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Some comments on the topological features of some 3-,4-connected networks and their relationships with the numbers e and π

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This paper describes an approximate mathematical formula that relates the two transcendental mathematical constants e and π to each other through the use of only four other simple integers. The formula arises out of a consideration of the topological character of certain 3-,4-connected networks.

KEY WORDS: numbers e and π, 3-,4-connected networks, topological character

1. Introduction

Some considerations of the topological character of certain 3-,4-connected networks lead to rather quite interesting connections with the two transcendental numbers e and π [1,2]. These results have come as an outgrowth of earlier results [1] on the computation of certain structural properties of properly scaled versions of some commonly known structure-types in crystal chemistry including the cubic diamond and Waserite networks.

The mathematical constants e and π are ubiquitous in mathematical and scientific formulae. In addition they are known as the transcendental numbers as they are infinitely, non-repeating continued fractions [3,4]. A formula has been devised herein where these two transcendental mathematical constants are related to each other by only four other simple integers: 3, 4, 5, and 7.

This formula arises out of considerations of the structural character of certain 3-,4-connected networks including the Pt_3O_4 structure-type, which was first reported by Waser et al. [5]. In particular, it is the topological character of such networks, in which a 3-to-4 stoichiometry of 4-connected vertices to 3-connected vertices, respectively holds, that gives rise to this formula. Therefore, the formula

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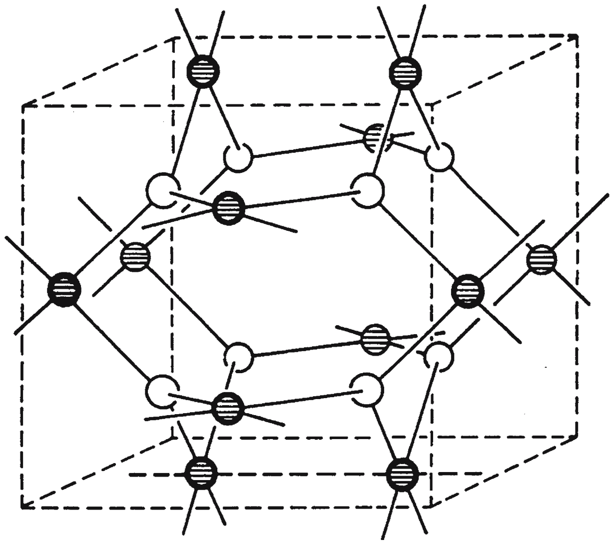

Figure 1. Crystal structure of Waserite or the Pt_3O_4 lattice.

arises through the identification of the corresponding connectivity index, p, of such 3-,4-connected networks [1,2]. The connectivity index, p, for these networks has been shown, in a separate publication, to be equal to a weighted average over the valences of the vertices in the unit cell of a structure [2]. For the Pt_3O_4 lattice, shown in figure 1, the following relations (1a) and (1b) hold.

$$p = (3\cdot 4 + 4\cdot 3)/7 \tag{1a}$$

$$p = (2/5)e\cdot\pi. \tag{1b}$$

It is relations (1a) and (1b) that form the basis for the formula described in section 2 below.

2. Mathematical formulae

The $e\cdot\pi$ relation, which is only approximate, and returns an equality to within > 99% of the true value of the product $e\cdot\pi$, is shown as equation (2) below:

$$e\cdot\pi = \frac{3 \times 4 \times 5}{7}. \tag{2}$$

It is interesting in this context that two transcendental numbers can be related to each other to within >99% accuracy through the use of only four simple integers. In fact, the relation can be factored such that it is entirely based upon the first 4 prime numbers 2, 3, 5, and 7.

Equation (3) shows a reformulation of the approximate $e{\cdot}\pi$ relation given in equation (1) to emphasize the strange nature of the equation:

$$1{\cdot}2.333333333333333\ldots\ldots \cdot e \cdot \pi = 4{\cdot}5. \tag{3}$$

In this case we see that 1 multiplied by the continued fraction represented as $2^{1/3}$, multiplied by e and π, leads to the product of 4 and 5. It is as if the integer sequence 1, 2, 3, 3, 3, 3, 3.....is transformed to the integer sequence 4,5 by the insertion of the mathematical factor $e{\cdot}\pi$.

Rearranged with the factor $2^{1/3}$ placed as a denominator, underneath the factor $4 \cdot 5$ on the righthand side of (2), the relation can be seen to be suggestive of the existence of the 5 Elements of Plato's *Timeas*. Plato equated the Ancient Greek element fire with the Platonic solid known as the tetrahedron. This occurs as the factor 1 on the lefthand side of (2), and can be seen to be self-reciprocal or self-dual, as is the tetrahedron (3,3) [1,2]. In the denominator on the righthand side of (2), the factor 2 in $2^{1/3}$ can be seen to represent the Ancient Greek element air, or the octahedron (3,4), which is reciprocal, or dual, to the Ancient Greek element earth, or the cube (4,3), that occurs in the numerator, in a reciprocity with the factor 2, as the factor 4. Finally, the decimal factor 0.3333....... in $2^{1/3}$ in the denominator of (2) can be seen to represent the Ancient Greek element water, or the icosahedron (3,5), which is reciprocal, or dual, to the Ancient Greek element called the quintessence, or the dodecahedron (5,3), that occurs as the factor 5 in the numerator in (2).

Therefore there are the first 5 simple integers in relation (2) that connect the transcendental numbers e and π to each other. These numbers: 1, 2, 3, 4, and 5, are thus seen to be suggestive of the 5 regular polyhedra constructed in the culmination of Euclid's *Elements* and employed as the Ancient Greek elements in Plato's cosmogony developed in his treatise called the *Timeas*.

3. Conclusion

We see in this communication that, in fact, simple topological considerations of certain crystallographically defined networks [5], or patterns, give rise to a connection to the fundamental constants of mathematics. Relations similar to that shown here have been derived, separately, to define the constants π and e by independent topological-geometrical considerations, based upon the intrinsic structural character of scaled versions of the cubic diamond and Pt_3O_4 lattices, as well as other lattices [1].

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