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The Electron Capture to Positron Emission Ratios in the Decay of ^{22}Na and ^{65}Zn

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The electron capture to positron emission ratios for allowed Gamow-Teller transitions from the decay of ^{22}Na and ^{65}Zn were measured. The values $\varepsilon/\beta^+ = 0.1128 \pm 0.0057$ and $K/\beta^+ = 31.3 \pm 2.0$ were obtained for ^{22}Na and ^{65}Zn, respectively. A detailed comparison with previous data is made. Theoretical ratios were calculated using the most recent Fermi and Coulomb functions. It was found an excellent agreement between our experimental data and the theoretical results evaluated without including contributions due to induced interactions.

1. Introduction

In the last years the interest in studies on electron capture (ε) to positron emission (β^+) ratios has been revived. This renewal was intensified after the recent review papers [1, 2] where the Glasgow Group have summarized available experimental data on ε/β^+ ratios for allowed transitions. In the compilation [1] the authors have included their own theoretical results too. Such evaluations were performed in the approximation independent of the form factor coefficients (FFC) F^N_{KLs}. Let us first make some comments on the meaning of this approach and, subsequently, discuss the agreement between theory and experiment.

Following the general procedures outlined in Ref. 3 one finds that the expression for the K-electron capture to β^+ decay ratio is

$$\frac{\lambda_K}{\lambda_{\beta^+}} = \frac{K}{\beta^+} = \frac{f_K}{f_{\beta^+}} \cdot \frac{C_K}{\bar{C}_{\beta^+}}. \tag{1}$$

Here f_K and f_{β^+} are statistical rate functions defined in [3] and $C_K/\bar{C}_{\beta^+}$ is the ratio of the shape factors of the K-electron capture and β^+-spectrum. The bar in $\bar{C}_{\beta^+}$ denotes the average over the energy W of the positrons. It is possible to demonstrate that neglecting higher order contributions of FFC, the ratio $C_K/\bar{C}_{\beta^+}$ is equal to unity (cf. [4]). Under such assumption: (i) one obtains the approximation used in Refs. 1, 2 and, (ii) our Equation (1) is reduced to

$$\frac{K}{\beta^+} = \frac{f_K}{f_{\beta^+}} = \frac{\pi}{2} \frac{\beta_K^2 (W_0 + W_K)^2 B_K}{\int_1^{W_0} F(-Z, W)\, W p (W_0 - W)^2 dW}, \tag{2}$$

where all the symbols have their usual meaning (cf. [3]).

From the comparison of the experimental data and theoretical values of K/β^+ listed in Table 1 of Ref. 1 one realizes that discrepancies still remain for a large number of nuclei. Especially, the differences are remarkable for nuclei with medium and high charge number Z. In a very recent paper Campbell et al. [5]

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reported a careful measurement of the ε/β^+ ratio in the decay of ^{120}Sb. The authors of [5] found that the experimental value disagrees with the theoretical prediction given by Equation (2). The uncertainty of the measurement is about 3% and the experimental K/β^+ ratio lies about 16% lower than the theoretical prediction. This result deepened the discussions of whether Equation (2) can satisfactorily describe the processes involved.
In some works (see [2] and references quoted therein) it was suggested that the disagreement could be removed by introducing terms proportional to the axial second class current which violate the G-parity and which are characterized by the coupling constants f_T. However, quite recently Behrens and Bühring [4] showed that the corrections provided by terms proportional to f_T could hardly explain deviations of about 10% from Equation (2). Bearing in mind this contradictory situation it is worthwhile to undertake a study of any transition where there is a minimal shadow of a doubt.
The purpose of the present work is to study in detail, both experimentally and theoretically, the K/β^+ ratios of the Gamow-Teller decays 3^+ (545 keV β^+) 2^+ and $\frac{5}{2}^-$ (329 keV β^+) $\frac{3}{2}^-$ of ^{22}Na and ^{65}Zn, respectively. For both transitions there are available previous experimental results. However, the agreement between theory and experiment is far from being satisfactory. A detailed review of the previous experimental data is made in Section 2. Our measurements are described in Section 3. The subsequent analysis and discussion are provided in Section 4.

2. Previous Experimental Works

The available experimental data for both ^{22}Na and ^{65}Zn nuclei were obtained through measurements performed using both direct and indirect methods. We call a procedure "indirect method" when the electron-capture-radiation and/or the positrons are inferred from other measured quantities. In Tables 1 and 2 we summarize the results of the previous experimental works [6–19]. The information quoted in our Tables 1 and 2 is somewhat more complete than that of Ref. 1, because they contain more experimental data. We should mention that, the weighted averages included in all the tables reported in this paper were calculated with the following prescription. The weighted mean of a series of data was determined considering that the weights are the reciprocals of the squares of the standard errors. The error of the mean was evaluated using Equation (1) of [20]. The associated χ^2/ν is the chi-square function [see Eq. (21) of [21]] per degree of freedom.

The theoretical values calculated by Fitzpatrick et al. [1] were corrected by the exchange correction coefficients B_K as evaluated by Vatai [22] and listed in Tables 1 and 2. Let us now discuss separately the situation for each nucleus.

2.1. *The ^{22}Na Nucleus*

In this case the experimental results obtained by using indirect methods can be grouped according to the error assigned to measured values. In such a respect there exists a clear difference between the measurements performed in the fifties and the sixties. We list the experimental data according to the criterion mentioned above in Table 1, where a calculated weighted average for each group is also included. The results of the fifties are in excellent agreement with the theoretical prediction given by Equation (2), whereas the most precise data (experimental error less than 1%) obtained in the sixties are in drastic disagreement. On the other hand, the only one experimental result obtained by means of a direct method reported by McCann and Smith [14] agrees with the theoretical prediction. Thus, in this case our aim is to clear up such a situation measuring the K/β^+ ratio using an indirect method.

2.2. *The ^{65}Zn nucleus*

All previous experimental results of the K/β^+ ratio corresponding to the $\frac{5}{2}^-$ (329 keV β^+) $\frac{3}{2}^-$ transition from ^{65}Zn are consistent one with each other independently of the method of measurement. Due to this feature, we list the previous date in Table 2 without any special classification. A quick inspection of this table indicates that the experimental data the errors of which are less than 15% are inconsistent with the prediction given by Equation (2). In particular the deviation of the weighted mean is about 15%, whereas the error of the mean is about 3.7%. The present contradictory status of the matter encouraged us to measure the K/β^+ ratio for this transition.

3. Measurements and Results

The measurements reported in this work were carried out using indirect methods. Since the experimental procedures utilized in the measurements on ^{22}Na and ^{65}Zn were different we describe separately both experiments.

3.1. *Measurement on ^{22}Na*

We are interested in the K/β^+ ratio of the decay from the ground state 3^+ of ^{22}Na to the first excited state

Table 1. Electron capture to positron emission ratio in the decay of ^{22}Na

	ε/β^+	K/β^+	Authors	Ref.
Experimental values				
Data (1950–59)	0.110 ±0.006		Sherr and Miller	[6]
	0.124 ±0.011		Kreger	[7]
	0.122 ±0.010		Allen et al.	[8]
	0.109 ±0.008		Konijn et al.	[9]
	0.112 ±0.004		Ramaswany	[10]
Weighted mean ($\chi^2/\nu=0.59$)	0.1128±0.0029	0.1043±0.0027		
Data (1960–69)	0.1041±0.0007		Williams	[11]
	0.1048±0.0007		Leutz and Weninger	[12]
	0.1037±0.0013		Vatai et al.	[13]
	0.1051±0.0015		Vatai et al.	[13]
Weighted mean ($\chi^2/\nu=0.34$)	0.1044±0.0005	0.0965±0.0004		
		0.1050±0.0090	McCann and Smith	[14]
	0.1128±0.0057	0.1043±0.0052	Present work	
Theoretical predictions				
		0.1059±0.0004	Fitzpatrick et al.	[1]
		0.1041±0.0005	Present work Equation (2)	

Table 2. Electron capture to positron emission ratio in the decay of ^{65}Zn

	K/β^+	Authors	Ref.
Experimental values			
	32.5±6.0	Sakai and Aubert	[15]
	28.0±1.8	Perkins and Haynes	[16]
	26.0±3.0	Avignon	[17]
	24.5±2.1	Gleason	[18]
	27.7±1.5	Hammer	[19]
Weighted mean ($\chi^2/\nu=0.72$)	27.1±1.0		
	31.3±2.0	Present work	
Theoretical values			
	30.5±0.4	Fitzpatrick et al.	
	31.6±0.5	Present work Equation (2)	

2^+ of ^{22}Na. Since the decay scheme of the ^{22}Na nucleus is very simple one can readily establish some useful relations. The sum of the total capture (all *K*-, *L*-, *M*-, ... electrons) and the β^+ decays to the 2^+ state in ^{22}Na is equal to the number of γ-rays of 1274 keV [N_γ (1274)] of the γ-transition to the ground state of ^{22}Na. In addition, we may have the 511 keV γ-rays ($N_{\gamma\pm}$) arising from the annihilation of the positrons. If we measure the coincidence between the N_γ (1274) and $N_{\gamma\pm}$ radiations, such a quantity $N_{\gamma\gamma\pm}$ is proportional to the β^+ events. A straightforward calculation indicates that the total ε/β^+ratio can be obtained by the formula

$$\frac{\varepsilon}{\beta^+}=2\,\frac{N_\gamma(1274)\,\varepsilon_{\gamma\pm}\,P_{\gamma\pm}}{N_{\gamma\gamma\pm}}-1. \qquad (3)$$

Here $\varepsilon_{\gamma\pm}$ is the total absolute efficienty of the detector for the 511 keV γ-rays and $P_{\gamma\pm}$ is the corresponding peak-to-total ratio. One should note that Equation (3) is independent of:
(i) the desintegration rate of the source (N_0), (ii) the branching ratio of the β^+-decay to the ground state of ^{22}Na, and (iii) the characteristics of the detector for the 1274 keV γ-rays.
The radioactive sample was purchased from the Comisión Nacional de Energía Atómica, Argentina. The source of ^{22}Na was placed into a copper cylindrical holder. The thickness of the copper was enough to ensure the positrons being annihilated within the walls.
We performed our measurements utilizing two NaI(Tl) scintillators to obtain a good efficiency. The

annihilation radiation $N_{\gamma^{\pm}}$ was detected in a Harshaw 3″×3″ NaI(Tl) crystal with integral line Type 12S12L/E, while the N_{γ} (1274) radiation was detected in a 2″×2″ NaI(Tl) crystal Type 8S8. Since the measurement was an absolute one we have drawn a special attention in the determination of $\varepsilon_{\gamma^{\pm}}$ and $P_{\gamma^{\pm}}$. The total efficienty $\varepsilon_{\gamma^{\pm}}=(0.661\pm0.002)\times10^{-2}$ for the 3″×3″ NaI(Tl) crystal corresponding to the source-to-detector distance of 20 cm, as well as the peak-to-total ratio $P_{\gamma^{\pm}}=0.530\pm0.002$ were determined using Ref. 23.

The angle between both NaI(Tl) detectors was about $\theta\simeq140°$ to avoid the detection of sum-peaks. In this experiment the total attenuation effect was negligible due to cancellation of the individual attenuations.

A series of 14 measurements of ε/β^{+} was carried out. Following the idea of Campbell et al. [5] we present the complete set of data. The results together with the weighted average are listed in Table 3. The errors quoted in Table 3 are only the statistical ones. The errors due to the uncertainties of $\varepsilon_{\gamma^{\pm}}$ and $P_{\gamma^{\pm}}$ were added quadratically to the error of the weighted mean. This result is included in Table 1, where it can be compared with previous data.

3.2. *Measurement on* ^{65}*Zn*

The aim of this experiment was to measure the K/β^{+} ratio for the Gamow-Teller transition between the ground states of ^{65}Zn and ^{65}Cu. The $\frac{5}{2}^{-}$ ground state of ^{65}Zn decays to the $\frac{3}{2}^{-}$ ground state of its daughter in about 48 % of all decays while about 52 % goes to the first excited state $\frac{5}{2}^{-}$ at 1115 keV. The transition $\frac{5}{2}^{-}\rightarrow\frac{5}{2}^{-}$ is a pure electron capture process because the positron emission is forbidden due to energy requirements. Thus, if one measure the annihilation radiation $N_{\gamma^{\pm}}$ of a ^{65}Zn source the amount of these events represents directly the positron emission in the transition of our interest. The electron capture events can be measured by detecting the K X-ray radiation N_{X}. However, the total number of N_{X} contains information about both electron capture transitions $\frac{5}{2}^{-}\rightarrow\frac{3}{2}^{-}$ and $\frac{5}{2}^{-}\rightarrow\frac{5}{2}^{-}$. To remove the contribution coming from the transition to the first excited state $\frac{5}{2}^{-}$ a coincidence technique should be used. If we measure the coincidence between the γ-rays of 1115 keV [labeled N_{γ} (1115)] and N_{X}, such a quantity $N_{\gamma\mathrm{X}}$ is proportional to the events which should be subtracted from N_{X} to obtain the K X-rays corresponding to the electron capture of the $\frac{5}{2}^{-}\rightarrow\frac{3}{2}^{-}$ transition alone.

The K/β^{+} ratio was derived from the quantities N_{X}, $N_{\gamma\mathrm{X}}$ and $N_{\gamma^{\pm}}$. The N_{X} radiation was detected by a Si(Li) crystal with a beryllium window. The absorption of this window is negligible for the 9.659 keV K X-rays of ^{65}Zn. On the other hand, some X-rays might fail to reach the sensitive area of the NaI(Tl) detector due to attenuation in the aluminium window

Table 3. Measured electron capture to positron decay ratios

	ε/β^{+} for ^{22}Na	K/β^{+} for ^{65}Zn
	0.1110±0.0063	31.03±0.69
	0.1090±0.0065	32.29±1.51
	0.1189±0.0087	30.12±0.72
	0.1095±0.0063	32.80±1.29
	0.1215±0.0066	32.66±1.48
	0.1052±0.0063	31.42±1.44
	0.1199±0.0067	30.76±0.77
	0.1101±0.0065	32.49±1.29
	0.1195±0.0063	33.57±1.13
	0.1075±0.0056	31.75±0.68
	0.1076±0.0073	32.57±1.29
	0.1150±0.0065	32.50±1.37
	0.1187±0.0062	31.61±1.15
	0.1080±0.0078	30.92±0.65
		30.52±0.54
Weighted mean ($\chi^2/\nu=0.74$)	0.1128±0.0018	31.47±1.28
		31.31±1.16
		30.40±0.73
		31.96±1.26
		31.78±0.66
		32.63±1.19
	Weighted mean ($\chi^2/\nu=0.95$)	31.33±0.20

present on the frontal surface of the crystal. When the $N_{\gamma\pm}$ radiation was measured the radioactive source was surrounded by two copper plates and it was necessary to consider the absorption within them. Taking into account these corrections the following relation was obtained for the K/β^+ ratio:

$$\frac{K}{\beta^+}=\frac{N_X}{N_{\gamma\pm}}\frac{2\varepsilon_{\gamma\pm}P_{\gamma\pm}}{\omega_K\,\varepsilon_X\,\Omega_X}$$
$$\cdot\left[1-\frac{N_{\gamma X}}{N_X}\cdot\frac{1}{\varepsilon_\gamma P_\gamma \exp(-\mu_\gamma\cdot x'\cdot\rho_{Al})}\right]$$
$$\cdot\exp(-\mu_{\gamma\pm}\cdot x\cdot\rho_{Cu}-\mu_{\gamma\pm}\cdot x'\cdot\rho_{Al}), \quad (4)$$

where ω_K is the K X-rays fluorescence yield, Ω_X is the solid angle subtended by the radioactive source to the K X-rays detector, μ_γ are the attenuation coefficients, x is the copper width of the source holder and x' is the width of the aluminium window. The remaining symbols have the same meaning as in the case of ^{22}Na. The K/β^+ ratio is again independent of the source intensity N_0 as in the experiment of ^{22}Na. The radioactive sample was purchased from the Comisión Nacional de Energía Atómica, Argentina. The source was deposited on a mylar foil.

The X$-\gamma$ coincidence spectrometer used in the present experiment consisted of the 3″×3″ NaI(Tl) counter for detecting γ-rays and a Si(Li) crystal model 1000 of Kevex coupled to a preamplifier Kevex model 2000 A for the detection of the low energy fluorescence transitions. The radioactive source was placed at source-to-detector distances of 10 cm and 1.9 cm, respectively. The characteristics of the Si(Li) detector were: cross-sectional area 30 mm^2, thickness 5 mm and resolution 500 eV at 10 keV. The resolving time 2τ of the spectrometer coincidence system was 50 nsec. The $N_{\gamma X}$ coincidence spectrum was registered in a multichannel analyser. A logic system was employed in order to display the true-plus-chance coincidences spectrum in a 256-channel submemory, and the chance coincidences in the other 256-channel submemory. A channel by channel subtraction gave the true coincidence spectrum.

The measurement of the annihilation gamma rays $N_{\gamma\pm}$ was carried out putting the radioactive source between two copper plates of 1 mm width, placed at 10 cm from the NaI(Tl) detector mentioned above.

The efficiencies and the peak-to-total ratios were obtained from Refs. 23, 24; their values are:

$$\varepsilon_{\gamma\pm}=(21.5\pm0.1)\times10^{-3},\quad \varepsilon_\gamma=(17.2\pm0.2)\times10^{-3},$$

$$\varepsilon_X=1,\quad P_{\gamma\pm}=0.645\pm0.002$$

and

$$P_\gamma=0.392\pm0.002.$$

The solid angle was $\Omega_X=(17.3\pm0.2)\times10^{-3}$ and the fluorescence yield $\omega_K=0.445\pm0.009$ was taken from Ref. 25.

A series of 21 measurements of K/β^+ was carried out. The results together with the weighted average are listed in Table 3. The errors quoted in Table 3 are only the statistical ones. The errors due to the uncertainties of ε, P, Ω, ω_K, x and x' were added quadratically to the error of the weighted mean. This result is included in Table 2, where it can be compared with previous data.

4. Analysis and Discussion

In Tables 1 and 2 we included the theoretical calculations of f_K/f_{β^+} made by Fitzpatrick et al. [1]. In order to study the sensitivity of the ratios f_K/f_{β^+} to slightly different calculations, we performed our own evaluation of these ratios (i) using more recent estimations of the Coulomb amplitudes β_K and (ii) computing f_{β^+} employing directly the routines written for Tables III and IV of Ref. 3. These computer routines take into consideration the finite size of the nucleus as well as screening corrections. The numerical values of β_K used along this work are based on the relativistic Hartree-Fock calculations by Mann and Waber [26]; a complete source of them can be found in [27]. The energy of the bound electron W_K and the exchange corrections B_K were extracted from the tables by Lederer et al. [28] and the work by Vatai [22], respectively. The maximum energy W_0 of the positron spectrum of ^{22}Na was evaluated as a weighted average of the data collected in a review work by Behrens and Szybisz [29], while the value W_0 for ^{65}Zn was taken from [1]. All the quantities mentioned in this paragraph are listed in Table 4, where the ratios f_K/f_{β^+} are also reported.

Table 4. Values of the quantities involved in the ratios f_K/f_{β^+} and $C_K/\bar{C}_{\beta^+}$

	^{22}Na$(Z=11)$	^{65}Zn$(Z=30)$
E_0 (keV)	545.0 ± 0.6	328.7 ± 1.1
W_0 (mc^2)	2.0665 ± 0.0012	1.6432 ± 0.0022
$\bar{W}$ (mc^2)	1.421	1.280
r_0 (fm)	1.37	1.27
$R(\text{n.u.})=r_0\cdot A^{1/3}$	0.00994	0.01322
f_{β^+}	0.2815 ± 0.0013	0.01860 ± 0.00027
β_K	4.4867×10^{-2}	0.23416
β_{L_1}	1.0984×10^{-2}	…
$W_K=1-\lvert E_K\rvert$	0.9979	0.9811
$W_{L_1}=1-\lvert E_{L_1}\rvert$	0.9999	…
B_K	0.9871	0.9915
B_{L_1}	1.3428	…
f_K	2.9310×10^{-2}	0.5881
f_K/f_{β^+}	0.1041 ± 0.0005	31.62 ± 0.46
A_1	0.0049	-0.0002
$C_K/\bar{C}_{\beta^+}$	1.0033	0.9999
K/β^+	0.1044 ± 0.0005	31.62 ± 0.46

We included our theoretical results for f_K/f_{β^+} in Tables 1 and 2 to make possible a quick comparison with those of [1]. One can see that there are slight differences between both evaluations. Such discrepancies are of about 1.7% for ^{22}Na and 3.7% for ^{65}Zn, however, in both cases the theoretical estimations are in mutual agreement if one increases the uncertainties to two standard deviations.

It is usual in the literature to compare the experimental datum of an electron-capture to positron emission ratio with the theoretical K/β^+ ratio. Thus, sometimes it is necessary to obtain a K/β^+ ratio from the measured ε/β^+ ratio. This is the situation of our measurement on ^{22}Na and, therefore, in the following we describe how the K/β^+ ratio was derived from the experimental ε/β^+ ratio. In general we can write ε/β^+ as

$$\frac{\varepsilon}{\beta^+}=\frac{K}{\beta^+}\left(1+\frac{L}{K}+\frac{M}{L}\cdot\frac{L}{K}+\cdots\right), \tag{5}$$

where the total capture ε is written as the sum of the separate shell captures $(K, L, \ldots)$. In the case of ^{22}Na the capture ratio M/L can be neglected when compared with L/K and, in addition, the capture ratio L_{II}/K can be safely disregarded against L_{I}/K. So, from Equation (5) we arrive at

$$\frac{\varepsilon}{\beta^+}=\frac{K}{\beta^+}\left(1+\frac{L}{K}\right)=\frac{K}{\beta^+}\left(1+\frac{L_{\mathrm{I}}+L_{\mathrm{II}}}{K}\right)$$
$$=\frac{K}{\beta^+}\left(1+\frac{L_{\mathrm{I}}}{K}\right). \tag{6}$$

The ratio L_{I}/K can be evaluated using the Equation (61) of [3]

$$\frac{L_{\mathrm{I}}}{K}=\frac{\beta_{L_{\mathrm{I}}}^2(W_0+W_{L_{\mathrm{I}}})^2 B_{L_{\mathrm{I}}}}{\beta_K^2(W_0+W_K)^2 B_K}=0.0816. \tag{7}$$

This numerical result was obtained using the values for W_0, β_K, W_K and B_K listed in Table 4. The factor 1/1.0816 was used to calculate the K/β^+ ratios from the corresponding ε/β^+ ratios for all the cases presented in Table 1.

Our experimental value of the ratio K/β^+ for ^{22}Na quoted in Table 1 disagrees with the ratios measured in the sixties. However, it is in excellent agreement with: (i) the data of the fifties, (ii) the value obtained by McCann and Smith [14] by a direct method and (iii) both theoretical predictions the ours and that of [1].

Inspection of the experimental results listed in Table 2 indicates that our experimental value of K/β^+ for ^{65}Zn is in disagreement with the previous data the errors of which are less than 15%. However our datum agrees strikingly well with the theoretical predictions.

Although our experimental data for both nuclei, ^{22}Na and ^{65}Zn, support strongly the theoretical predictions given by Equation (2), we find interesting to estimate the corrections to $C_K/\bar{C}_{\beta^+}$ due to the contributions of the higher order FFC. When terms connected with ${}^AF_{121}^N\ (l, m, n, \rho)$ and FFC of rank 2 are neglected it is possible to demonstrate that for allowed Gamow-Teller transitions the ratio of shape factors can be written as (cf. [27, 30])

$$\frac{C_K}{C_{\beta^+}}=1+\tfrac{2}{3}A_1 \tag{8}$$

with

$$\begin{aligned}A_1=&\sqrt{\tfrac{2}{3}}\{2(W_K+\bar{W})-(1+\gamma_1/\bar{W})\}R\cdot{}^VF_{111}^0/{}^AF_{101}^0\\&+\sqrt{\tfrac{1}{3}}(1+\gamma_1/\bar{W})R\,{}^AF_{110}^0/{}^AF_{101}^0\\&-(W_K+\bar{W})R\alpha Z[{}^AF_{101}^1(1,1,1,1)/9\\&+{}^AF_{101}^1(1,2,2,1)]/{}^AF_{101}^0\\&-\tfrac{1}{9}W_0R^2\{10(W_K+\bar{W})-(1+\gamma_1/\bar{W})\}\cdot{}^AF_{101}^1/{}^AF_{101}^0,\end{aligned} \tag{9}$$

and

$$\gamma_1=\sqrt{1-(\alpha Z)^2}, \tag{10}$$

where $\bar{W}$ is the energy W averaged over the β^+ spectrum and α is the fine structure constant. The ratios of relativistic over nonrelativistic FFC, i.e. ${}^VF_{111}^0/{}^AF_{101}^0$ and ${}^AF_{110}^0/{}^AF_{101}^0$, depend rather sensitively on the nuclear structure and are difficult to calculate precisely. However, these ratios can be approximately estimated in a nuclear-structure independent way. The other ratios ${}^AF_{101}^1/{}^AF_{101}^0$, ${}^AF_{101}^1(1,1,1,1)/{}^AF_{101}^0$, and ${}^AF_{101}^1(1,2,2,1)/{}^AF_{101}^0$ can be calculated easily when we assume a uniform nucleon distribution. For the evaluation of A_1 we used the following values for the ratios

$${}^VF_{111}^0/{}^AF_{101}^0=4.7\sqrt{6}/2\,MR\,\lambda=0.00251/R, \tag{11a}$$

$${}^AF_{110}^0/{}^AF_{101}^0=\sqrt{3}/2\,MR=0.00047/R, \tag{11b}$$

$${}^AF_{101}^1/{}^AF_{101}^0=3/5, \tag{11c}$$

$${}^AF_{101}^1(1,1,1,1)/{}^AF_{101}^0=27/35, \tag{11d}$$

$${}^AF_{101}^1(1,2,2,1)/{}^AF_{101}^0=57/70. \tag{11e}$$

Here M is the nucleon mass in mc^2 units and $\lambda=-C_A=1.250\pm0.009$ is the axial vector coupling constant (cf. [31]). The results for A_1 and the ratio $C_K/\bar{C}_{\beta^+}$ are included in Table 4. In A_1 there exists a cancellation between the first and the third term. This cancellation is stronger for higher Z. Thus we have shown that the correction to $C_K/\bar{C}_{\beta^+}$ is about 0.33% for ^{22}Na, and only about 0.01% for ^{65}Zn. In general, we can state that within the approximation used in this work the corrections introduced by Equation (8) are not very important.

Since the contributions due to the induced pseudoscalar and pseudotensor terms are not well understood until now and they are not needed at all for the interpretation of the experimental data reported in the present work, we omit their analysis.

Finally, as a result of this work we can conclude that the discrepancies between theory and experiment pointed out in the review publications [1, 2] are no longer valid for the transitions studied in the present paper.

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Note Added in Proof. The exchange and overlap corrections B_K and B_{L_I} evaluated by Vatai [22] have been recently recalculated in [27]. These new values $B_K = 0.959$ and $B_{L_I} = 1.272$ for ^{22}Na and $B_K = 0.983$ for ^{65}Zn change slightly the results reported in the text. Thus, on the one hand, our experimental value derived for the ratio K/β^+ for ^{22}Na reads now $K/\beta^+ = 0.1045 \pm 0.0052$ and, on the other hand, the theoretical predictions given by Equation (2) are $K/\beta^+ = 0.1012 \pm 0.0004$ and $K/\beta^+ = 31.3 \pm 0.5$ for ^{22}Na and ^{65}Zn, respectively. However the overall conclusions remain the same.