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A Comment on Spiral Motions in Projective Relativity

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Astronomical evidence has been inadequately invoked to support projective relativity. The spiral structure cannot be explained just by the existence of spiral orbits, and the use of Oort's constant to support the theory is also a missunderstanding. Besides, some mathematical inaccuracies make the application invalid.

1. INTRODUCTION

Pessa [1] examined the two-body problem in projective relativity and found that, under reasonable assumptions, particles gravitating near a large central mass follow spiral trajectories. He extrapolated that result to conclude that it explains the spiral structure and kinematical properties of galaxies.

His conclusions are based on missunderstandings of some astronomical facts and in the present note we wish to clarify the situation.

2. THE SPIRAL STRUCTURE OF GALAXIES

Let us begin by noting that if spiral orbits were the answer to the problem of spiral structure of galaxies, there are quite simple ways to

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obtain them; for example, if a galaxy loses mass at a moderately low rate the eccentricities of the orbits do not change but their semimajor axes grow larger, i.e., the orbits become spirals (see, e.g., [2]).

Nevertheless, spiral orbits are not the answer and, although in the past some investigators tried to explain the spiral structure of galaxies through the actual distribution of the galactic mass in a spiral pattern, that view is no longer acceptable. Kalnajs [3] showed that it is possible to construct purely kinematical models of spiral structure with classical elliptic orbits, and modern theories explain spiral structure either as a density wave (see, e.g., [4]), or through autopropagated stellar formation (see, e.g. [5]) in rotating stellar disks with no systematic radial motion.

Observations have failed to detect any gross concentration of mass in the spiral arms or voids in the interarm regions (see, e.g., [6]). What is nowadays recognized is that the spiral arms display large concentrations of *young objects*; in fact, the arms, or their immediate neighborhood, are the most active places of stellar formation. Since the young objects are on the average much brighter than the older population we get a strong concentration of *light* (and not necessarily of *mass*) towards the spiral arms. In other words, the spiral aspect displayed in photographs of spiral galaxies (particularly those taken in blue light) is indicative of a concentration of luminous objects in the spiral arms, but by no means does it imply that the bulk of the galactic mass lies in the arms nor that the stellar orbits are spirals.

3. OORT'S CONSTANT A

Let us consider now another misunderstanding found in the paper by Pessa [1]. He seems to believe that the expression of Oort's constant,

$$A = -\frac{1}{2}\rho\frac{d\omega}{d\rho} \tag{1}$$

implies that the second term is a constant [we used Pessa's notation in (1), but we introduced the minus sign that he omitted]. Actually, Oort's constant is just a term that appears when one makes a series expansion of the kinematical formulas, valid for small distances from the Sun, and it should be evaluated at a specified value of ρ, namely, the distance of the Sun to the galactic center (see, e.g., [7]). In other words, A is a constant because it evaluates the second term of (1) at a particular value of ρ; by no means does the expression (1) imply the constancy of the second term for any ρ. Actually, if one defines A for arbitrary ρ, and not just for the solar neighborhood, one will obtain different values of A at different distances of the galactic center. Incidentally, Pessa [1] states that the value of A is near 0.004″, but the units are in fact arc second *per year* and the value is closer to 0.003″/yr.

4. MATHEMATICAL DIFFICULTIES

There are several mathematical inaccuracies as well in Pessa's [1] work. First of all, he neglected small terms in his equation (23) to obtain his equation (26). Those terms are of the order of $(\dot{\rho}/c)^2$ and $(\rho/r)^2$. However, he kept the term $2t\rho\dot{\rho}/r^2$ which happens to be of the same order. Since, as Pessa himself indicated, it is always $ct < r$, it follows immediately that

$$2t\rho\dot{\rho}/r^2 < 2(\dot{\rho}/c)(\rho/r) \tag{2}$$

i.e., comparable to the orders of magnitude he had neglected. Without that term equation (26) is essentially what one obtains in Newtonian mechanics. The same results if one accepts Pessa's solution (27):

$$\rho = \rho_0(1 + \alpha t), \qquad \alpha t \ll 1 \tag{3}$$

because then it is

$$2t\rho\dot{\rho}/r^2 = 2\left(\frac{\alpha t}{1 + \alpha t}\right)\left(\frac{\rho}{r}\right)^2 \tag{4}$$

Notice, however, that Pessa's [1] equation (26) has an exact solution, namely,

$$t = A \cdot \mu^{B^2K^2c^2} \cdot \exp(-B^2\mu) \tag{5}$$

where A is an arbitrary constant and

$$B = \varepsilon_0/r(m_0^2c^4 - \varepsilon_0^2)^{-1} \tag{6a}$$

$$\mu = (m_0^2c^4 - \varepsilon_0^2)\rho^2 + K^2c^2 \tag{6b}$$

and that from such solution it follows immediately that the first derivative of ρ with respect to t is always infinite at $t = 0$, except for the very particular case chosen by Pessa. In other words solutions of the form proposed by Pessa are valid only for the very particular relationship between the angular momentum K and the quantity ε_0, related to the energy, implied by Pessa's equations (28a) and (28b); i.e., not every orbit would be a logarithmic spiral.

In order to investigate the orbits within the theory of projective relativity one should then consider cases where $(\dot{\rho}/c)^2$ and $(\rho/r)^2$ are not negligible and solve Pessa's equation (23) numerically. Obviously, this would imply considering length and velocity scales much larger than those prevalent within galaxies and, again, they seem of little value to investigate the spiral structure of galaxies.

5. CONCLUSION

Spiral trajectories do not, by themselves, explain the spiral structure of galaxies. Pessa [1] missunderstood the meaning of Oort's constant and his use of it to support his hypothesis is invalid.

The mathematical inaccuracies included in Pessa's work made it to appear adequate for galactic scales. In fact, once those inaccuracies are removed, it is evident that even accepting the validity of projective relativity, to have observable effects from that theory we need much larger length and velocity scales than galactic ones, i.e., intergalactic scales.

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