# The calculation of $2 \boldsymbol{\nu} \boldsymbol{\beta} \boldsymbol{\beta}$ matrix elements within the collective description *) 

D.R. Bes<br>Dept. de Física, CNEA, Av. Libertador 8250, (1429) Bs. As., Argentina and<br>Dept. de Física, Univ. Favaloro, Belgrano 1723, (1093) Bs. As., Argentina<br>O. Civitarese<br>Dept. de Física, UNLP, C.C. 67, (1900) La Plata, Argentina

Received 31 January 2002
The problem of particles coupled by an isovector pairing interaction is solved by means of a collective treatment. The Hamiltonian and the beta-decay operators acting within the np-subspace are constructed. The procedure reproduces exact results for a schematic model. The realistic calculation of the Fermi amplitude for the double-beta decay of ${ }^{76} \mathrm{Ge}$ confirms the current expectation about its smallness relative to the Gamow-Teller decay.

PACS: 21.60.Jz, 22.40.Hc, 21.60.Fw
Key words: collective variables, Fermi and Gamow-Teller transitions

## 1 Outline of the talk

A well known procedure in many-body problems is to include part of the interaction as a mean field that does not exhibit the same symmetries as the original problem. However, the naive application of the procedure often leads to unstable results. This is the case for the calculation of double-beta decay matrix elements in medium and heavy nuclei. In the first part of the lecture we shall illustrate with a very simple example how to proceed in such cases. In the second part we shall apply this procedure to the pairing field and to the calculation of schematic cases. Finally we shall present the results of some realistic calculations.

## 2 The model Mobol

A ball is constrained to move along a ring with radius $r_{0}$. Although the solution of this problem is simple enough in the laboratory frame,

$$
\begin{equation*}
x^{(l)}=r_{0} \cos \omega t, \quad y^{(l)}=r_{0} \sin \omega t \tag{1}
\end{equation*}
$$

it is even simpler in a frame rotating with the ball, namely

$$
\begin{equation*}
x=r_{0}, \quad y=0 \tag{2}
\end{equation*}
$$

[^0]The absence of angular motion in the intrinsic frame must be compensated by the introduction of the angular coordinate $\theta$ relating the motion of the intrinsic frame to the laboratory frame. From here on it is denoted as a collective coordinate, as opposed to the original variable in the moving frame, which is called intrinsic. We are faced with the same difficulties encountered, for instance, in nuclear physics, namely:
i. the solution violates the original symmetry of the problem;
ii. it has more degrees of freedom than those of the initial problem;
iii. there is no restoring force in the $y$-direction (the origin of the instabilities may be traced back to this fact);
iv. there is an "order parameter", represented by $r_{0}$, which should be larger than the fluctuations in the $y$-direction.

As known from the treatment of descriptions from intrinsic frames (see, for instance, Ref. [1]), we must introduce constraints, which in case of Mobol read

$$
\begin{align*}
j-I & =0  \tag{3}\\
H & =\frac{1}{2 m} p_{y}^{2}=\frac{1}{2 \Im} j^{2}, \quad \Im=m r_{0}^{2} . \tag{4}
\end{align*}
$$

Here $I$ generates collective rotations $([\theta, I]=i)$ and $j$, intrinsic rotations $\left(j=r_{0} p_{y}\right)$. Eq. (3) expresses the fact that the description obtained by moving the ball trough a certain angle is completely equivalent to that one moving the rotating frame through the opposite angle. The Hamiltonian (4) must be solved taking into account the constraint (3). Physical states satisfy the constraint; unphysical states do not. According to Dirac [2], the constraints are taken into account by demanding the vanishing of the variable $B$ conjugate to the Lagrange multiplier

$$
\begin{align*}
B & =0, \quad[\lambda, B]=\mathrm{i}  \tag{5}\\
H_{\text {Dirac }} & =\frac{1}{2 \Im} J^{2}-\lambda(J-I) . \tag{6}
\end{align*}
$$

The Dirac Hamiltonian is equivalent to the initial one for physical states. Along the process of quantifying gauge fields, Becchi, Rouet, Stora and Tyutin (BRST) [3] have further developed these ideas, by introducing additional (ghost) variables. The previous constraints are substituted by the requirement

$$
\begin{align*}
Q & =-\eta(J-I)+\bar{\pi} B=0, \quad[\eta, \pi]_{+}=[\bar{\eta}, \bar{\pi}]_{+}=1  \tag{7}\\
H_{\mathrm{BRST}} & =H_{\mathrm{coll}}+H_{\mathrm{sp}}  \tag{8}\\
H_{\mathrm{coll}} & =\frac{1}{2 \operatorname{Im}} I_{0}^{2} \\
H_{\mathrm{sp}} & =\frac{1}{2 \Im}\left(J-I_{0}\right)^{2}-\lambda J+\omega^{2}\left(B \theta-\frac{1}{2 \Im} B^{2}\right)+\mathrm{i}\left(\pi \bar{\pi}+\omega^{2} \eta \bar{\eta}\right) .
\end{align*}
$$

The BRST Hamiltonian displays a collective term representing a rotor with angular momentum $I_{0}$. Since the original Mobol problem has only one (rotational) degree of
freedom, all the ones appearing in $H_{\text {sp }}$ are spurious. It is important to mention that it is possible to construct a vacuum for the spurious sector which is also the only physical state in this sector. The application of the BRST invariance to many-body problems was carried out in Ref. [4]. In summary:
i. the collective coordinate has become a real variable. As a trade off, the $y$ degree of freedom in the intrinsic frame has been transferred to the spurious sector;
ii. the final Hamiltonian involves one real and four spurious degrees of freedom. However, the last ones become isolated even in more complicated cases. They can be treated in perturbation theory, using $r_{0}^{-1}$ as the expansion parameter;
iii. the lost symmetries have been restored at the collective level;
iv. the singularities inherent to the absence of restoring forces in the $y$-direction are eliminated.
v. the introduction of collective coordinates is the first step of an exact manybody treatment which must be carried out consistently.

We may also allow some fluctuations in the $x$-direction, which in this model correspond to real, intrinsic excitations. In such a case the Mobol potential becomes the Mexican hat potential. Such extension represents a non trivial caricature of a realistic situation in which both intrinsic and collective real excitations are present and in which there is an isolated spurious sector [4].

## 3 The pairing-isospin case

Since the possible double-beta decay emitters are medium and heavy nuclei, one has to resort to mean field treatments, such as the BCS + RPA. Within such an approach, it was shown that the inclusion of pairing-type proton-neutron interactions resulted in the suppression of the double-beta decay matrix elements $[5,6]$. Although this suppression was also obtained with several other approaches [7] and it was confirmed by the few available shell model calculations [8], the reliability of the theoretical predictions has been hampered by instabilities in the treatment. An alternative approach based on group theoretical methods has confirmed the existence of zero-energy states $[9,10]$ which were interpreted as the signature of a phase transition [11]. In this presentation the problem is tackled along the lines described in Section 2. Instead of the simple model we must deal with a many-body problem including both intrinsic and collective degrees of freedom. Such unified models play a fundamental role in nuclear physics [12]. Dr. Civitarese will talk about the model encompassing the single-particle spherical field and the translational motion in the following lecture. I am going to deal now with the problem of simultaneous pairing and isospace deformations [13]. In this case there are four constraints

$$
\begin{equation*}
\tau_{k}-T_{k}=0 \tag{9}
\end{equation*}
$$

corresponding to the three components of the intrinsic isospin $\tau_{q}$ and to the number of pairs of particles $\tau_{a}$. The operators $T_{q}$ and $T_{a}$ are the corresponding collective generators.

### 3.1 The Hamiltonian

The following Hamiltonian has been frequently used in the literature.

$$
\begin{equation*}
H=\epsilon_{v j} \tau_{v j}-g_{v} S_{v}^{+} S_{v}-\frac{1}{2} g_{\perp} S_{\perp}^{+} S_{\perp} \tag{10}
\end{equation*}
$$

where $\epsilon_{v j}$ and $\tau_{v j}$ are the single-particle energy and number operators of protons ( $v=\mathrm{p}$ ) or neutrons ( $v=\mathrm{n}$ ) in the $j$-shell. We denote by $S_{\mathrm{p}}^{+}, S_{\mathrm{n}}^{+}$and $S_{\perp}^{+}$the pairing operators creating a proton pair, a neutron pair and a proton-neutron pair coupled to angular momentum zero.

$$
\begin{equation*}
S_{v}^{+}=c_{v j m}^{+} c_{v j \bar{m}}^{+}, \quad S_{\perp}^{+}=c_{\mathrm{p} j m}^{+} c_{\mathrm{n} j \bar{m}}^{+}+c_{\mathrm{n} j m}^{+} c_{\mathrm{p} j \bar{m} \overline{+}}^{+} \tag{11}
\end{equation*}
$$

The Hamiltonian allows for the differences between proton and neutron singleparticle energies and pairing strengths, and for an (arbitrary) strength of the neutron-proton isovector pairing component. As a consequence of the presence of isovector and isoquadrupole terms in (10), this Hamiltonian does not, in general, conserve isospin. Therefore we are faced with a problem that, to our knowledge, had not been tackled before: to disentangle unphysical violations of the symmetries introduced by the formalism (through the use of the basic set of states determined by the field approximation) from those violations produced by the lack of invariance of the effective nuclear Hamiltonian. Since the calculations are performed in the intrinsic system, the Hamiltonian (as any other operator), should be transformed to this frame. We obtain

$$
\begin{align*}
H= & H_{0}+H_{1}+H_{2}  \tag{12}\\
H_{0}= & \epsilon_{a j} \tau_{a j}-g_{0}\left(S_{\mathrm{p}}^{+} S_{\mathrm{p}}+S_{\mathrm{n}}^{+} S_{\mathrm{n}}+\frac{1}{2} S_{\perp}^{+} S_{\perp}\right) \\
H_{1}= & \epsilon_{0 j} D_{0 \sigma}^{1} \tau_{\sigma j}-g_{1}\left[D_{00}^{1}\left(S_{\mathrm{p}}^{+} S_{\mathrm{p}}-S_{\mathrm{n}}^{+} S_{\mathrm{n}}\right)\right. \\
& \left.-\frac{1}{\sqrt{2}} D_{01}^{1}\left(S_{\mathrm{p}}^{+} S_{\perp}+S_{\perp}^{+} S_{\mathrm{n}}\right)+\frac{1}{\sqrt{2}} D_{0 \overline{1}}^{1}\left(S_{\mathrm{n}}^{+} S_{\perp}+S_{\perp}^{+} S_{\mathrm{p}}\right)\right] \\
H_{2}= & -g_{2}\left[D_{00}^{2}\left(S_{\mathrm{p}}^{+} S_{p}+S_{\mathrm{n}}^{+} S_{\mathrm{n}}-S_{\perp}^{+} S_{\perp}\right)\right. \\
& +\sqrt{\frac{3}{2}} D_{01}^{2}\left(S_{\perp}^{+} S_{\mathrm{n}}-S_{\mathrm{p}}^{+} S_{\perp}\right)+\sqrt{\frac{3}{2}} D_{0 \overline{1}}^{2}\left(S_{\perp}^{+} S_{\mathrm{p}}-S_{\mathrm{n}}^{+} S_{\perp}\right) \\
& \left.+\sqrt{6}\left(D_{02}^{2} S_{\mathrm{p}}^{+} S_{\mathrm{n}}+D_{0 \overline{2}}^{2} S_{\mathrm{n}}^{+} S_{\mathrm{p}}\right)\right]
\end{align*}
$$

We may verify that the transformed Hamiltonian commutes with the four constraints of the problem, as befits physical operators. This is not the case for (10), which is unphysical. In (12) we have used

$$
\begin{align*}
g_{0} & =\frac{1}{3}\left(g_{\mathrm{p}}+g_{\mathrm{n}}+g_{\perp}\right), \quad g_{1}=\frac{1}{2}\left(g_{\mathrm{p}}-g_{\mathrm{n}}\right), \quad g_{2}=\frac{1}{6}\left(g_{\mathrm{p}}+g_{\mathrm{n}}-2 g_{\perp}\right), \\
\epsilon_{a j} & =\epsilon_{\mathrm{p} j}+\epsilon_{\mathrm{n} j}, \quad \epsilon_{0 j}=\epsilon_{\mathrm{p} j}-\epsilon_{\mathrm{n} j},  \tag{13}\\
\tau_{a j} & =\frac{1}{2}\left(\tau_{\mathrm{p} j}+\tau_{\mathrm{n} j}\right), \quad \tau_{0 j}=\frac{1}{2}\left(\tau_{\mathrm{p} j}-\tau_{\mathrm{n} j}\right) .
\end{align*}
$$

Up to now the Hamiltonian (12), together with the constraints (9), constitutes an exact complication of the original problem. As in the case of Mobol, we look for
simplification through the definition of the intrinsic system (cf. Eq. (2)). Since there are four angular variables in the collective subspace, we may choose four conditions defining the intrinsic frame [14]. The selection

$$
\begin{equation*}
\operatorname{Im}\left\langle S_{\mathrm{p}}\right\rangle=\operatorname{Im}\left\langle S_{\mathrm{n}}\right\rangle=\left\langle S_{\perp}\right\rangle=0 \tag{14}
\end{equation*}
$$

leads to the usual Bogolyubov-Valatin transformation between identical particles. Albeit this selection of a gauge constitutes a violation of isospin and gauge symmetries, it is performed in the intrinsic frame. The np-pairing is incorporated through the collective rotations in isospace and gauge space. The two remaining expectation values $\left\langle S_{\mathrm{p}}\right\rangle$ and $\left\langle S_{\mathrm{n}}\right\rangle$ are real and considered to be the "order parameters", i.e. the large quantities of the system. If the isospin $T$ is also large, the $D$ functions may be treated within a boson description through a generalization of the HolsteinPrimakoff algebra [15]. If we assume that the constraints (9) hold (and we know how to enforce them through the Dirac-BRST procedure, as in the case of Mobol), the leading orders of the the Hamiltonian simplify to

$$
\begin{align*}
& \langle H\rangle=\epsilon_{v j}\left\langle\tau_{v j}\right\rangle-g_{v}\left\langle S_{v}\right\rangle^{2}, \\
& H^{(20)}-\left\langle\lambda_{v}\right\rangle \tau_{v}^{(20)}=e_{v j} \tau_{v j}^{(20)}-g_{v}\left\langle S_{v}\right\rangle\left(S_{v}^{+(20)}+S_{v}^{(20)}\right),  \tag{15}\\
& H^{(11)}-\left\langle\lambda_{v}\right\rangle \tau_{v}^{(11)}=e_{v j} \tau_{v j}^{(11)}-g_{v}\left\langle S_{v}\right\rangle\left(S_{v}^{+(11)}+S_{v}^{(11)}\right), \\
& H_{\|}=-g_{v} S_{v}^{+(20)} S_{v}^{(20)},  \tag{16}\\
& H_{\perp}=+\omega_{\perp \iota} \Gamma_{\perp_{\iota}}^{+} \Gamma_{\perp_{\iota}}+\omega_{\xi} \xi^{+} \xi,  \tag{17}\\
& H_{\text {mix }}=-g_{2} \frac{3}{T}\left\langle S_{\mathrm{p}}\right\rangle\left\langle S_{\mathrm{n}}\right\rangle\left(\beta^{4} \xi^{+2}+\beta^{-4} \xi^{2}\right)  \tag{18}\\
& -\left(\beta^{2} \xi^{+} \Gamma_{\perp \iota}+h c\right) \phi_{a \iota}+\left(\beta^{2} \xi^{+} \Gamma_{\perp \iota}^{+}+h c\right) \phi_{b \iota}, \\
& H_{\mathrm{sp}}=-\frac{1}{2 \Im_{\perp}}\left[\tau_{1}^{(20)}, \tau_{\overline{1}}^{(20)}\right]_{+} . \tag{19}
\end{align*}
$$

Equations (15) and (16) yield the Hamiltonians acting separately between neutrons and protons, including the respective random-phase-approximation (RPA) interactions. The Lagrange multipliers $\left\langle\lambda_{v}\right\rangle$ are introduced in order to fix the average number of neutrons and protons. The RPA proton-neutron residual pairing interaction in (10), $-\frac{1}{2} g_{\perp} S_{\perp}^{+} S_{\perp}$, is responsible for the instabilities that have appeared in previous calculations. In the present treatment such interaction is substituted by an isospin-independent nuclear interaction which gives rise to the well behaved bosons $\Gamma_{\perp_{\iota}}^{+}$, creating the so called antianalogue states. The operator $\xi^{+}$acts in the collective space by increasing the value of $m=T-M$ (i.e., it excites the band of analogue states having $M=T$ for the ground state). Therefore the "badly behaved" operator $\tau_{1}$ has been substituted by the "well behaved" operator $\xi^{+}$. The excitation frequency $w_{\xi}$ includes the single-particle Coulomb displacements

$$
\begin{equation*}
\omega_{\xi}=-\frac{1}{T} \epsilon_{0 j}\left\langle\tau_{0 j}\right\rangle+\frac{3 g_{2}+g_{1}}{T}\left\langle S_{\mathrm{p}}\right\rangle^{2}+\frac{3 g_{2}-g_{1}}{T}\left\langle S_{\mathrm{n}}\right\rangle^{2} . \tag{20}
\end{equation*}
$$


$A, M=T$
A, $M=T-1$
A, $M=T-2$

Fig. 1. Lowest energy states of a system with A nucleons. The set of quantum numbers $\left(A, T, M,\left(n_{\xi}, n_{\perp \iota}, n_{d \iota}\right)\right)$ needed to specify a state is indicated for each state. Solid lines represent allowed Fermi transitions and dash lines correspond to allowed Gamow-Teller transitions. Dash-dotted lines indicate the mixing between states induced by the action of the Hamiltonian of Eqs. (15)-(19) (see the text).

The (quadratic) Hamiltonians (15)-(17) provide a basis having the convenient feature that both the isospin $T$ and its projection $M$ over the laboratory z-axis are good quantum numbers. In addition there are states with a different symmetry, whose properties are not affected by the present formalism, as the states $I^{\pi}=1^{+}$ created by the operators $\Gamma_{d \iota q}^{+}$with frequencies $\omega_{d \iota}$ and magnetic quantum number $q$. The spectrum of states associated with the neutron-proton sector is labeled by the quantum numbers $\left(A, T, M,\left(n_{\xi}, n_{\perp \iota}, n_{d \iota}\right)\right)$ and is represented in Fig. 1. It displays a signature

$$
\begin{equation*}
(-1)^{\sum_{\iota}\left(n_{\perp \iota}+n_{d \iota}\right)+T-A / 2}=1 . \tag{21}
\end{equation*}
$$

The isospin mixing terms are included in $H_{\text {mix }}$. The operator $\beta^{2}$ increases the value of the isospin by one unit $\left(\beta^{2}|T\rangle=|T+1\rangle\right)$. Therefore, the product $\beta^{2} \xi^{+}$conserves the projection $M$ in the laboratory frame. This is true for all isospin mixing terms in (18). The operator $\beta^{4} \xi^{+2}$ mixes the ground state of a nucleus having isospin $T-2$ and projection $T-2$ with the double IAS with spin $T$. It is proportional to the isoquadrupole strength $g_{2}$. The operator $\beta^{2} \xi^{+} \Gamma_{\perp_{\imath}}^{+}$creates the antianalogue states $\Gamma_{\perp_{\iota}}^{+}|0\rangle$ in the neighbor odd-odd nucleus with isospin $T-1$, laboratory projection $M=T-1$, simultaneously with the IAS carrying isospin $T, M=T-1$. There are also transitions in which an antianalogue state is destroyed, while the analogue is created. The matrix elements of $H_{\text {mix }}$ are represented with dash-dotted lines.

The coefficients $\phi_{a \iota}, \phi_{b \iota}$ arise both from the single-particle and the isovector and isoquadrupole pairing contributions. Therefore, we have been able to disentangle the proper isospin mixing terms from the spurious ones through the application of the collective formalism. On the contrary, the naive RPA defines phonons which carry a mixture of isospin values, leading to unpredictable consequences. There remains the last line in (17). It goes straightforwardly to the spurious sector as the $p_{y}^{2}$ term in Mobol. Due to limitations in time, that is all of what I can tell about the spurious sector in this presentation.

### 3.2 The $\boldsymbol{\beta}^{-}$transition operators

The $\beta^{-}$transition operators, being isovectors, should be also transformed to the intrinsic frame. For the Fermi and Gamow-Teller GT operators, we obtain

$$
\begin{align*}
\beta^{(\mathrm{F}-)}= & -\sqrt{2} \tau_{1} \rightarrow-\sqrt{2 T} \xi^{+},  \tag{22}\\
\beta_{q}^{(\mathrm{GT}-)}= & \sigma_{q 1} \equiv \frac{1}{\sqrt{3}}\left\langle j_{1}\|\sigma\| j_{2}\right\rangle\left[c_{p j_{1}}^{+} c_{n j_{2}}\right]_{q}^{1} \\
& \rightarrow \beta^{-2}\left(q_{f_{\iota}} \Gamma_{d \iota q}^{+}+(-1)_{b \iota}^{q} \Gamma_{d \iota(-q)}\right) . \tag{23}
\end{align*}
$$

We note that for Fermi transitions, the operator $\tau_{1}$ has been again substituted by $\xi^{+}$. Therefore there are only Fermi beta decay processes within the isobaric analogue band. However, the isospin mixing admixtures $H_{\text {mix }}$ make possible the transition between the initial state $(A T T)$ to the final state $(A(T-2)(T-2))$. It may proceed through either the intermediate IAS $(A T(T-1)(1,0,0))$ or through the states $(A(T-1)(T-1)(0,1,0))$. The reduced matrix elements of the spin operator are included in the GT operator (23). In that expression there has appeared the isospin decreasing operator $\beta^{-2}$, which allows the occurrence of double-beta decay process without recourse to the isospin non-conserving terms of the Hamiltonian. The coefficients $q_{f \iota}, q_{b \iota}$ are obtainable within an RPA calculation of $I^{\pi}=1^{+}$ excitations.

## 4 Applications to Fermi double-beta-decay transitions

### 4.1 Schematic models

To illustrate the above introduced formalism, we have performed calculations in a model space consisting of a single $j$-shell. In this case antianalogue states are not present. We use the values $j=9 / 2$ and $j=19 / 2$. The other parameters entering in the calculations are taken from [13], i.e., $\epsilon_{0 j}=0.8 \mathrm{MeV}, g_{\mathrm{n}}=g_{\mathrm{p}}=0.4 \mathrm{MeV}, A=10$, $T=3$. Figure 2 displays the IAS energies $\omega_{\xi}$ (Eq. (20)) as a function of $g_{2} / g$ (upper boxes). The point where $\omega_{\xi}$ vanishes and the point where the symmetry is restored are different. This (exact) result cannot be reproduced by other approximations as shown in [10]. The amplitude $M_{1}$ of the Fermi transition to the intermediate IAS and the amplitude $M_{2}$ from the IAS to the final state $(A(T-2)(T-2))$ are displayed in the lower boxes of Fig. 2. $M_{2}$ is proportional to the admixture of the


Fig. 2. IAS energy and matrix elements for Fermi transitions from the initial ground state to the IAS $\left(M_{1}\right)$ and from the IAS to the final ground state $\left(M_{2}\right)$. Exact values (solid lines) and perturbative values (dotted lines) are given as functions of the ratio $g_{2} / g$ and for the two model spaces $j=9 / 2,19 / 2$ considered in the text.
double IAS $(A, T,(T-2)(2,0,0))$ in the final state (Eq. (17)). Figure 3 shows the exact and collective, Fermi double-beta-decay matrix elements. As expected if $T$ is conserved, the exact results show the suppression of the matrix element around the point $g_{2}=0$. The result is reproduced in the naive RPA (QRPA) and in the collective approach, but not by the so called renormalized RPA (RQRPA). On the other side, the naive RPA cannot go through the unstable region corresponding to small $g_{2}$ values. Ikeda's sum rule is also reproduced by the collective approach but not by the RQRPA.

### 4.2 Realistic calculations

We have performed a realistic calculation for the double-beta decay in ${ }^{76} \mathrm{Ge}$. Singleparticle energies are obtained from the parametrization of the Woods-Saxon potential recommended in [12], including the proton electrostatic field. The calculation


Fig. 3. Theoretical matrix elements for allowed Fermi double-beta-decay transitions. Exact (solid lines) and perturbative (dotted lines) values are shown, together with the results of the conventional QRPA method and the results of the RQRPA method.
has been performed for the two pairs of strength values $g_{\mathrm{n}}=g_{\mathrm{p}}=0.289 \mathrm{MeV}$ ( $g_{1}=0$ ) and $g_{\mathrm{n}}=0.289 \mathrm{MeV}, g_{\mathrm{p}}=0.342 \mathrm{MeV}$ (so that the proton gap becomes slightly larger than the neutron one). The Fermi amplitudes are represented as a function of the isoquadrupole strength $g_{2}$ in Figs. 4a and b. Two curves are displayed in each figure, corresponding to the contribution in which the intermediate state is the IAS and to the sum of all other contributions from the physical roots $(\perp \iota)$. We note:
i. In the region $-0.040 \leq g_{2}$, the contribution due to the admixture of the double IAS in the final state dominates, as evidenced from the linearity of the corresponding matrix element with $g_{2}$. Thus the results in Fig. 4 resemble those of Fig. 3.


Fig. 4. Matrix elements corresponding to allowed double Fermi transitions, as a function of the coupling constant $g_{2}$ and for the values of $g_{n}$ and $g_{p}$ indicated in the figure. The contributions from transitions which proceed by the IAS (curves $I A S$ ) and other states $\left(\Sigma_{\iota}\right)$ are shown by solid and dashed lines, respectively. a) $\left.g_{\mathrm{p}}=g_{\mathrm{n}}, \mathrm{b}\right) g_{\mathrm{n}} \neq g_{\mathrm{p}}$.
ii. The sum over the finite frequency roots $(\perp \iota)$ is incoherent but for the most negative $g_{2}$ values, for which the validity of first order perturbation theory becomes doubtful.

We have obtained the $I^{\pi}=1^{+}$states in the odd-odd system through an RPA diagonalization of an isoscalar pairing interaction and a spin-isospin interaction.


Fig. 5. Matrix elements for allowed double Gamow-Teller transitions, as a function of $g_{d}$ (see the text) and for two different values of $g_{1}$.

Their relative intensities are fixed through the condition that the GT resonance lies at the empirical energy. Figure 5a displays the GT double-beta-decay matrix elements [6] as a function of the isoscalar paring strength $g_{d}$. We verify:
i. The GT amplitudes are an order of magnitude larger than those of the F contributions. Therefore this comparison confirms the existing belief about the dominance of the GT amplitudes in double-beta-decay processes.
ii. The calculation of both the F and the GT amplitudes is hampered by their sensitivity to a unique parameter, $g_{2}$ and $g_{d}$, respectively. Unfortunately, the value of these parameters is uncertain.

## 5 Conclusions

We have developed a collective treatment for motion in isospace and gauge space. The system is described from a moving frame of reference using both collective variables (determining the orientation of the moving frame) and intrinsic variables (describing the motion of the particles with respect to the moving frame). Since there are four collective variables, there are also four constraints (9) to be satisfied and four conditions (14) that fix the body relatively to the moving frame. They may be chosen such that there is only pairing between identical particles in the moving frame (16). The solution to this last problem constitutes a mature subject in nuclear physics. However, the main purpose of this work is to study the neutronproton subspace. Two difficulties have to be overcome: i) in the above mentioned
basis, a zero-frequency RPA mode and subsequent infrared problems should be expected (at least if the Hamiltonian is an isoscalar); ii) the Hamiltonian generally used in the literature for our region of interest is usually not an isoscalar, and thus we must disentangle the real isospin mixing effects produced by the Hamiltonian from those produced by our isospin-violating treatment. In the present paper we show how to overcome both difficulties, under the assumption that the contraints (9) hold. In particular, we obtain the quadratic Hamiltonian and the beta-decay operators acting in the neutron-proton subspace. We do so without constructing explicitly the mechanisms through which the constraints become validated. The solution is checked against exact results for the case of particles moving in a single- $j$ shell and coupled by the isovector pairing interaction. The agreement is very satisfactory, which is not the case for other procedures used previously to treat the same problem. This is true for the calculation of the energy differences between odd-odd and even-even nuclei, and for single-beta-decay and double-beta-decay matrix elements. We have made also realistic calculations for the Fermi contributions to double-beta-decay processes in ${ }^{76} \mathrm{Ge}$. The expectation that such processes are negligible relative to the Gamow-Teller contributions was never tested correctly because of the above mentioned difficulties in the calculation. Our results support conclusively this expectation.

Concerning realistic applications of the formalism, we would like to emphasize its potential, particularly, in dealing with the calculations of electroweak scalar currents.

This work was supported in part by the CONICET through the Carrera del Investigador Científico; by the Fundacion Antorchas and by the University Favaloro (project 010/99). The contribution of Dr. N.N. Scoccola to some of the above presented results is gratefully acknowledged.

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[^0]:    ${ }^{*}$ ) Presented by D.R. Bes at the Workshop on calculation of double-beta-decay matrix elements (MEDEX' 01), Prague, Czech Republic, June 11-15, 2001.

