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ELEMENTARY PARTICLES AND FIELDS
Theory

Quark Matter and Meson Properties in a Nonlocal $SU(3)$ Chiral Quark Model at Finite Temperature*

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Abstract—We study the finite temperature behavior of light scalar and pseudoscalar meson properties in the context of a three-flavor nonlocal chiral quark model. The model includes mixing with active strangeness degrees of freedom, and takes care of the effect of gauge interactions by coupling the quarks with a background color field. We analyze the chiral restoration and deconfinement transitions, as well as the temperature dependence of meson masses, mixing angles, and decay constants.

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1. INTRODUCTION

The behavior of strongly interacting matter at finite temperatures and/or densities has become an issue of great interest in recent years. From the theoretical point of view, one way of addressing this subject is through the development of effective models that provide a qualitatively good description of hadron phenomenology, keeping the symmetry properties of the underlying QCD theory. The predictions arising from these models can be contrasted with those obtained within lattice QCD calculations, with the possibility to be extended into regions not accessible by lattice techniques. Our aim in this sense is to go one step beyond previous analyses, studying the properties of strongly interacting matter in the context of a three-flavor nonlocal chiral model that includes mixing with active strangeness degrees of freedom, and takes care of the effect of gauge interactions by coupling the quarks with a background color gauge field. This model, which can be viewed as an extension of the Polyakov loop Nambu–Jona-Lasinio (PNJL) model [1], takes into account the nonlocality that arises naturally in the context of several successful approaches to low-energy quark dynamics, as, e.g., the instanton liquid model, the Schwinger–Dyson resummation techniques, and also lattice QCD itself. Here, we consider the case of finite temperature and zero chemical potential, studying the main features of the chiral restoration and deconfinement transitions. Then we focus on the light scalar and pseudoscalar meson sector, analyzing how masses, mixing angles, and decay constants get modified when mesons propagate in a hot medium.

2. FORMALISM

We consider the Euclidean action for a nonlocal chiral quark model in the case of three light flavors, including the coupling to a background color field A_0. The traced Polyakov loop, which is taken as order parameter of confinement, is given by $\Phi = \frac{1}{3}\mathrm{Tr}\exp(i\beta\phi)$, where $\beta = 1/T$, $\phi = iA_0$. We work in the so-called Polyakov gauge, in which the matrix is given by a diagonal representation $\phi = \phi_3\lambda_3 + \phi_8\lambda_8 = \mathrm{diag}(\phi_r, \phi_g, \phi_b)$.

In our model the mean field thermodynamical potential Ω_{MFA} is found to be

$$\Omega_{\mathrm{MFA}}(T) = -2\sum_{f,c}\int_{p,n}\mathrm{Tr}\ln\left[p_{nc}^2 + \Sigma_f^2(p_{nc})\right] - \frac{1}{2}\left[\sum_f\left(\bar{\sigma}_f\bar{S}_f + \frac{G}{2}\bar{S}_f^2\right) + \frac{H}{2}\bar{S}_u\bar{S}_d\bar{S}_s\right] + \mathcal{U}(\Phi, T), \quad (1)$$

where $f = u, d, s$; $c = r, g, b$, and the shorthand notation

$$\int_{p,n} = \sum_n \int d^3p/(2\pi)^3$$

has been used. We have also defined $p_{nc} = (\mathbf{p}, \omega_n - \phi_c)$, where ω_n stand for the fermionic Matsubara frequencies. The (momentum dependent) quark

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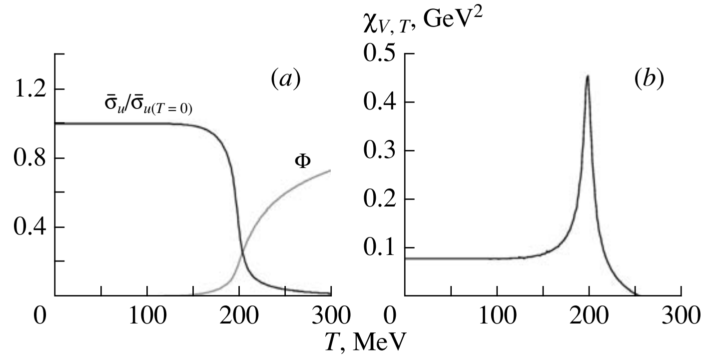

Fig. 1. (a) Normalized mean field value $\bar{\sigma}_u/\bar{\sigma}_{u(T=0)}$ and Polyakov loop Φ as functions of T. (b) Chiral susceptibility $\chi_{V,T}$ as function of T.

constituent masses $\Sigma_f(p_{nc})$ are given by $\Sigma_f(p_{nc}) = m_f + \bar{\sigma}_f g(p_{nc})$, where $g(p)$ is a nonlocal form factor (taken here for simplicity as a Gaussian function) and $\bar{\sigma}_f$ are the mean field values of scalar fields introduced through the bosonization of the original fermionic theory. We have also introduced the auxiliary fields S_f, whose mean field values can be obtained within the stationary phase approximation from the values of $\bar{\sigma}_f$ and the constants G and H [2]. In addition, in Eq. (1) we include an effective potential $\mathcal{U}(\Phi, T)$ that accounts for color field self-interactions. Its explicit form can be found in [3]. Owing to the charge conjugation properties of the QCD Lagrangian, and assuming that ϕ_3 and ϕ_8 are real-valued, one has $\phi_8 = 0$, $\Phi = [2\cos(\phi_3/T) + 1]/3$.

From the minimization of the thermodynamical potential (which has to be properly regularized) one obtains a set of three coupled "gap equations"

$$\frac{\partial \Omega_{\mathrm{MFA}}}{(\partial\bar{\sigma}_u, \partial\bar{\sigma}_s, \partial\phi_3)} = 0. \tag{2}$$

These equations determine the mean field values $\bar{\sigma}_u$, $\bar{\sigma}_s$, and ϕ_3 at a given temperature.

3. MESON PROPERTIES AT FINITE T

In order to obtain the meson mass spectrum and other properties one has to consider the mesonic fluctuations around the mean field values. The resulting quadratic contribution to the finite temperature bosonized effective action can be written as

$$S_E^{\mathrm{quad}} \tag{3}$$
$$= \frac{1}{2}\int_{q,m} \sum_{M=S,P} G_M(\mathbf{q}^2, \nu_m^2) M(q_m) M(-q_m),$$

where $M = S, P$ corresponds to scalar ($S = a_0$, κ, σ, f_0) and pseudoscalar ($P = \pi$, K, η, η') mesons, and $q_m = (\mathbf{q}, \nu_m)$, where $\nu_m = 2m\pi T$ are bosonic Matsubara frequencies.

The functions $G_M(\mathbf{q}^2, \nu_m^2)$ are given by loop integrals. One has, e.g.,

$$G_{\pi(K)}(\mathbf{q}^2, \nu_m^2) \tag{4}$$
$$= \left[\left(G + \frac{H}{2}\bar{S}_{s(u)}\right)^{-1} + C^-_{uu(su)}(\mathbf{q}^2, \nu_m^2)\right],$$

where the functions $C^-_{ff'}$ are defined by [4]

$$C^-_{ff'}(\mathbf{q}^2, \nu_m^2) = -8\sum_c \int_{p,n} g(p_{nc} + q_m/2) \tag{5}$$
$$\times \frac{p_{nc}^2 + p_{nc}q_m + \Sigma_f(p_{nc}+q_m)\Sigma_{f'}(p_{nc})}{D_f(p_{nc}+q_m) D_{f'}(p_{nc})},$$

with $D_f(s) \equiv s^2 + \Sigma_f^2(s)$. In this way, one can obtain the lowest meson screening masses by solving the equations $G_M(-m_M^2, 0) = 0$.

In the η–η' sector, the physical fields are in general related to the $U(3)$ states η_8 and η_0 through two mixing angles θ_η and $\theta_{\eta'}$ defined in such a way that there is no η–η' mixing at the level of the quadratic action. In a similar way, the states σ and $f_0(980)$ are linear combinations of σ_8 and σ_0, with mixing angles θ_σ and θ_{f_0}. The mixing angles (that are in general different from each other) can be obtained from the functions $G_M(\mathbf{q}^2, \nu_m^2)$ [4].

Finally, in the case of the pseudoscalar mesons other important quantities are the weak decay constants f_{ab}, defined by

$$\langle 0|A^a_\mu(0)|\pi_b(q)\rangle = i f_{ab} q_\mu, \tag{6}$$

where A^a_μ is the a-component of the axial current. For $a, b = 1, \ldots, 7$, f_{ab} can be written as $\delta_{ab} f_P$, with $P = \pi$ for $a = 1, 2, 3$ and $P = K$ for $a = 4$ to 7. In contrast, the decay constants become mixed in the $a = 0, 8$ sector. Details on how to obtain the expressions for the currents in the presence of nonlocal fields can be found, e.g., in [2, 5].

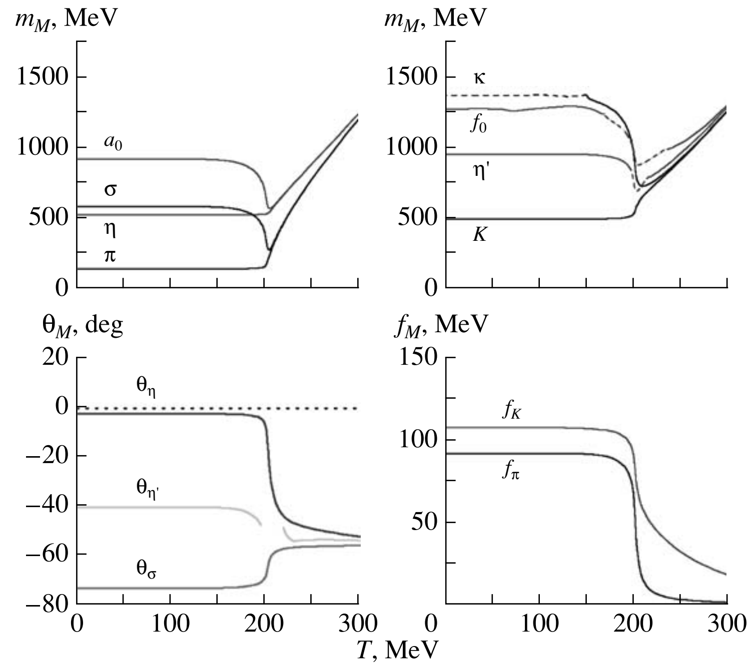

Fig. 2. Meson properties as functions of the temperature. In upper panels we display the meson masses, while in the lower panels we quote the values of mixing angles (left) and decay constants (right).

4. NUMERICAL RESULTS

In order to determine the parameters of the model, we consider for simplicity a Gaussian-like form factor, and take as input values the empirical π, K, and η' meson masses and the pion decay constant f_π. In this way, it is seen that one obtains a very good description of meson observables at $T = 0$ [2, 4]. Then, when extending the model to finite T, it is found that u, d chiral condensates and effective quark masses show a sharp decrease at a critical temperature $T_{\rm cr} \simeq 200$ MeV. This is shown in Fig. 1, where we plot $\bar\sigma_u/\bar\sigma_{u(T=0)}$ (Fig. 1*a*) and the chiral susceptibility $\chi_{V,T}$ (Fig. 1*b*) as functions of the temperature. Both the characteristics of the transition and the value of $T_{\rm cr}$ are consistent with some recent lattice calculations [6] (in fact, the interaction with the Polyakov loop Φ provides a substantial enhancement of the otherwise too low value of $T_{\rm cr}$ [7]). In addition, it is seen that Φ rises considerably in the same region (see Fig. 1*a*), thus both chiral restoration and deconfinement transitions take place at approximately the same temperature.

Next, in Fig. 2 we show the behavior of some meson properties as functions of T. In the upper panels we display the meson mases (defined as in the previous section), while in the lower panels we quote the values of mixing angles (left) and decay constants f_π and f_K (right) [4]. As a further indication of the chiral restoration in the nonstrange sector, it is seen that the masses of chiral partners (π, σ) and (η, a_0) become degenerate immediately after the transition. In contrast, (K, κ) and (η', f_0) masses match at a somewhat higher temperature, due to their larger strange quark content. Indeed, it is seen that as T increases the η_8–η_0 mixing angles approach the "ideal mixing" values, hence the η meson becomes almost nonstrange, while the opposite happens to the η'. Finally, the pseudoscalar decay constants show the expected behavior, i.e. a sharp decrease at the chiral transition temperature.

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REFERENCES

1. K. Fukushima, Phys. Lett. B **591**, 277 (2004); E. Megias, E. Ruiz Arriola, and L. L. Salcedo, Phys. Rev. D **74**, 065005 (2006); C. Ratti, M. A. Thaler, and W. Weise, Phys. Rev. D **73**, 014019 (2006).
2. A. Scarpettini, D. Gómez Dumm, and N. N. Scoccola, Phys. Rev. D **69**, 114018 (2004).
3. S. Rossner, C. Ratti, and W. Weise, Phys. Rev. D **75**, 034007 (2007).
4. G. A. Contrera, D. Gómez Dumm, and N. N. Scoccola, Phys. Rev. D **81**, 054005 (2010).
5. R. D. Bowler and M. C. Birse, Nucl. Phys. A **582**, 655 (1995); R. S. Plant and M. C. Birse, Nucl. Phys. A **628**, 607 (1998); D. Horvatic, D. Blaschke, D. Klabucar, and O. Kaczmarek, Phys. Rev. D **84**, 016005 (2011).
6. Y. Aoki et al., JHEP **0906**, 088 (2009); A. Bazavov et al., Phys. Rev. D **80**, 014504 (2009).
7. G. A. Contrera, D. Gómez Dumm, and N. N. Scoccola, Phys. Lett. B **661**, 113 (2008).