

# Accretion on to strange-matter pulsars

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## ABSTRACT

We study the effects of the accretion of normal matter on to strange-matter pulsar models. It is assumed that, because of the high strangeness barrier, normal matter is inert in contact with  $Q_\alpha$  matter. For this reason, normal matter accretion is able to form a thick outer layer with densities far above neutron drip. Accretion can make the superfluid quark-alpha  $Q_\alpha$  layer disappear by solidification but, for the same reason, a superfluid neutron layer is formed. The fractional moment of inertia of the latter is large enough to fulfil the vortex-creep glitch model requirements.

The high rotational stability of millisecond pulsars is discussed in the framework of the strange-matter hypothesis. If the validity of the vortex-creep model is assumed, it is shown that, for the case of accreted strange-pulsar models, this stability cannot be interpreted as due to the solidification of the superfluid  $Q_\alpha$  layer. Nevertheless, ‘standard’ homogeneous strange stars remain as interesting candidates for the internal composition of these objects.

The main conclusions of this work should be unaltered if  $Q_\alpha$  is not preferred to strange matter at low pressures, but some other strange complex plays a similar role.

**Key words:** accretion, accretion discs – dense matter – equation of state – stars: neutron – pulsars: general.

## 1 INTRODUCTION

Pulsar physics has progressively become a fascinating arena for observers and theoreticians. The interplay between the growing body of observations and the theoretical model-building that attempts to knit a reliable picture of these objects is nowadays stronger than ever. However, major questions remain to be answered concerning not only the pulsar emission mechanism, but also the evolution and internal structure of pulsars. Actually, there is no lack of theoretical proposals about the latter, and increasingly accurate observational facts are the most helpful tool for determining the nature of these compact stars. One of the latest developments along this line has been the suggestion that there may be only a few neutrons in the ‘neutron’ star if the ground state of cold hadronic matter happens to be in the form of degenerate quark-gluon plasma, commonly termed *strange matter* (Farhi & Jaffe 1984; Witten 1984). This

hypothetical state would leave little room for the conventional structure based on neutron-matter equations of state (hereafter EOS) (see, e.g., Shapiro & Teukolsky 1983 and references therein), because the conversion to the strange state ought to be effective on a short time-scale of order  $\sim 1$  s, as it is determined by the temperature of the hot protoneutron star (Horvath, Benvenuto & Vucetich 1992). In such a situation, we are forced to seek an explanation for the full pulsar phenomenology in the framework of the strange-matter theory, because it is unlikely that a ‘mixed’ population of neutron and strange stars exists (Alcock, Farhi & Olinto 1986; Horvath & Foglia 1992).

It has previously been recognized that the simplest structure of the so-called strange stars (Alcock et al. 1986; Haensel, Zdunik & Schaeffer 1986; Benvenuto & Horvath 1989a; Chakrabarty 1991) has a serious shortcoming when faced with explaining the ‘glitch’ observations collected from several pulsars (Shapiro & Teukolsky 1983). Simply stated, the homogeneity of their interiors (essentially a quark Fermi liquid) does not allow the sudden change in some internal component required to produce the observed jumps in the pulsar angular velocity  $\Omega$  and angular velocity derivative  $\dot{\Omega}$

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(Haensel et al. 1986; Alpar 1987). To overcome this problem, it is imperative to have some differentiated layer(s) present in the structure or, in other words, some discontinuity of the derivative of the pressure against chemical potential. Following Michel's suggestion (1988), the 18-quark neutral boson termed the quark-alpha (hereafter  $Q_\alpha$ ) has been postulated to play the role of a 'nucleus' of strange matter and has been introduced into the stellar models (Benvenuto & Horvath 1990; Benvenuto, Horvath & Vucetich 1990, 1991), which became layer-structured. The existence of a (super)fluid  $Q_\alpha$  layer was tentatively identified as responsible for the glitch events, although the precise mechanism producing them is still not understood. Provided that this picture is correct, but quite irrespective of the details of the glitch dynamics, it is clear that other observational tests should be applied to address the viability of the  $Q_\alpha$  layer.

It is the purpose of this work to discuss the consequences of accretion of normal matter on to strange-matter pulsar models (hereafter SPM). The key point for the present work is that, because of the high strangeness barrier, normal matter is inert against conversion into  $Q_\alpha$  matter, and so the structure of the resulting objects is *accretion-dependent*. Because of the large uncertainty in the value of the mass  $M_{Q_\alpha}$  of the  $Q_\alpha$  particle, we shall employ two values (as in our previous work on this topic), namely  $M_{Q_\alpha} = 5$  and  $5.5$  GeV. We pay particular attention to the interplay between the normal-matter crust and the superfluid layer(s) of the star, which is believed to be strongly related to the glitch phenomenon.

In Section 2, we briefly review the main features of SPM. In Section 3, we discuss the process of accretion on to SPM. In Section 4, we present analytical and numerical calculations of accreted SPM. In Section 5, we discuss the results and the main conclusions of the present work.

## 2 THE STRUCTURE OF STRANGE-MATTER PULSARS

Let us recall the main features of the structure of an SPM (Benvenuto & Horvath 1990; Benvenuto et al. 1990, 1991). The adopted EOS has a complex structure: at low pressures it predicts a  $Q_\alpha$  superfluid which, at a pressure of  $0.06 \text{ MeV fm}^{-3}$ , solidifies in an f.c.c. lattice. Later, at much higher pressures ( $\sim \text{few MeV fm}^{-3}$ , somewhat dependent on the assumed mass  $M_{Q_\alpha}$  of the  $Q_\alpha$  particle<sup>1</sup>), the  $Q_\alpha$  particles themselves are squeezed into strange matter. The structure of the pulsar is determined by this sequence of phase transitions: below a thin crust of normal matter, there is a layer of  $Q_\alpha$  superfluid (in this model, related to the glitch mechanism), a thick  $Q_\alpha$  solid mantle immediately below (with a small contamination of charged particles) and a strange-matter core. A plausible scenario for the formation of these models, based on the detonation to strange matter mechanism for type II supernovae, has been proposed by Benvenuto & Horvath (1989b), Benvenuto, Horvath & Vucetich (1989) and Benvenuto et al. (1989, 1991).

When the supernova core is burnt to strange matter, the explosive conversion becomes a standard shock wave at a

<sup>1</sup> $9.33 \text{ MeV fm}^{-3}$  for  $M_{Q_\alpha} = 5 \text{ GeV}$  and  $2.61 \text{ MeV fm}^{-3}$  for  $M_{Q_\alpha} = 5.5 \text{ GeV}$ , respectively.

density  $\rho \sim 2\rho_0$  (where  $\rho_0$  is the nuclear-matter saturation density), which should be enough to blow off the mantle and envelope of the massive star. At this point the combustion process left behind the shock does not stop, however, but becomes a deflagration, transforming most of the star into strange matter. Since the efficiency of the process is high but not perfect, a small quantity of normal matter forming the crust (and 'contaminating' the solid  $Q_\alpha$  mantle) remains. A normal-matter crust is necessary for the formation of a magnetosphere, while the residual charged particles trapped in the solid provide a rather strong coupling of the latter to the rest of the star. The slow rate of weak interactions keeps this normal matter *out* of chemical equilibrium with  $Q_\alpha$  matter.

In the frame of this EOS, a new branch of stellar objects is predicted:  $Q_\alpha$  stars. These objects are light enough that they do not reach the pressure of phase transition to bulk strange matter in their cores. For this reason, the entire structure of the star is composed of  $Q_\alpha$  matter.

## 3 ACCRETION ON TO STRANGE PULSARS

Let us consider an SPM. As described in Section 2, its structure shows a superfluid layer of  $Q_\alpha$  bosons, which lies above a solid-phase and strange-matter core, and below a normal-matter crust extending all the way up to the surface. Because of the negligible rate of the reactions  $6n \rightarrow Q_\alpha$  (which involves a high-order strangeness-changing, weak-interaction process), chemical equilibrium is absent between the normal crust and the  $Q_\alpha$  superfluid interface. Now imagine that accretion on to the pulsar (from, for example, a companion star) begins. As the accreted matter accumulates above the superfluid layer, the pressure exerted by the former on top of the latter steadily increases. Eventually, when this pressure reaches the value  $P_c = 0.06 \text{ MeV fm}^{-3}$ , the  $Q_\alpha$  superfluid *solidifies*, precluding any further timing noise and glitches produced by it. It is interesting to note that the value of  $P_c$  is independent of the uncertain mass  $M_{Q_\alpha}$  of the  $Q_\alpha$  particle.

There is a conspicuous type of astronomical object for which accretion on to compact objects is expected: millisecond pulsars. Since their discovery in 1982, millisecond pulsars have posed a number of challenging questions to the theoreticians (see van den Heuvel & Battacharaya 1991 for a comprehensive review). Anticipated as early as 1974 (Bisnovatyi-Kogan & Konberg 1974) as the natural result of a mass-transfer episode, most of them are now widely believed to be spun-up compact stars. Besides the short periods ( $P \leq 10 \text{ ms}$ ), they share some other common features, mainly their low magnetic fields,  $B \approx 10^9 \text{ G}$ . Also, most of them belong to binary systems, but appear to be otherwise similar to the isolated radio pulsar population. However, and in contrast with the latter, they show extremely regular pulsation patterns, being equivalent to clocks at least as good as the best man-made ones (Davis et al. 1985). In particular, no glitch has been ever observed from a millisecond pulsar (Lyne 1991, private communication; Taylor 1992, private communication), and only upper limits for timing-noise activity have been established (Dewey & Cordes 1989), showing that they are objects with extremely stable rotation rates. More recently, Stinebring et al. (1990) have observed noise in the timing residuals of PSR 1937+24. This noise has a power-law spectrum,

$P(f) \propto f^{-\alpha}$ , with index  $\alpha = 3.2 \pm 1.3$  ( $\alpha = 2-4$  may indicate clock instabilities, ephemeris errors, interstellar medium propagation effects or rotational instabilities), and is qualitatively similar to the noise observed in other pulsars. Nevertheless, it has been impossible to determine the origin of this noise. More observational data seem to be necessary for elucidating this important problem.

Analysis of the restless timing behaviour of radio pulsars suggests that the latter may be attributed to irregularities of the superfluid component coupling (Jones 1991). If a link between the pulsar history (i.e. recycling accretion) and the internal structure is found, it is possible to imagine that the lack of glitches and timing noise can be interpreted as an expected feature. It is important for the present work to note that the estimated quantity of accreted matter necessary for accelerating a millisecond pulsar (like PSR 1937+24) to its present period is at least  $\sim 0.1 M_{\odot}$  (Srinivasan 1989).

#### 4 MODELS OF ACCRETED STRANGE-MATTER PULSARS

In this section, we present analytical and numerical models for the main features of accreted SMP.

Let us study the structure of a normal-matter layer formed by accretion which has a pressure  $P_b$  at its bottom. We shall calculate the mass  $M_A$  of this layer, the moment of inertia fraction of the neutron drip superfluid included in this layer, and also the mass-radius relationship for the models.

To a very good approximation, we can assume that normal matter in the range  $\rho \leq \rho_0$  (where  $\rho_0 = 2.7 \times 10^{14} \text{ g cm}^{-3}$  is the nuclear-matter saturation density) can be described by a simple polytropic EOS,

$$P = K \rho^{\gamma}, \quad (1)$$

where  $K$  is a constant, and  $\gamma = 5/3$  for non-relativistic conditions. If we assume the layer of accreted matter to be very thin ( $\Delta R/R \ll 1$ , where  $\Delta R$  is the thickness of this layer, and  $R$  is the radius of the star), we can calculate its structure by employing the very simple equation

$$\frac{dP}{dr} = -g\rho, \quad (2)$$

where

$$g = \frac{GM}{R^2} \left(1 - \frac{2GM}{Rc^2}\right)^{-1}; \quad (3)$$

the symbols have their usual meanings, and their values correspond to the surface in which the normal-matter layer is located. Solving this equation, we get the density profile of the crust,

$$\rho = \rho_b \left(1 - \frac{\Delta r}{\Delta R}\right)^{\frac{1}{\gamma-1}}, \quad (4)$$

where  $\rho_b$  is the density at the bottom of the crust,  $\Delta r$  is the height from the bottom of the accreted crust, and

$$\Delta R = \frac{\gamma}{\gamma-1} \frac{K}{g} \rho_b^{\gamma-1}. \quad (5)$$

Now, we can compute the mass of the crust as

$$M_A = 4\pi R^2 \int_0^{\Delta R} \rho dr = \frac{4\pi R^2}{g} P_b. \quad (6)$$

Writing this equation in convenient units, we have

$$\frac{M_A}{M_{\odot}} = 7.53 \times 10^{-2} \left(\frac{R}{10^6 \text{ cm}}\right)^4 \left(\frac{P_b}{1 \text{ MeV fm}^{-3}}\right) \left(\frac{M}{M_{\odot}}\right)^{-1} \times \left[1 - 0.2952 \left(\frac{M}{M_{\odot}}\right) \left(\frac{R}{10^6 \text{ cm}}\right)^{-1}\right] - \frac{M_i}{M_{\odot}}, \quad (7)$$

where  $M_i$  is the mass of normal matter before accretion started.

Let us calculate the amount of accreted matter necessary to make the  $Q_{\alpha}$  superfluid disappear, i.e.,  $P_c = P_b = 0.06 \text{ MeV fm}^{-3}$ . Assuming  $M_i = 0$ , the simple analytic equation (7) predicts, for a  $1.5\text{-}M_{\odot}$  object,  $M_A = 1.4 \times 10^{-2} M_{\odot}$  for the stellar model with  $M_{Q_{\alpha}} = 5 \text{ GeV}$  and  $M_A = 4.75 \times 10^{-3} M_{\odot}$  for one with  $M_{Q_{\alpha}} = 5.5 \text{ GeV}$ , which bracket the relevant range of the quark-alpha mass  $M_{Q_{\alpha}}$  (see below).

For the moment of inertia  $I_A$ , the expression is (in the same approximation)

$$I_A = \frac{8\pi}{3} \frac{R^6 P_b}{g}, \quad (8)$$

which, with the former scaling, reads

$$I_A = 10^{44} \left(\frac{R}{10^6 \text{ cm}}\right)^6 \left(\frac{P_b}{1 \text{ MeV fm}^{-3}}\right) \left(\frac{M}{M_{\odot}}\right)^{-1} \times \left[1 - 0.2952 \left(\frac{M}{M_{\odot}}\right) \left(\frac{R}{10^6 \text{ cm}}\right)^{-1}\right] \text{ g cm}^2. \quad (9)$$

Assuming  $P_b = 0.06 \text{ MeV fm}^{-3}$ , equation (9) predicts, for a  $1.5\text{-}M_{\odot}$  object,  $I_A = 4.8 \times 10^{43} \text{ g cm}^2$  for the stellar model with  $M_{Q_{\alpha}} = 5 \text{ GeV}$ , and  $I_A = 9.8 \times 10^{42} \text{ g cm}^2$  for the one with  $M_{Q_{\alpha}} = 5.5 \text{ GeV}$ . The total moments of inertia of these models are (Benvenuto & Horvath 1990)  $2.7 \times 10^{45}$  and  $1.9 \times 10^{45} \text{ g cm}^2$ , respectively. If we assume that all the matter is in the state of superfluid neutron drip, we can calculate the moment of inertia fraction for these models, whose values are 1.79 and 0.52 per cent (see below).

We have also calculated the structure of the accreted SPM numerically. Our main interest in doing so is to investigate the values of  $M_A$  and  $I_A$ , and also the mass-radius relationship of the models. It is useful not only in comparing with the very simple analytic calculations given above, but also in computing their structure at values of  $P_b$  higher than those used above. We have employed the fully relativistic Tolman-Oppenheimer-Volkoff equations and, for the EOS of the normal-matter layer, we have employed the equation given in Mølnvik & Østgaard (1985), which is valid for particle number densities  $n < 0.182 \text{ fm}^{-3}$  ( $n = 0.16 \text{ fm}^{-3}$  at  $\rho = \rho_0$ ),

$$\varepsilon = \rho c^2 = 939.45 n(1 + 4.52 \times 10^{-2} n^{0.6}) \text{ MeV fm}^{-3}, \quad (10a)$$

$$P = 25.78 n^{1.6} \text{ MeV fm}^{-3}. \quad (10b)$$

We have assumed the parameters for strange-matter EOS and the solid  $Q_\alpha$  EOS as in Benvenuto & Horvath (1990) and Benvenuto et al. (1990).

We have integrated the structure by means of a standard Runge-Kutta technique from the centre of the star (composed of strange matter or solid  $Q_\alpha$  matter; see below) until the star reaches the assumed value of  $P_b$ . From this point up to the surface, we have used the neutron-matter EOS. The sequences have been constructed for central pressures from 3 to 350  $\text{MeV fm}^{-3}$  and for values of  $P_b$  of 0.06, 0.12, 0.24, 0.48, 0.96 and 1.92  $\text{MeV fm}^{-3}$  for each of the assumed masses of the  $Q_\alpha$ , namely  $M_{Q_\alpha} = 5$  and 5.5 GeV, respectively. The main results are shown in Figs 1(a), 2(a) and 3(a) (corresponding to  $M_{Q_\alpha} = 5$  GeV) and Figs 1(b), 2(b) and 3(b) (corresponding to  $M_{Q_\alpha} = 5.5$  GeV), respectively, and in Tables 1 and 2 for objects of  $1.44 M_\odot$ .

In Figs 1(a) and (b), we show the logarithm of the accreted mass as a function of the total mass of the object for each value of  $P_b$ . As the different curves have been computed doubling  $P_b$  because of equation (7), we expect them to be equispaced in a logarithmic plot, which is in good agreement with the numerical calculation. It is worth noting that the analytically predicted values of the accreted matter are in very good agreement with the numerically computed ones. Note the spikes at a mass value of  $\sim 1.5 M_\odot$  in Fig. 1(a). They arise because at low masses (and hence low central pressures) we have stars with solid  $Q_\alpha$  cores and, at the values indicated by the spikes, the core of the star is dense enough to form strange matter (see Section 2). The central pressure at these points is  $P_c = 9.33 \text{ MeV fm}^{-3}$ , which is higher than the central pressures at the beginning of the sequences. Nevertheless, the spikes are not present in Fig. 1(b) because, in this case, the pressure of strange-matter formation is only  $P_c = 2.61 \text{ MeV fm}^{-3}$ , and so we have neither accreted  $Q_\alpha$  stars nor spikes in these sequences. The lowest sequences, corresponding to the accretion necessary to solidify the superfluid  $Q_\alpha$  layer, show that we need a very small amount of matter to do so.

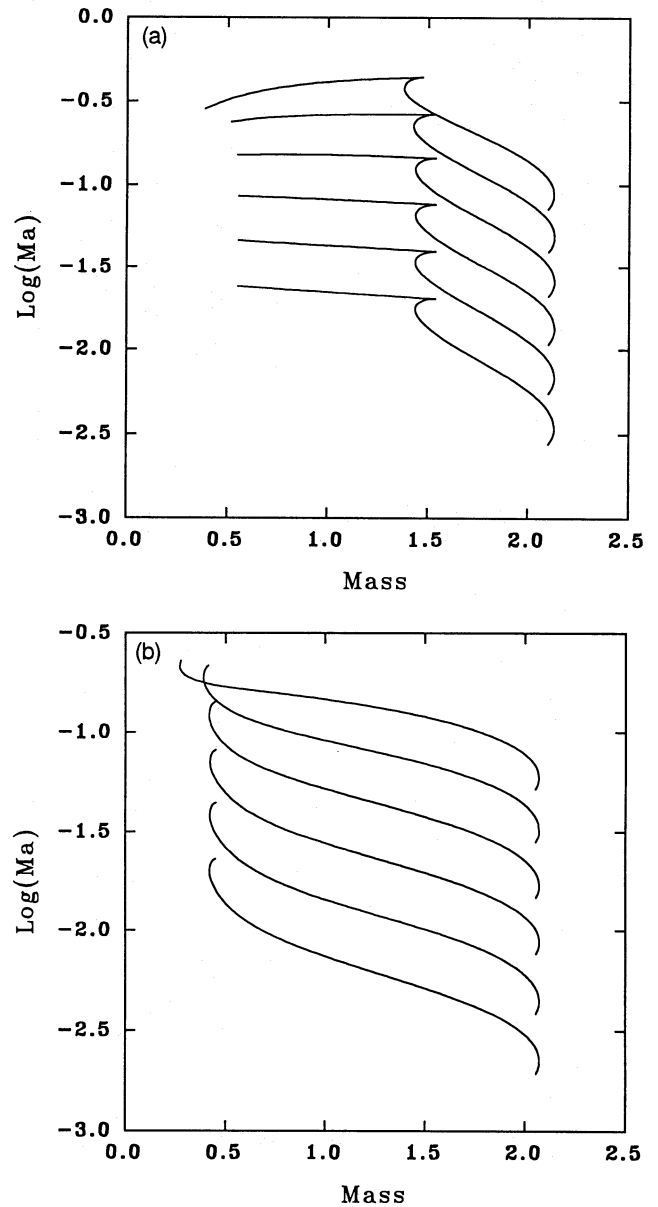
In Figs 2(a) and (b), we show the logarithm of the percentage of the total moment of inertia of the star included in the neutron drip superfluid. To compute this, we have assumed that all the matter with densities  $\rho \geq \rho_d$  (where

**Table 1.** Accreted  $1.44 M_\odot$  SPM models: the case of  $M_{Q_\alpha} = 5$  GeV.

$P_b$ (MeV/fm <sup>3</sup> )	$\log(M_A/M_\odot)$	Redshift	Radius (Km)	$\log(I_{sf}/I)$
0.06	-1.785	0.158	16.750	-1.627
0.12	-1.491	0.157	16.813	-1.339
0.24	-1.205	0.157	16.838	-1.062
0.48	-0.935	0.158	16.715	-0.806
0.96	-0.697	0.163	16.303	-0.586
1.98	-0.513	0.175	15.425	-0.422

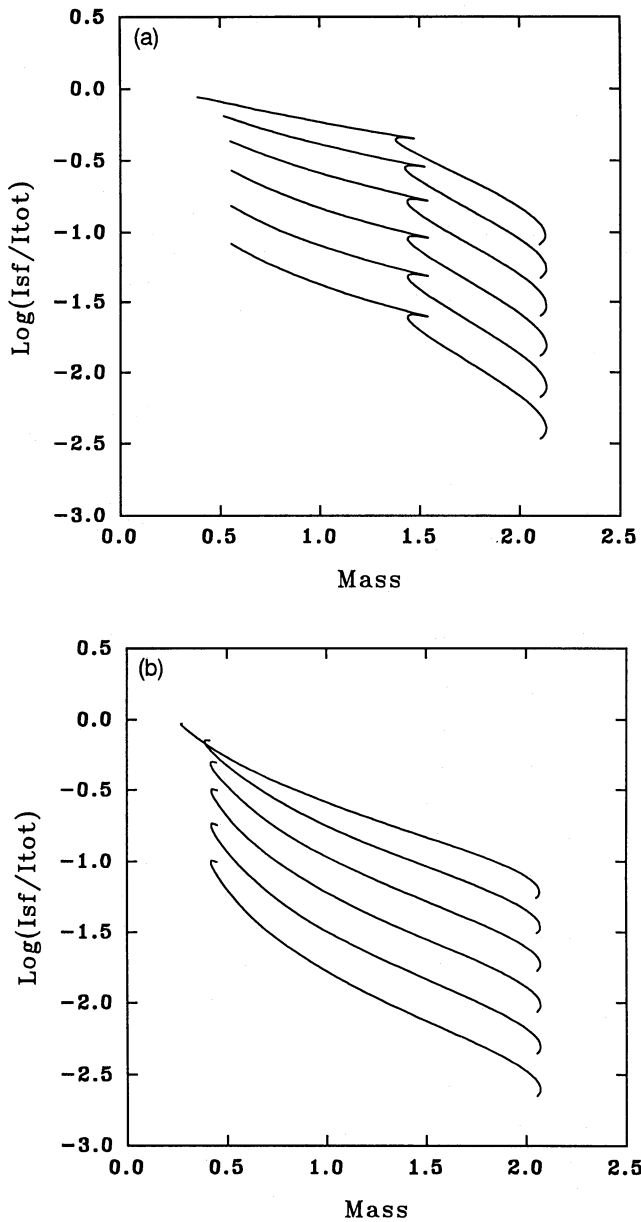
**Table 2.** Accreted  $1.44 M_\odot$  SPM models: the case of  $M_{Q_\alpha} = 5.5$  GeV.

$P_b$ (MeV/fm <sup>3</sup> )	$\log(M_A/M_\odot)$	Redshift	Radius (Km)	$\log(I_{sf}/I)$
0.06	-2.272	0.221	12.883	-2.092
0.12	-1.978	0.220	12.915	-1.802
0.24	-1.689	0.220	12.929	-1.519
0.48	-1.410	0.221	12.900	-1.251
0.96	-1.146	0.224	12.774	-1.006
1.98	-0.912	0.231	12.462	-0.802



**Figure 1.** (a) Logarithm of the accreted mass (in  $M_\odot$ ) as a function of the total mass of the object (in  $M_\odot$ ). The values of  $P_b$  are 0.06, 0.12, 0.24, 0.48, 0.96 and 1.92  $\text{MeV fm}^{-3}$  from bottom to top. The value of  $M_{Q_\alpha}$  is  $M_{Q_\alpha} = 5$  GeV. For low-mass stars up to the spikes, we have pure  $Q_\alpha$  stars, and for more massive objects we have accreted SPM. (b) As (a), but for  $M_{Q_\alpha} = 5.5$  GeV. Note the absence of spikes.

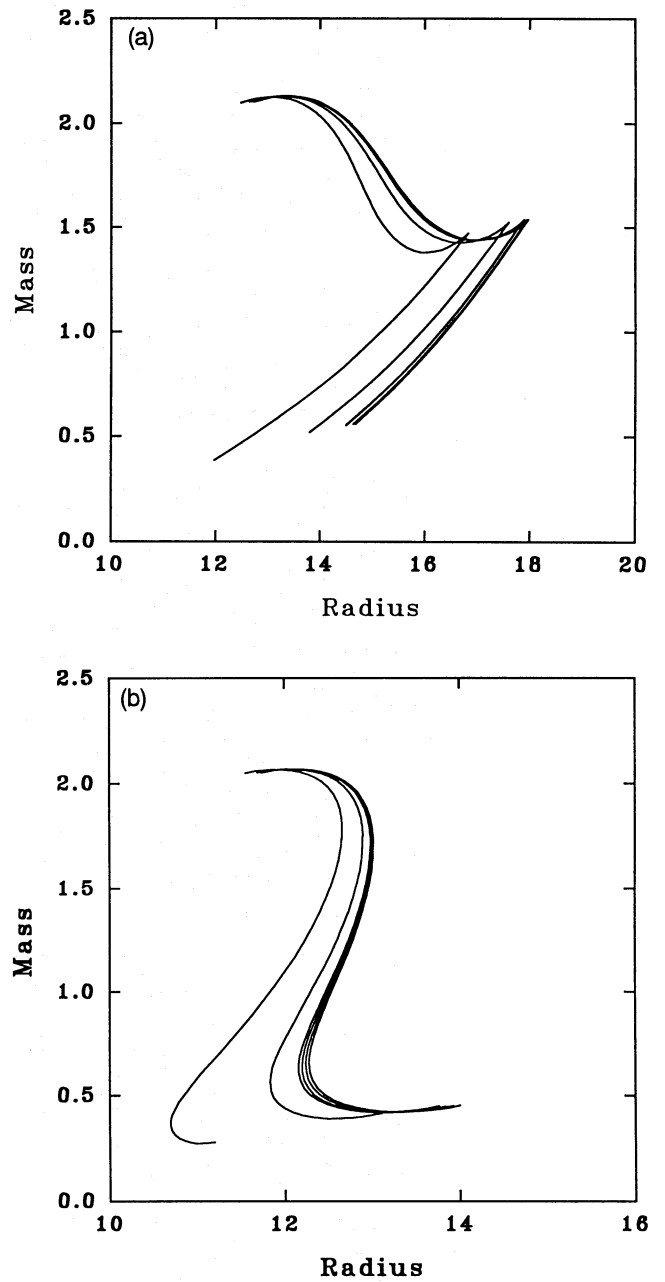
$\rho_d = 4 \times 10^{11} \text{ g cm}^{-3}$  is the neutron drip density) is in the superfluid state. This is a very good approximation for our purposes. As in the case of Figs 1(a) and (b), we again explain the equispaced curves by equation (9), which fits the numerical results very nicely. It is very important to remark that, even for the lowest values employed for  $P_b$ , we have a significant part of the total moment of inertia of the star in the neutron drip superfluid layer. The percentages are high enough that we may expect glitches and timing noise driven by this accreted layer (the glitch requirement is  $I_{sf}/I_{\text{tot}} \approx 10^{-3} - 10^{-2}$ ). Accretion can therefore solidify the  $Q_\alpha$



**Figure 2.** (a) Logarithm of the percentage of the total moment of inertia of the star included in the neutron drip superfluid layer as a function of the total mass of the object (in  $M_{\odot}$ ). Note that, for accreted SPM (below the spikes), the moment of inertia fraction is large enough to accommodate the glitch phenomenology in the framework of the vortex-creep model. (b) As (a), but for  $M_{Q_a} = 5.5$  GeV. Note the absence of spikes.

layer, but even in this case it will *unavoidably* form another new, non-negligible neutron drip superfluid layer. It is not yet clear whether some change in the pattern of irregularities should be expected as a result of this phenomenon.

In Figs 3(a) and (b), we show the mass–radius relationships. These are very similar to the ones presented in Benvenuto & Horvath (1990), especially for the lowest values of  $P_b$ . This behaviour justifies the assumption of  $\Delta R/R \ll 1$ , at least in these cases. For higher values of  $P_b$ , the accreted matter layer is thick and, because it is more compact than the solid  $Q_a$  layer, it will accommodate an



**Figure 3.** (a) The mass–radius relation (mass in  $M_{\odot}$ , radius in km). The curves move to the left for increasing  $P_b$ . Note that, for the lower values of  $P_b$ , the effect of the accretion upon this relation is negligible (i.e., the layer fulfils the condition  $\Delta r/R \ll 1$ ). (b) As (a), but for  $M_{Q_a} = 5.5$  GeV. Note the absence of spikes.

equal quantity of matter in a smaller volume. Hence we have the sequences moving to the left (the more accretion, the more compact the star).

It is important to stress that, if we allow for a very massive accreted layer (a high  $P_b$ ) to form, we can obtain an inversion of the density profile of the star (i.e.  $d\rho/dr > 0$ ). This is a consequence of the steeper dependence of  $P$  on  $\rho$  for the solid  $Q_a$  EOS compared to neutron matter. To have equilibrium between the bottom of the accreted layer and the outer part of the solid mantle, we need the pressure to be continuous. However, this is not necessary for the density. If

$P_b$  is high enough, the density of normal matter will be greater than that of the  $Q_\alpha$  solid. We can compute the values above which the quoted inversion begins to occur when  $P_{\text{solid}} = P_{\text{neutrons}}$  and  $\rho_{\text{solid}} = \rho_{\text{neutrons}}$  (in an obvious notation). The results for each of the cases considered in the present paper are as follows: for  $M_{Q_\alpha} = 5$  GeV the inversion occurs at  $P = 0.8412$  MeV fm $^{-3}$  and  $\rho = 2.126 \times 10^{13}$  g cm $^{-3}$ , and for the case of  $M_{Q_\alpha} = 5.5$  GeV it occurs at  $P = 0.8908$  MeV fm $^{-3}$  and  $\rho = 2.329 \times 10^{13}$  g cm $^{-3}$ . Note that the two sequences with  $P_b = 0.96$  and  $1.92$  MeV fm $^{-3}$  include this inversion.

## 5 DISCUSSION AND CONCLUSIONS

In this work we have studied the problem of accretion of normal matter on to SPM. We have found that accretion is able to solidify the  $Q_\alpha$  superfluid layer by compression. However, it forms a new neutron drip superfluid layer. If we allow for the amounts of accretion of the order of magnitude necessary for accelerating a millisecond pulsar, we find that the new superfluid layer is, in principle, thick enough to drive glitches and timing noise.

It is generally agreed that both phenomena are the result of variations of the superfluid vortex lines coupling to the nuclear lattice that coexists with it (Alpar et al. 1984; Pines & Alpar 1985). Because of the sensitivity of the pinning forces to the microscopic parameters (e.g. energy gap), the density at which pinning to nuclei begins is somewhat uncertain (see, for example, Ainsworth, Pines & Wambach 1989 for details of the calculations which yield lower values than earlier gaps), but it may be safely asserted that  $\rho \geq 10^{13}$  g cm $^{-3}$  is favoured (Epstein & Baym 1988). It is important to remark that even the pinned-state picture is not necessarily correct; detailed calculations by Jones (1991) yield not pinning but corotating vortices which persist through the lifetime of the pulsar. Thus a completely different mechanism for glitch production (Jones 1990a) and timing noise generation (Jones 1990b) may be operating in compact stars.

Irrespective of the details of the actual glitch mechanism, it will be able to operate in conventional neutron stars as well as in accreted SPM. For this reason, accreted SPM have no obvious advantage compared to neutron stars in explaining the stability in the rotation rates observed in millisecond pulsars. This is the main conclusion of the present work. Nevertheless, homogeneous strange stars are still attractive candidates for explaining this behaviour, because they do not have any neutron drip layer (Jones 1990b). Note that the objection of Alpar (1987) to the existence of strange stars cannot be applied in identifying millisecond pulsars with homogeneous strange stars, because at present we have not detected any glitches in these objects.

Regarding the inversion of density, it appears just at the solid  $Q_\alpha$ -normal matter interface. It is important to note that, at the densities reached at the bottom of the normal layer ( $\rho \leq \rho_0/10$  for the considered models), it is composed of a Coulombic lattice nucleus permeated by a neutron drip superfluid. On the other hand, the  $Q_\alpha$ s are arranged in an f.c.c. solid, mainly because of excluded volume effects rather than cohesive forces; moreover, these forces (which were neglected in computing the EOS of the corresponding phases) should be present, but are expected to be weak due to the closed shell structure of  $Q_\alpha$ s. In this situation, a nucleus at the interface will feel the electromagnetic

repulsion of the neighbouring nuclei and will tend to move towards the diluted phase and, because of the same effect,  $Q_\alpha$ s will tend to move to the denser phase. In this way we expect a diffusive mixing between these two layers, which should tend to smear out the density inversion.

A nucleon can occupy only the inter-hard sphere space, and thus the smaller the space the higher the energy. We therefore expect a large amount of contaminating nucleons at lower densities. The only way to solve the problem of a mixed  $Q_\alpha$ -neutron EOS is to start from thermodynamics. In this case, this approach would give us only equilibrium conditions, but it is certainly not clear whether the time-scales of relaxation of the system are smaller than the evolutionary (accretion) time-scales. If not, the thermodynamical results would not be useful. In any case, the present neutron superfluid will be more than enough to drive the observed rotational instabilities, because we will need only a tiny fraction of the accreted normal matter to contaminate the underlying layer (see Figs 2a and b and above).

In this work, we have assumed a particular candidate for low-pressure 'drops' of strange matter, namely the  $Q_\alpha$  particle. This is a hypothetical state, and perhaps not the actual case in nature. If some other particle playing a similar role to that assumed here for the  $Q_\alpha$  exists, the main results of this paper will remain unchanged. This conclusion follows from the general need for the existence of a high strangeness barrier between normal matter and the putative particle (see Michel 1988 for alternative candidates) in order to avoid conflict with basic results of laboratory physics.

After the completion of our work, we became aware of the work of Gilson & Jaffe (1993) on the structure of very small strangelets. They solve Dirac's equation for non-interacting quarks in the M.I.T. bag, finding that shell effects strongly enhance the stability of strangelets, for example for  $A = 6$ , which just corresponds to the  $Q_\alpha$  particle (as is expected in Michel 1988, and is the main argument for postulating this particle). This work gives some support for the hypothesis we have employed in the present work, in spite of the fact that the treatment is very rough. Clearly, we would need more refined models in order to have better predictions about the binding energy of the  $Q_\alpha$  particle.

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