Exact and approximate analytical solutions of groundwater response to tidal fluctuations in a theoretical inhomogeneous coastal confined aquifer

Leonardo B. Monachesi · Luis Guarracino

Abstract The influence of hydraulic conductivity heterogeneity on tide-induced head fluctuations is presented for a theoretical coastal confined aquifer. The conceptual model assumes that the hydraulic conductivity increases linearly with the distance from the coastline. This type of heterogeneity has been observed in many alluvial coastal aquifers. An exact analytical solution that predicts induced head fluctuations is obtained in terms of a Hankel function. The exact solution can be approximated by a simple mathematical expression, valid for small rates of increase of hydraulic conductivity. Both exact and approximate solutions show significant differences from the classical solution obtained for a homogeneous aquifer. Near the coastline the amplitude of the induced head fluctuation is damped but it is enhanced as the distance to the coast increases. The time-lag between sea tide and induced head fluctuation in the aquifer is not linear; it behaves as a square-root type function leading to a faster transmission of the tidal fluctuation. Hypothetical examples show that the influence of hydraulic conductivity heterogeneity can be significant and should be considered for a correct description of the groundwater response.

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Introduction

The interaction between groundwater and seawater induced by tidal fluctuations has been extensively analyzed through both analytical and numerical methods. Since the 1950s, many analytical solutions to describe this interaction have been derived. Jacob (1950) and Ferris (1951) were the first to obtain an analytical equation for a single homogeneous confined aquifer. Due to its simplicity, this equation has been widely used to estimate hydraulic parameters in coastal aquifers (e.g., Carr and van der Kamp 1969; Drogue et al. 1984; Serfes 1991; Erskine 1991; Millham and Howes 1995; Trefry and Johnston 1998; Jha et al. 2003). In recent years, more complex analytical solutions have been obtained for two-layer systems consisting of an aquifer confined by a semipermeable layer. These analytical solutions allow for the study of leakage and storage effects on the tide-induced head fluctuations (e.g., Jiao and Tang 1999; Li and Jiao 2001a,b; Li and Jiao 2002a,b; Li et al. 2002; Jeng et al. 2002; Li and Jiao 2003b; Song et al. 2007; Li et al. 2008; Sun et al. 2008). All the aforementioned theoretical results are obtained under the assumption of homogeneity of the aquifer system layers. This assumption has significant discrepancy from real aquifers, which usually exhibit inhomogeneity and anisotropy in their hydraulic properties (Li and Jiao 2003a; Trefry and Bekele 2004).

The study of heterogeneity on tide-induced head fluctuations using analytical solutions has been addressed by several researchers. Trefry (1999) presented comprehensive solutions for a finite aquifer consisting of an arbitrary number of contiguous homogeneous zones subjected to sinusoidal linear boundary conditions. Guo et al. (2010) derived an analytical solution for a semi-infinite single aquifer comprising two different homogeneous zones. Chuang et al. (2010) extended this conceptual model to a leaky aquifer system divided into a finite number of horizontal regions. Li et al. (2007), Guo et al. (2007), Xia et al. (2007), Rotzoll et al. (2008) and Geng et al. (2009) included the effect of an outlet capping in submarine aquifer systems. However, to the authors' knowledge, there are no analytical solutions that consider a continuous variation of the hydraulic properties with distance. In particular, an analytical solution that considers a continuous increase of hydraulic conductivity can be useful for studying alluvial coastal aquifers. In alluvial aquifers, progressively finer sediments are usually deposited on the downstream part of the depositional zone, giving as a result a continuous increase of hydraulic conductivity with the distance to the coastline (Freeze and Cherry 1979; Lunt et al. 2004; Carol et al. 2009; Cardenas 2010; Chuang et al. 2010). In some cases, the rate of increase of hydraulic conductivity can be significant. Montalto et al. (2006) have reported linear variations of approximately two orders of magnitude along transects of 50 m in a flooded tidal marsh in the Hudson River estuary, USA.

The objective of this technical note is to present both exact and approximate analytical solutions for tide-induced head fluctuations in a coastal confined aquifer with hydraulic conductivity that linearly increases with the distance to the coastline. Hypothetical examples are designed to test the analytical solutions and to analyze the effect of the hydraulic conductivity heterogeneity on tide-induced head fluctuations.

Mathematical model and exact analytical solution

Consider a coastal aquifer laying between two impermeable layers as shown in Fig. 1. Both the aquifer and the impermeable layers end at the coastline and extend landward infinitely. Layers are horizontal and the seaward boundary is assumed to be vertical. For the mathematical description of the problem, let the x-axis be perpendicular to the coastline, horizontal and positive landward, with its origin at the coastline. The datum of the induced head fluctuation is chosen to be the mean water level.

In order to derive an analytical solution, the following assumptions are made: the flow in the confined aquifer is horizontal and obeys Darcy's law; the effect of density variations on water flow is neglected; the

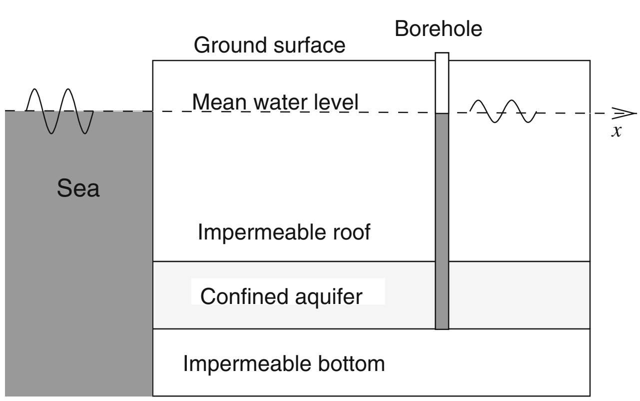

Fig. 1 Schematic representation of the confined coastal aquifer. The x-axis is represented by a *dashed line*

hydraulic conductivity increases with the horizontal distance; the specific storativity is constant. According to the aforementioned assumptions, the governing equation for the head fluctuations within the confined aquifer can be written as (Bear 1988):

$$\frac{\partial}{\partial x}\left(K(x)\frac{\partial h}{\partial x}\right) = S_s \frac{\partial h}{\partial t} \tag{1}$$

where $h(x,t)$ is the groundwater head [L], S_s the specific storativity [L^{-1}] and $K(x)$ the hydraulic conductivity [LT^{-1}], x the distance to the coastline [L], and t the time [T]. Note that Eq. (1) applies strictly to a confined aquifer. However when the fluctuations in h are small compared to the aquifer thickness, it can also be applied to unconfined aquifers (Bear 1988; Townley 1995, Guo et al. 2010).

The boundary condition at the interface of the sea and the aquifer is expressed as:

$$h(0,t) = A\cos(\omega t) \tag{2}$$

where $h(0,t)$ is the head at $x=0$, A the tidal amplitude [L] and ω the tidal angular frecuency [T^{-1}]. At infinity, the following no-flow boundary condition is used:

$$\lim_{x\to\infty} K(x)\frac{\partial h}{\partial x} = 0 \tag{3}$$

Here, only the periodic solution is considered, so $-\infty < t < \infty$ and no initial conditions are needed.

Assume a linear increase of the hydraulic conductivity with the distance x:

$$K(x) = K_0(1 + bx) \tag{4}$$

where K_0 [LT^{-1}] is the hydraulic conductivity of the aquifer at $x = 0$ and b [L^{-1}] is the rate of increase ($b \geq 0$). A linear model for $K(x)$ could be considered arbitrary, but is an initial step towards a more realistic description of some type of heterogeneous aquifers. For example, Cardenas (2010) uses Eq. (4) for describing some features of groundwater dynamics in a fluvial island that can not be accurately represented by a constant hydraulic conductivity. It is also worth mentioning that the hydraulic conductivity given by Eq. (4) tends to infinity when x tends to infinity. This is an unrealistic value for K; however, the analytical solution is not affected by the values of hydraulic conductivity far away from the coast, as will be shown in the next section.

The exact analytical solution of the boundary value problem Eqs. (1)–(4) is presented in the Appendix and further expanded in the electronic supplementary material (ESM), and is given by:

$$h(x,t) = \mathrm{Re}\left[A\frac{H_0^{(1)}(2\Lambda\sqrt{1+bx})}{H_0^{(1)}(2\Lambda)}e^{i\omega t}\right] \tag{5}$$

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where Re denotes the real part of the expression, $H_0^{(1)}$ is the first kind Hankel function of zero order, $\Lambda = a(-1+i)$ / b, and a [L^{-1}] the tidal propagation parameter defined as:

$$a = \sqrt{\frac{\omega S_s}{2K_0}}. \tag{6}$$

For a homogeneous aquifer ($K(x) = K_0$), the inverse of a is the characteristic dampening distance for which the amplitude of the induced head fluctuation decays to A/e~$0.36A$.

Discussion of the exact analytical solution

To explore the influence of the hydraulic conductivity heterogeneity on tide-induced head fluctuations, the following hypothetical example is designed. The hydraulic parameters of the confined aquifer are assumed to be K_0= 1 m/h, S_s=10^{-5} m^{-1} and b=10^{-2} m^{-1}. The sea tide is considered semidiurnal (period of 12.4 h) with an amplitude A of 1 m. The tidal propagation parameter computed using Eq. (6) is a=1.59 10^{-3} m^{-1}. For the sake of simplicity distances are expressed in dimensionless form multiplying x by a.

Figure 2 shows the sea tide and head fluctuations for both heterogeneous and homogeneous aquifers at two representative points located near (ax=0.25) and far (ax=2.0) from the coast. At ax=0.25 the induced head fluctuation for the heterogeneous aquifer (curve for b in Fig 2a) has a smaller amplitude than the homogeneous one. However, far from the coast (ax=2.0), the amplitude of the fluctuation given by Eq. (5) is greater than the homogeneous case and a significant time-lag between both responses is also observed. This simple example shows that the effect of heterogeneity on head fluctuations is significant and strongly depends on the distance to the coast.

For a better understanding of the influence of heterogeneity on induced head fluctuations, both the amplitude and time-lag as functions of the dimensionless distance ax are analyzed. The amplitude (h_{max}) and time-lag (t_{lag}) of head fluctuations have the following expressions:

$$h_{max}(x) = A\sqrt{B_r^2 + B_i^2} \tag{7}$$

$$t_{lag}(x) = -\frac{1}{\omega}\tan^{-1}\left(\frac{B_i}{B_r}\right) \tag{8}$$

where:

$$B_r = \mathrm{Re}\left[\frac{H_0^{(1)}(2\Lambda\sqrt{1+bx})}{H_0^{(1)}(2\Lambda)}\right], B_i = \mathrm{Im}\left[\frac{H_0^{(1)}(2\Lambda\sqrt{1+bx})}{H_0^{(1)}(2\Lambda)}\right] \tag{9}$$

Figure 3 shows the amplitudes computed using Eq. (7) for three different rates of increase of hydraulic conductivity b= 10^{-1}, 10^{-2}, 10^{-3} m^{-1}. As a reference, the figure includes the amplitude of a homogeneous aquifer which has the following analytical expression: Ae^{-ax} (Jacob 1950). In comparison with the homogeneous model, the linear heterogeneity produces more damped amplitudes for distances less than approximately $1/a$ (characteristic dampening distance). In this region near the coast, the damping effect increases with the values of b. An opposite behavior is observed for increasing inland distances: the amplitudes of the heterogeneous aquifer are enhanced, leading to larger intrusion of induced fluctuations in the aquifer. This enhancing effect significantly increases with the inland distance, particularly for large values of b.

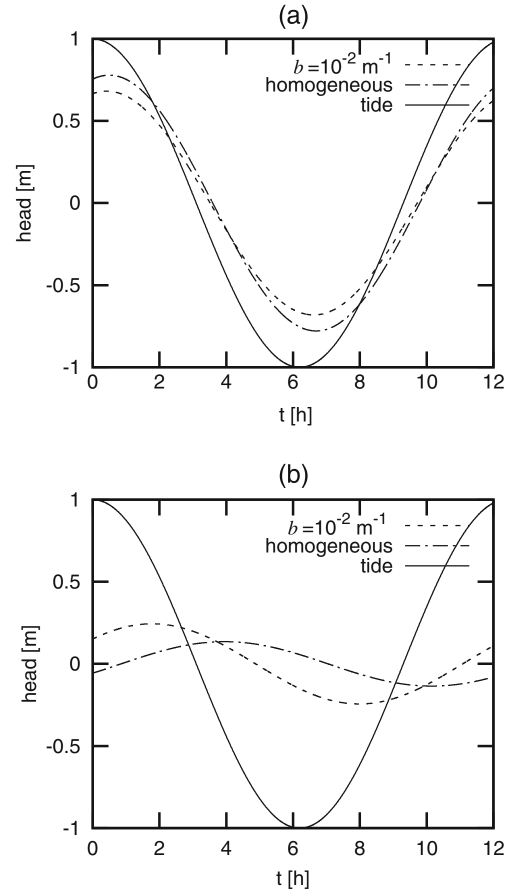

Fig. 2 Tide-induced head fluctuation versus time at different observation points: **a** a=0.25 and **b** a=2.0

The time-lag between the sea tide and the induced head fluctuation on the heterogeneous aquifer can be computed from Eq. (8). Figure 4 shows time-lags as a function of ax for b= 10^{-1}, 10^{-2}, 10^{-3} m^{-1}. Time-lags of the heterogeneous aquifers are smaller than the time-lag of the homogeneous aquifer, which show a linear increase with distance. The time-lag of the heterogeneous aquifer behaves as a square-root type function, giving as a result a faster transmission of the tidal fluctuation. As it would be expected, the propagation velocity of the induced tide in the aquifer increases with b.

In order to analyze the effect of unrealistic values of K predicted by Eq. (4) when x tends to infinity, an analytical

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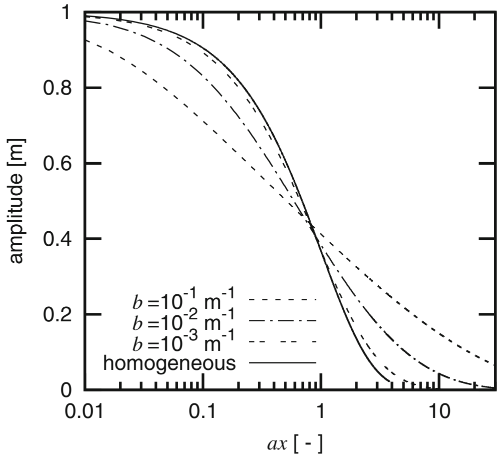

Fig. 3 Amplitude versus dimensionless distance *ax* in log scale for different rates of increase of hydraulic conductivity

solution for a finite aquifer of extension L is derived. In this case, Eq. (1) is solved in a finite domain with the tidal condition Eq. (2) and the following no-flow boundary condition in the right edge of the aquifer:

$$K(x)\frac{\partial h}{\partial x} = 0, \text{in } x = L \tag{10}$$

Based on a similar reasoning to derive Eq. (5), it can be shown that the analytical solution of the boundary value problem defined by Eqs. (1), (2) and (10) is given by:

$$h(x,t) = \text{Re}\left[A\left(C_1 J_0(2\Lambda\sqrt{1+bx}) + C_2 Y_0(2\Lambda\sqrt{1+bx})\right)e^{i\omega t}\right] \tag{11}$$

where

$$C_1 = \frac{Y'_0(2\Lambda\sqrt{1+bL})}{J_0(2\Lambda)Y'_0(2\Lambda\sqrt{1+bL}) - Y_0(2\Lambda)J'_0(2\Lambda\sqrt{1+bL})} \tag{12}$$

$$C_2 = \frac{-J'_0(2\Lambda\sqrt{1+bL})}{J_0(2\Lambda)Y'_0(2\Lambda\sqrt{1+bL}) - Y_0(2\Lambda)J'_0(2\Lambda\sqrt{1+bL})} \tag{13}$$

with J_0 and Y_0 being the first and second kind Bessel functions of zero order.

Figure 5 shows how the amplitudes of induced head fluctuations change with the distance to the coast for both an infinite aquifer and finite aquifers of dimensionless extension aL=10 and 30. In all cases, the hydraulic parameters of the aquifer are the same as those used in Fig. 2. It can be seen that with increasing of the extension aL, the amplitudes predicted by Eq. (11) quickly tend to the amplitude of the infinite aquifer given by Eq. (5). This test demonstrates that induced head fluctuations are mainly determined by the values of hydraulic conductivity near the coast and Eq. (5) is valid even though the values of K predicted by Eq. (4) are unrealistic.

Asymptotic approximation of the exact solution

In this section, an approximate expression of the exact solution (Eq. 5) is obtained for relatively small rates of increase of hydraulic conductivity. The approximate analytical solution is valid for:

$$b << 2^{7/2}a \approx 10a \tag{14}$$

For these values of b the arguments of the Hankel function of Eq. (5) satisfy:

$$\left|2\Lambda\sqrt{1+bx}\right| >> \frac{1}{4}, \quad |2\Lambda| >> \frac{1}{4} \tag{15}$$

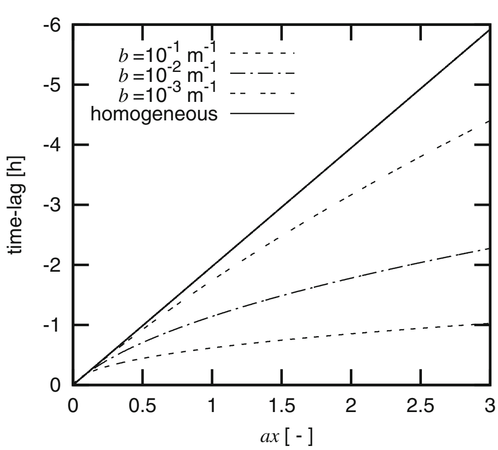

Fig. 4 Time-lag versus dimensionless distance *ax* for different rates of increase of hydraulic conductivity

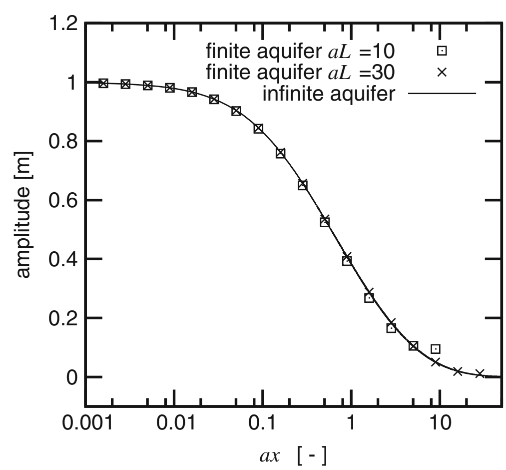

Fig. 5 Amplitude versus dimensionless distance *ax* in log scale for finite and infinite aquifers

and the following asymptotic approximations of $H_0^{(1)}$ hold (Arfken and Weber 2005):

$$H_0^{(1)}\left(2\Lambda\sqrt{1+bx}\right) \approx \left[\pi\Lambda\sqrt{1+bx}\right]^{-1/2} e^{i\left[2\Lambda\sqrt{1+bx}-\frac{1}{4}\pi\right]} \tag{16}$$

$$H_0^{(1)}(2\Lambda) \approx \pi\Lambda^{-1/2} e^{i\left[2\Lambda-\frac{1}{4}\pi\right]} \tag{17}$$

By replacing Eqs. (16) and (17) in Eq. (5), the following approximate solution is obtained:

$$h(x,t) = A\frac{e^{-2\frac{a}{b}\left(\sqrt{1+bx}-1\right)}}{(1+bx)^{\frac{1}{4}}}\cos\left[-2\frac{a}{b}\left(\sqrt{1+bx}-1\right)+\omega t\right] \tag{18}$$

Although the validity of Eq. (18) is limited to values of b given by Eq. (14), its mathematical expression is simple and can be used for a qualitative analysis of induced head fluctuations.

When the heterogeneity of the hydraulic conductivity is negligible, i.e., $b \rightarrow 0$ it can be shown that Eq. (18) becomes:

$$h(x,t) = Ae^{-ax}\cos(-ax+\omega t) \tag{19}$$

which is the analytical solution obtained by Jacob (1950) for a homogeneous confined aquifer.

Figures 6 and 7 show the amplitudes and phase-lags, respectively, of exact and approximate solutions (Eqs. 5 and 18) for two different magnitudes of the rate of increase of hydraulic conductivity b. As it is expected, for a relatively small rate of increase ($b=10^{-3}$ m^{-1}), the values of amplitude and phase-lag predicted by Eq. (18) are in excellent agreement with the ones of the exact solution Eq. (5). On the other hand, for $b=10^{-1}$ m^{-1}, the condition (Eq. 14) is not satisfied and significant discrepancies are observed between values predicted by both analytical solutions.

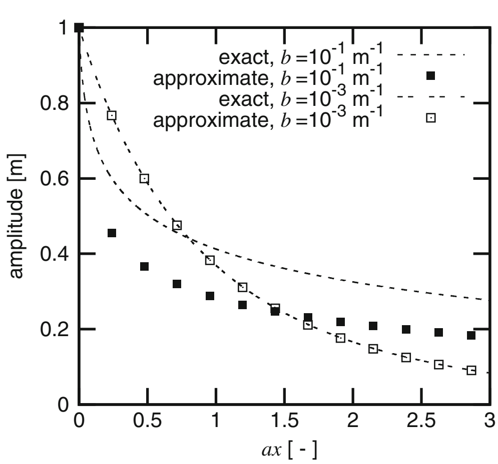

Fig. 6 Amplitude of exact and approximate analytical solutions versus dimensionless distance *ax*

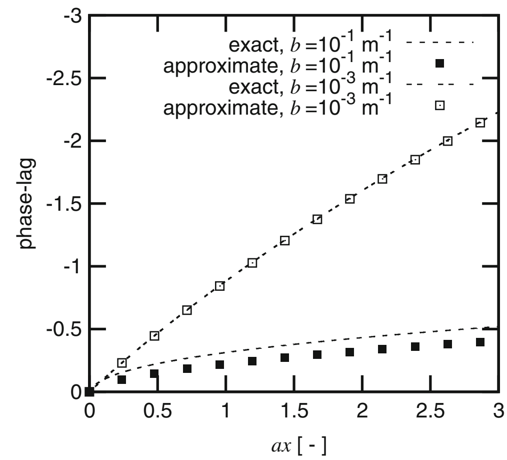

Fig. 7 Phase-lags of exact and approximate analytical solutions versus dimensionless distance *ax*

Conclusions

This technical note investigates tide-induced head fluctuations in a confined coastal aquifer whose hydraulic conductivity linearly increases with the distance to the coast. An exact analytical solution that predicts head fluctuations is derived in terms of a Hankel function. For small rates of increse of hydraulic conductivity, an approximate analytical solution with a simple mathematical expression is also obtained. In general terms, it can be concluded that the linear heterogeneity in hydraulic conductivity produces the following effects on the induced head fluctuations: (1) dampened amplitudes for distances less than the characteristic dampening distance $1/a$, (2) enhanced amplitudes for increasing inland distances ($x > 1/a$) and (3) a faster transmission of the tidal effect with a time-lag that can be approximated with a square-root type function. Hypothetical examples show that the influence of the hydraulic conductivity heterogeneity can be significant and should be included in the study of coastal aquifers where this type of heterogeneity has been reported.

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Appendix

Let $H(x,t)$ be the solution of the following boundary value problem:

$$\frac{\partial}{\partial x}\left(K_0(1+bx)\frac{\partial H}{\partial x}\right) = S_s\frac{\partial H}{\partial t} \tag{20}$$

$$H(0,t) = Ae^{i\omega t} \tag{21}$$

$$\lim_{x\to\infty} K(x)\frac{\partial H}{\partial x} = 0 \tag{22}$$

Then, the solution of Eqs.(1)–(4) satisfies:

$$h(x,t) = \mathrm{Re}[\, H\ (x,t)] \tag{23}$$

where Re denotes the real part of the followed complex expression. In order to verify the boundary condition Eq. (21), $H(x,t)$ must be expressed as:

$$H(x,t) = AX(x)e^{i\omega t} \tag{24}$$

where $X(x)$ is a complex function.

Substituting Eq.(24) in Eqs.(20)–(22), the following boundary value problem is obtained:

$$\frac{d}{dx}\left((1+bx)\frac{dX}{dx}\right) + b^2\Lambda^2 X = 0 \tag{25}$$

$$X(0) = 1 \tag{26}$$

$$\lim_{x\to\infty}(1+bx)\frac{dX}{dx} = 0 \tag{27}$$

where:

$$\Lambda^2 = -i\frac{\omega S_s}{b^2 K_0} = -i\,2\left(\frac{a}{b}\right)^2 \tag{28}$$

In order to find the general solution of Eq.(25), the following change of variables is proposed:

$$u = 2\Lambda\sqrt{1+bx} \tag{29}$$

where $\Lambda = a(-1+i)\,/b$. Replacing Eq.(29) in Eq.(25) gives:

$$u^2\frac{d^2X}{du^2} + u\frac{dX}{du} + u^2 X = 0 \tag{30}$$

The ordinary differential Eq.(30) is the zero order Bessel equation and its general solution can be written as (Abramowitz and Stegun 1965):

$$X(u) = C_1 J_0(u) + C_2 Y_0(u) \tag{31}$$

where J_0 and Y_0 are the first and second kind Bessel functions of zero order, and C_1 and C_2 are complex constants.

Using asymptotic expressions for the derivatives of J_0 and Y_0 (Arfken and Weber 2005), it can be shown that the boundary condition Eq.(27) is satisfied when:

$$(C_1 + iC_2) = 0 \tag{32}$$

Then:

$$X(u) = C_1(\,J_0(u) + i\,Y_0(u)) = C_1 H_0^{(1)}(u) \tag{33}$$

where ${H_0}^{(1)}$ is the Hankel function of zero order and first kind (Abramowitz and Stegun 1965). Now, imposing the boundary condition Eq.(26) to Eq.(33) yields:

$$C_1 = H_0^{(1)}(2\Lambda)^{-1} \tag{34}$$

Then the solution of the boundary value problem Eqs. (25)–(27) is:

$$X(x) = \frac{H_0^{(1)}(2\Lambda\sqrt{1+bx})}{H_0^{(1)}(2\Lambda)} \tag{35}$$

Finally, in virtue of Eqs.(23) and (24):

$$h(x,t) = \mathrm{Re}\left[A\frac{H_0^{(1)}(2\Lambda\sqrt{1+bx})}{H_0^{(1)}(2\Lambda)}e^{i\omega t}\right] \tag{36}$$

which is the exact analytical solution for the boundary value problem Eqs.(1)–(4).

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