–0^+ β transitions were published in late 1975, new experimental improvements have enabled the experimental errors of the relevant measurable magnitudes to be significantly reduced. In other

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words, the half-lives t_0 and the maximum energies of the positron spectra, W_0, can now be measured with increased precision, making it possible to calculate more precisely the quantities involved in the evaluation of the $f't$ values, namely,

$$f' = \bar{f}\,\overline{C_\beta(W)}[1+\overline{\delta_R(W)}], \tag{1}$$

and

$$t = \frac{t_0}{\mathrm{BR}}\left(1+\frac{\varepsilon}{\beta^+}\right). \tag{2}$$

Here, $\bar{f}$ is the integrated statistical rate function defined as in [4], $\overline{C_\beta(W)}$ is the shape factor averaged over the energies W of the positron spectrum, $\overline{\delta_R(W)}$ is the "outer" model-independent radiative correction which includes corrections of order α, $Z\alpha^2$, and $Z^2\alpha^3$ (α being the fine structure constant, and Z the nuclear charge of the daughter nucleus), BR stands for the branching of the superallowed β transition, and ε/β^+ indicates the electron-capture-to-positron-decay ratio.

Let us now summarize the relevant formulas for the superallowed Fermi β transitions. The $f't$ values obey the equation

$$f't = \frac{K}{G_\beta'^2({}^{\mathrm{V}}F^0_{000})^2}, \tag{3}$$

with

$$G'_\beta = G_\beta(1+\Delta_R)^{1/2}, \tag{4}$$

where K is a combination of physical constants (see, e.g., [4]) which reduces to $K=1.230618 \times 10^{-94}\,\mathrm{erg}^2\,\mathrm{cm}^6\,\mathrm{s}$ in cgs units, ${}^{\mathrm{V}}F^0_{000}$ is the Fermi form-factor coefficient (FFC), and Δ_R is the "inner" model-dependent radiative correction. The FFCs are given in terms of the nuclear matrix elements (NMEs) in Table 6 of [7]. Using that formula for the case of β^+ transitions, we arrive at

$${}^{\mathrm{V}}F^0_{000} = f_V\,{}^{\mathrm{V}}\mathcal{M}^0_{000} + f_S \int \left[W_0 - \frac{\alpha|Z|}{R}U(r)\right]\beta\,T_{000}, \tag{5}$$

where ${}^{\mathrm{V}}\mathcal{M}^0_{000}$ is the Fermi NME, $U(r)$ is the potential of the nuclear charge distribution, and R is the nuclear radius. It has been shown [5, 6] that keeping only the large components and assuming, as usual a uniform charge distribution, the Fermi FFC can be reduced to

$${}^{\mathrm{V}}F^0_{000} = f_V\,{}^{\mathrm{V}}\mathcal{M}^0_{000}[1+(f_S/f_V)\,\Delta_{IS}], \tag{6}$$

with

$$\Delta_{IS} = -W_0 + \frac{6}{5}\frac{\alpha|Z|}{R}. \tag{7}$$

Finally, substituting the expression (6) in (3), we obtain

$$f't = \frac{K}{G_V'^2({}^{\mathrm{V}}\mathcal{M}^0_{000})^2[1+(f_S/f_V)\,\Delta_{IS}]^2}, \tag{8}$$

with

$$G'_V = f_V\,G'_\beta = f_V\,G_\beta(1+\Delta_R)^{1/2}. \tag{9}$$

The evaluation of the Fermi NME remains as the most outstanding theoretical problem. The superallowed Fermi process involves component states of the $T=1$ isospin multiplet. Assuming that both initial and final states are pure isospin states, ${}^{\mathrm{V}}\mathcal{M}^0_{000}$ is equal to $\sqrt{2}$. However such an assumption is no longer valid when one includes Coulomb or charge-dependent short-range forces in the nuclear Hamiltonian. Then a small but important breaking of the SU(2) nuclear symmetry occurs. Therefore in the literature the squared Fermi NME is currently written as

$$({}^{\mathrm{V}}\mathcal{M}^0_{000})^2 = 2(1-\delta_c), \tag{10}$$

where δ_c accounts for the mismatch of the nuclear states involved. Introducing this expression in (8), we obtain

$$f't = \frac{K}{2G_V'^2(1-\delta_c)[1+(f_S/f_V)\,\Delta_{IS}]^2}. \tag{11}$$

From the beginning of the systematic studies of 0^+–0^+ superallowed β transitions, several authors have evaluated δ_c, obtaining results which were not completely consistent with one another. Nevertheless such uncertainties in δ_c were negligible compared with the relative errors of the experimental $f't$ values. However, nowadays the $f't$ values can be determined with enough precision to make accurate knowledge of δ_c crucial. Several efforts have therefore recently been devoted to evaluating δ_c. In Sect. 2 we discuss the available estimates of δ_c. In Sect. 3 we revise the methods in the light of the most up-to-date surveys of experimental data to obtain (a) a value for G'_V, and (b) a limit for the IS coupling constant f_S.

2. The Isospin Impurity Correction δ_c

The quantity δ_c is nuclear-model dependent and therefore somewhat uncertain. A summary of calculations performed until 1971 can be found in the review articles by Blin-Stoyle [1] and Behrens [8]. Here we concentrate mainly on subsequent work. In the recent literature one can find two different ways of tackling the problem of the evaluation of the isospin impurity correction. One is a microscopic approach, where the charge-dependent effects are taken into account be direct computation. The second procedure is a phenomenological one, based on a general theorem on renormalization of coupling constants. We outline both these estimates in this section.

2.1. Microscopic Calculations of δ_c

For the numerical computation, δ_c is broken into two parts: one, δ_{c1}, is due to small differences in the single-particle neutron and proton radial wave functions which cause the radial overlap integral of the parent and daughter nucleus to be smaller than unity; the other, δ_{c2}, arises from charge-dependent configuration mixing with other 0^+ states. Although these two aspects cannot be separated completely, such a division has been made in all the available calculations.
Several evaluations of δ_{c1} have been reported. They can be grouped into two approaches:

Approach A: This consists in computing the effect of the one-body Coulomb force in admixing, to the states in question, other states of the same or different isospin. Four estimates of this kind are available, published by Damgaard [9], Fayans [10], Lane and Mekjian [11], and Towner, Hardy, and Harvey [12]. We shall not describe those calculations here; for more details, the reader is derived to the original papers. However we shall point out that in all four calculations, the radial integrals were carried out using harmonic-oscillator wave functions. Instead of tabulating the results of [9–12], we prefer to plot the values of δ_{c1} as a function of Z: see Fig. 1a. This figure shows a definite trend of δ_{c1} with Z, which is similar for all the calculations. Starting from few hundredths of a percent for ^{14}O, δ_{c1} increases regularly up to few tenths of percent (0.34%–0.56%) for ^{54}Co. Since δ_{c1} is due mainly to the one-body Coulomb potential, it is reasonable to expect on general grounds that it should increase with Z, as is observed in Fig. 1a. It can be seen that for all Z, the estimates by Damgaard [9] are larger than the others. It is important to point out that the authors of [12] have followed the procedure of Damgaard [9] introducing some modifications: (a) using the radius R of the uniformly charged sphere and the harmonic-oscillator parameter $\hbar\omega$ appropriate for each nucleus, instead of adopting the standard values $R = 1.2\,A^{1/3}$ fm and $\hbar\omega = 41\,A^{-1/3}$ MeV; and (b) including higher-order perturbation theory. The results obtained in both [9] and [12] agree for ^{14}O. For larger Z, the estimates by Towner et al. [12] are smaller

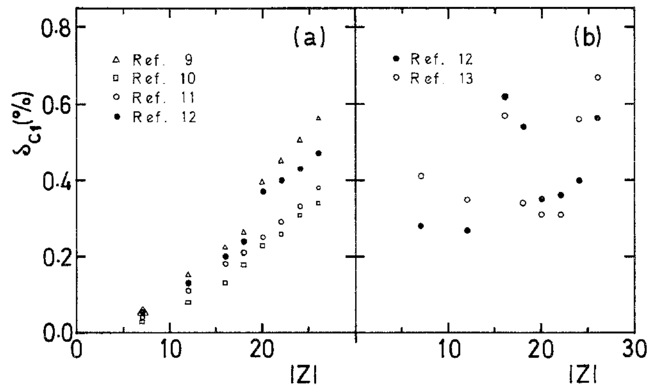

Fig. 1. The available estimates of the isospin impurity correction δ_{c1} as a function of the nuclear charge Z of the daughter nuclei

than those by Damgaard [9]; the difference increases with Z, and reaches 16% for ^{54}Co. However, the value $\delta_{c1} = 0.47\%$ for ^{54}Co reported in [12] is still 24% larger than the value $\delta_{c1} = 0.38\%$ obtained by Lane and Mekjian [11].

Approach B. In this case the overlap is calculated using wave functions generated in a standard Woods–Saxon potential including the Coulomb term for protons. Such single-particle wave functions account directly for the different binding energies of the proton and neutron involved by the β process. Configuration mixing has been taken into account by summing over the full parentage spectra in the $A-1$ nuclei. There are two calculations along these lines, those of Towner, Hardy, and Harvey [12], and those of Wilkinson [13]. The results are displayed in Fig. 1b. Contrary to the feature found in Fig. 1a, no general trend can be traced in this case because of a rather random distribution of δ_{c1} versus Z.

For δ_{c2}, on the other hand, at present, the only values available are those obtained by Towner and Hardy [14] from calculations with two-body Coulomb forces and wave functions generated by the Oak Ridge-Rochester shell-model code. The numerical results of δ_{c2} reflect the shell effects, as can be expected from a two-body operator for such light nuclei.
Table 1 shows all the estimates of $\delta_c = \delta_{c1} + \delta_{c2}$ arising from the combination of the various evaluations of δ_{c1} mentioned above with the unique set of δ_{c2} reported in [14].

2.2. Phenomenological Approach to δ_c

The phenomenological approach to δ_c was suggested by Wilkinson [2, 15] and subsequently discussed by Wilkinson and coauthors in [16, 17]. It is based on a

Table 1. Estimates of the isospin impurity corrections δ_c in percent according to various approaches

Nucleus	Microscopic calculations $\delta_c=\delta_{c1}+\delta_{c2}$, δ_{c2} from [14]						Phenomenological approach $\delta_c=kZ^{1.86}$
	Approach A to δ_{c1}				Approach B to δ_{c1}		$k=(1.63\pm0.23)\times10^{-5}$ [a]
	δ_{c1} from [9]	[10]	[11]	[12]	[12]	[13]	
^{14}O	0.11	0.08	0.09	0.10	0.33	0.46	0.06 ± 0.01
^{26}Al^m	0.22	0.15	0.18	0.20	0.34	0.42	0.17 ± 0.02
^{34}Cl	0.46	0.36	0.41	0.43	0.85	0.80	0.28 ± 0.04
^{38}K^m	0.43	0.34	0.37	0.40	0.70	0.50	0.35 ± 0.05
^{42}Sc	0.52	0.36	0.38	0.50	0.48	0.44	0.43 ± 0.06
^{46}V	0.49	0.30	0.33	0.44	0.40	0.35	0.51 ± 0.07
^{50}Mn	0.54	0.34	0.36	0.46	0.43	0.59	0.60 ± 0.08
^{54}Co	0.60	0.38	0.42	0.51	0.60	0.71	0.70 ± 0.10

[a] This value was obtained fitting data of [17] (for more details see text)

Table 2. Results of the fits of the $f't$ values to the formula (13) $f't=(f't)_{Z=0}+a'Z^{1.86}$

Data	$(f't)_{Z=0}$ [s]	a' [$\times10^{-2}$]	k [$\times10^{-5}$]	χ^2/ν
Wilkinson et al. [17]	$3{,}084.8\pm1.9$	5.03 ± 0.71	1.63 ± 0.23	3.7
Vonach et al. [20] [a]	$3{,}088.7\pm2.1$	3.80 ± 0.86	1.23 ± 0.28	1.9
Towner and Hardy [22]	$3{,}085.0\pm2.5$	4.69 ± 0.86	1.52 ± 0.28	1.7
Wilkinson et al. [17] [b]	$3{,}083.5\pm1.9$	5.64 ± 0.71	1.83 ± 0.23	2.6

[a] In this survey the half-life of ^{46}V was corrected (see footnote of Table 3 and text)
[b] In this case the $f't$ values were corrected according to $f't\,[1-(\delta_{c2}-\bar{\delta}_{c2})]$

theorem developed by Behrends and Sirlin [18] and Ademollo and Gatto [19] (hence Wilkinson called this the BSAG approach) which states that the renormalization of coupling constants within symmetry multiplets goes as the square of the mass splittings. Figure 1 of [15] shows that the squares of the mass splitting ΔE between the analogous states in question are significantly well represented by $(\Delta E)^2\propto Z^{1.86}$. The BSAG approximation cannot provide an accurate value of δ_c; it only tells us that

$$\delta_c=kZ^{1.86}, \tag{12}$$

where k should be obtained from a fitting procedure. It is clear that using such a general theorem, it is not reasonable to expect correct values of δ_c for each transition; however it at least can give approximate values. Going back to (11), using (12) and setting $f_S/f_V=0$, we can write

$$f't=\frac{K}{2G_V'^2}(1+\delta_c)=(f't)_{Z=0}+a'Z^{1.86}, \tag{13}$$

where $(f't)_{Z=0}$ might be considered appropriate to the free nucleon and therefore used to derive G_V'. Equation (13) requires that the experimental $f't$ values follow an almost parabolic law as a function of Z. Such a behavior has been observed only after the precise measurements of W_0 carried out using the time-of-flight system at the Munich tandem [20]. For instance, this behavior can be seen looking at Fig. 1 of [17], where the data are fitted to expression (13). It is interesting to comment on some aspects of such an adjustment. Since the authors of [17] have not published the results for all the parameters, we repeated the fit with the aid of the package of subroutines MINUITS from CERN [21]. A plus-and-minus-one-standard-deviation fit yielded the results quoted in Table 2 with $\chi^2=22.3$. The value $k=(1.63\pm0.23)\times10^{-5}$ leads to δ_c listed in Table 1. A glance at this table indicates that the smooth trend of δ_c with Z obtained from BSAG follows approximately the values of $\delta_c=\delta_{c1}+\delta_{c2}$ with δ_{c1} taken from [9], of course without the fluctuations due to shell effects taken account of by δ_{c2}.

3. Analysis and Discussion

In this section we analyze the most up-to-date surveys of the eight best-measured cases – these are the decays where the errors of the $f't$ values are smaller than 0.5% – reported by Vonach et al. [20], Towner and Hardy [22], and Wilkinson, Gallmann, and Alburger [17]. It should be pointed out that the authors of [17, 20, 22] have adopted different criteria for selecting values of t_0 and W_0 corresponding to each nuclei. The technique used by Wilkinson et al. [17] was to accept the most recent precise measurement for each quantity, averaging-in only those earlier measurements with which it is statistically consistent. Vonach et al. [20] applied a procedure that consisted in taking all measurements whose quoted errors were not larger than ten times the error of the most accurate measurement, and rejecting the data which deviated from the weighted average by more than three times the quoted error. The first step of the method used by Towner and Hardy [22] is the same as that employed by Vonach et al.; however, instead of disregarding incompatible data, they inflated the uncertainties of the average value in a standard way (described in [3]) to cover incompatibility. Of course the inflated errors of the latter technique favor somewhat "better" χ^2/ν values for fitting procedures. It is interesting to compare the $f't$ values obtained using the methods described above. For example, the results obtained by Wilkinson et al. and Towner and Hardy are displayed in Fig. 1 of both papers [17, 22], respectively. A simultaneous inspection of both figures indicates that different choices of data led to sizeable changes of $f't$ values. The $f't$ values calculated by Vonach et al. [20] are rather closer to Towner and Hardy's data [22].

For our analysis we took the surveys of $f't$ values reported in [17, 22] just as they are in the original papers. On the other hand, we used the survey quoted in the note added in proof of [20] including one correction. Since [20] was published, two new precise measurements of the half-life of ^{46}V have been reported – by Squier et al. [23], and by Alburger and Wilkinson [24]. These new data differ significantly from the half-life adopted in [20]. In particular, Alburger and Wilkinson [24] state that their value published in a previous work [15] should be corrected. Therefore we calculated a weighted average using the values: 422.28 ± 0.23 ms from [23], 422.52 ± 0.45 ms [24], and 423.4 ± 2.0 ms [25]. This average (422.34 ± 0.21 ms instead of 424.0 ± 0.5 ms) was used for our study, yielding an $f't$ value equal to 3101 ± 2 s.

3.1. Search for a Reliable G'_V

Let us define the quantity $\mathscr{F}t$, which aside from the corrections contained in $f't$, also includes those due to isospin impurities:

$$\mathscr{F}t = f't(1-\delta_c) = \frac{K}{2G_V'^2[1+(f_S/f_V)\Delta_{IS}]^2}. \tag{14}$$

Assuming the validity of the CVC theory, the vector second-class current (SCC) does not exist. Under such an assumption, namely setting f_S to be zero, if δ_c is evaluated correctly, we should expect $\mathscr{F}t$ constant from nucleus to nucleus. In our previous investigations [5, 6] we tested the constancy of the $\mathscr{F}t$ values over the known β decays performing a least-squares analysis. Searching for a possible small dependence of $\mathscr{F}t$ on the charge of the daughter nucleus, we fitted the $\mathscr{F}t$ values to the formula

$$\mathscr{F}t = (\mathscr{F}t)_{Z=0}[1+a|Z|]. \tag{15}$$

No evidence for any lingering Z dependence in the experimental data was found in [5, 6]. It was thus reasonable [2–6] to evaluate a weighted average of $\mathscr{F}t$ values and subsequently derive a reliable value for the coupling constant G'_V. We should point out that the test of Z independence was not applied in the work of Wilkinson et al. [17] or of Towner and Hardy [22].

We calculated several sets of $\mathscr{F}t$ values using the $f't$ data quoted in the surveys mentioned above [17, 20, 22], and combined them with each of the microscopic estimates of δ_c listed in Table 1. We evaluated the weighted average $\overline{\mathscr{F}t}$ for each set and fitted the $\mathscr{F}t$ values to (15). Both the internal and external errors of $\overline{\mathscr{F}t}$ were computed according to the formulas quoted in the general instructions of the Nuclear Data Sheets. The internal error reflects the accuracy of the individual data, while the external error accounts also for the dispersion. The ideal situation occurs when both internal and external errors are equal; if not, the larger should be taken. For reference, we calculated the χ^2/ν of the straight mean of $f't$, setting $\delta_c = 0$. All the results are listed in Table 3.

A glance at Table 3 indicates that we can trace some general conclusions which are independent of a specific survey of $f't$ values. First of all we can stress that the χ^2/ν of the averages $\overline{\mathscr{F}t}$ corresponding to $\mathscr{F}t$ values evaluated adopting Approach A for δ_{c1} [$\overline{\mathscr{F}t}$(A)] are smaller than those χ^2/ν corresponding to calculations using Approach B [$\overline{\mathscr{F}t}$(B)]. This behavior has been already found by Wilkinson et al. [17] in the analysis of their own data. The estimate of [9] has not been mentioned at all by Wilkinson et al. [17]. The results obtained adopting Approach B for δ_{c1} present other unpleasant features. The external errors of $\overline{\mathscr{F}t}$(B) are much larger than the internal ones, and the values of χ^2/ν are in general scarcely improved from those obtained with $\delta_c = 0$; indeed, in the case of the survey [20], they are even worse.

Table 3. Calculated $\overline{\mathcal{F}t}$, $(\mathcal{F}t)_{Z=0}$, and a together with the corresponding χ^2/ν

Data survey	$\delta_c=\delta_{c1}+\delta_{c2}$		Weighted average				Fit to (15)		
	δ_{c1}	δ_{c2}	$\overline{\mathcal{F}t}$ [s]	$\Delta(\overline{\mathcal{F}t})$ [s] int.	ext.	χ^2/ν	$(\mathcal{F}t)_{Z=0}$ [s]	a [$\times 10^{-4}$]	χ^2/ν
Vonach et al. [20][a]	0	0	3,096.8	1.0	2.2	4.5	3,083.2±3.1	2.57±0.56	1.7
	Approach A [9]	[14]	3,084.4	1.0	1.0	0.9	3,084.2±3.1	0.03±0.56	1.1
	[10]		3,088.3	1.0	1.2	1.5	3,082.5±3.1	1.10±0.56	1.1
	[11]		3,087.3	1.0	1.2	1.4	3,081.9±3.1	1.03±0.56	1.1
	[12]		3,085.5	1.0	1.0	1.0	3,083.5±3.1	0.37±0.56	1.1
	Approach B [12]		3,080.6	1.0	2.3	5.3	3,069.8±3.1	2.06±0.56	4.2
	[13]		3,080.1	1.0	2.6	6.7	3,066.5±3.1	2.61±0.56	4.0
Towner and Hardy [22]	0	0	3,097.0	1.2	2.8	5.9	3,078.5±3.6	3.21±0.59	1.9
	Approach A [9]	[14]	3,084.3	1.2	1.4	1.4	3,080.9±3.6	0.58±0.59	1.5
	[10]		3,088.8	1.2	1.8	2.4	3,079.2±3.6	1.66±0.59	1.5
	[11]		3,087.9	1.2	1.8	2.3	3,078.8±3.6	1.58±0.59	1.5
	[12]		3,085.7	1.2	1.4	1.7	3,080.0±3.6	0.99±0.59	1.5
	Approach B [12]		3,083.4	1.2	2.0	3.1	3,071.1±3.6	2.12±0.59	1.5
	[13]		3,081.9	1.2	2.6	5.1	3,068.3±3.6	2.37±0.59	3.2
Wilkinson et al. [17]	0	0	3,096.8	0.9	2.8	10.7	3,077.3±2.8	3.48±0.48	3.6
	Approach A [9]	[14]	3,083.8	0.9	1.4	2.8	3,078.5±2.8	0.94±0.48	2.6
	[10]		3,087.9	0.9	1.9	5.1	3,076.6±2.8	2.02±0.48	3.0
	[11]		3,087.0	0.9	1.9	4.9	3,076.0±2.8	1.97±0.48	2.8
	[12]		3,085.1	0.9	1.6	3.4	3,077.6±2.8	1.34±0.48	2.7
	Approach B [12]		3,081.1	0.9	2.7	10.0	3,064.9±2.8	2.90±0.48	5.5
	[13]		3,080.3	0.9	2.7	9.9	3,063.7±2.8	2.99±0.48	5.2

[a] We analyzed the survey included in the note added in proof; however, instead of using the value 424.0±0.5 ms for the half-life of ^{46}V, we adopted the weighted average 422.34±0.21 ms calculated with more recent data (for more details see text)

Furthermore, the values of χ^2/ν are reduced by almost a factor two when the $\mathcal{F}t(\mathrm{B})$ values are fitted to (15). Such an adjustment indicates pronounced positive slopes and the results of $[\mathcal{F}t(\mathrm{B})]_{Z=0}$ are significantly smaller than the corresponding averages $\overline{\mathcal{F}t}(\mathrm{B})$. For instance, consider the set built using the survey by Wilkinson et al. [17] combined with their own δ_{c1} [13]. In this case we have $\overline{\mathcal{F}t}(\mathrm{B})=3{,}080.3 \pm 2.7$ s against an $[\mathcal{F}t(\mathrm{B})]_{Z=0}=3{,}063.7\pm2.8$ s with a slope $a=(2.99\pm0.48)\times10^{-4}$. The main trouble of the Approach B might lie in a possible overestimation of the corrections for the lighter nuclei, i.e., ^{14}O, ^{26}Al^m, and ^{34}Cl. All these features mentioned above make the sets of $\mathcal{F}t(\mathrm{B})$ values unsuitable for calculating a meaningful $\overline{\mathcal{F}t}$.

The situation is much more satisfactory when one analyzes the results obtained using Approach A for δ_{c1}. For all sets, the values of the χ^2/ν corresponding to $\overline{\mathcal{F}t}(\mathrm{A})$ are drastically better than those obtained with $\delta_c=0$. The best fit for each survey of $f't$ values is reached when the estimates by Damgaard [9] are used. Before focusing our discussion entirely on these results, let us note that when the survey by Vonach et al. [20] is used in conjunction with the estimates by Towner et al. [12], an excellent value of $\chi^2/\nu=1.0$ is also obtained. In these cases the goodness of the adjustment is not improved when the data are fitted to (15). The averages $\overline{\mathcal{F}t}(\mathrm{A})$ from the three surveys [17, 20, 22] are not merely consistent with one other within the quoted errors, but are practically the same. It can be also mentioned that the surveys [17, 22] give $\overline{\mathcal{F}t}(\mathrm{A})$ with external errors slightly larger than the internal ones, showing that the dispersion is larger than expected from errors of individual $\mathcal{F}t(\mathrm{A})$ values. Since the $\overline{\mathcal{F}t}(\mathrm{A})$ computed with δ_{c1} from [9] satisfy the minimal conditions to be suitable to provide a reliable effective vector coupling constant, we evaluated G'_V, getting the values listed in Table 4.

For the sake of completeness we tested whether the surveys by Vonach et al. [20] and by Towner and Hardy [22] could be fitted well to the quasi-parabolic formula (13). The numerical computation using the MINUITS code [21] yielded the results quoted in Table 2. The χ^2/ν of these fits are much better than those obtained with $\delta_c=0$, and the goodness of the adjustments indicates that the data can be represented satisfactorily well by (13). It was already suggested by Wilkinson et al. [17] that the goodness of the fits can be further improved if the $f't$ values are corrected by the shell effects due to the two-body charge-dependent potential. This can be done by correcting the individual $f't$ values by the departure

Table 4. The weighted averages $\overline{\mathscr{F}t}$ with the corresponding effective vector coupling G'_V and the values of the induced scalar coupling constant f_S/f_V with χ^2/ν of the fits to (17)

Data	δ_c	$\overline{\mathscr{F}t}$ [s]	G'_V [$\times 10^{-49}$ erg cm^3]	$(\mathscr{F}t)_{\Delta_{IS}=0}$ [s]	f_S/f_V [$\times 10^{-3}$]	χ^2/ν
Results of Ref. 6		$3{,}082.9 \pm 2.0$	1.41276 ± 0.00047	$3{,}075 \pm 25$	-0.40 ± 1.40	0.4
Vonach et al. [20][a]	$\delta_{c1}+\delta_{c2}$[b]	$3{,}084.4 \pm 1.0$	1.41242 ± 0.00023	$3{,}091 \pm 15$	0.32 ± 0.75	1.0
Towner and Hardy [22]		$3{,}084.3 \pm 1.4$	1.41245 ± 0.00031	$3{,}079 \pm 16$	-0.26 ± 0.86	1.6
Wilkinson et al. [17]		$3{,}083.8 \pm 1.4$	1.41256 ± 0.00033	$3{,}073 \pm 16$	-0.56 ± 0.76	3.2
	$kZ^{1.86}$				-0.16 ± 0.83[c]	4.4[c]

[a] In this survey the half-life of ^{46}V was corrected (see footnote of Table 3 and text)
[b] Here δ_{c1} was taken from [9] and δ_{c2}, from [14]
[c] These results were obtained fitting the $f't$ values to (19)

of the individual δ_{c2} values from $\bar{\delta}_{c2}$. In this way we can eliminate from the experimental data some effects which cannot be accounted for in the BSAG approach. Following this procedure, instead of using (13), we should use

$$f't[1-(\delta_{c2}-\bar{\delta}_{c2})]=(f't)_{Z=0}+a'Z^{1.86}. \quad (16)$$

Just as an example, the results of such a fit for the survey of [17] are also included in Table 2, where it can be seen that $\chi^2/\nu=3.7$ is improved to 2.6.

If all is well, the results obtained for $\overline{\mathscr{F}t}$ must agree with $(f't)_{Z=0}$. Actually, this is the case, since the reliable $\overline{\mathscr{F}t}$ values listed in Table 4 are practically the same as the value $(f't)_{Z=0}=3{,}084.1 \pm 1.9$ s recommended by Wilkinson et al. [17], and therefore the corresponding values of G'_V are also in excellent agreement.

3.2. *Determination of the Upper Limit for f_S*

To determine the upper limit for f_S/f_V, we followed the method described in [6] rather than the previous one reported in [5]. The technique introduced in [6] requires experimental $\mathscr{F}t$ values to be fitted to

$$\mathscr{F}t=(\mathscr{F}t)_{\Delta_{IS}=0}[1-2(f_S/f_V)\Delta_{IS}]. \quad (17)$$

The advantages of this fit over the straight average of departures from $\overline{\mathscr{F}t}$ proposed in [5] have been discussed elsewhere [6]. We calculated f_S/f_V for the sets which provide reliable averages $\overline{\mathscr{F}t}$ listed in Table 4. In the least-squares fit, we used the values of Δ_{IS} listed in Table 1 of the complete version of [6]. The results are included in Table 4. The quality of the adjustments are similar to those obtained with the fit to (15). The results of f_S/f_V for the three sets are consistent with one other and agree with the value of zero, thus supporting the CVC theory. The actual limit is provided by the quoted errors of f_S/f_V. The errors of these three values are practically equal. Therefore we can set as a limit for the IS coupling constant the straight mean value $\overline{f_S/f_V}=(-0.17 \pm 0.80) \times 10^{-3}$. For the sake of comparison we included in Table 4 the results of the previous work [6]. It can be seen that the errors obtained in the present work are reduced by a factor of two with respect to that reported before.

To have an alternative way to determine the limit for f_S/f_V we developed a new procedure using the BSAG approach to δ_c. Starting from (11) and adopting the phenomenological expression (12) for δ_c, we arrive at

$$f't=\frac{K[1+kZ^{1.86}]}{2G_V'^2[1+(f_S/f_V)\Delta_{IS}]^2}. \quad (18)$$

Since Δ_{IS} is about 3, and f_S/f_V is expected to be of the order 10^{-3}, this equation can be written as

$$\begin{aligned} f't&=(f't)_{Z=0}[1+kZ^{1.86}][1-2(f_S/f_V)\Delta_{IS}] \\ &=(f't)_{Z=0}[1+kZ^{1.86}-2(f_S/f_V)\Delta_{IS}]. \end{aligned} \quad (19)$$

To derive the parameters $(f't)_{Z=0}$, k, and f_S/f_V, we fitted the experimental $f't$ values to the expression (19). The numerical computation was performed with the aid of the MINUITS code [21]. As before a plus-and-minus-one-standard-deviation fit was employed. We obtained $(f't)_{Z=0}=3{,}082 \pm 19$ s, $k=(1.60 \pm 0.29) \times 10^{-5}$, and, for f_S/f_V, the value included in Table 4. The $\chi^2=22.2$ at the minimum indicates that the $\chi^2(=22.3)$ of the fit to (13) could not be improved introducing the extra parameter f_S/f_V. The result $f_S/f_V=(-0.16 \pm 0.87) \times 10^{-3}$ fixes a similar limit to those obtained from the former procedure. The fact that the limit for f_S/f_V is independent of the method of managing the experimental data is an attractive and stimulating result.

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