Spurious center of mass effects in electroweak decays *)

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The microscopic treatment of the spurious motion of the center of mass in nuclear many-body systems is reviewed. In this talk we present and compare the results of using the counterterms which are needed to fulfill the translational and Galilean invariances, and those obtained by using collective variables. The formalism is applied to the calculation of matrix elements of electroweak operators, as those which are active in the (μ^-, e^-) conversion process.

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1 Introduction

Among the nuclear structure elements which can influence the outcome of a Standard Model analysis [1–4], the problem of symmetry violations and its consequences upon the calculation of nuclear matrix elements of electroweak operators is relevant. We have centered our attention on the restoration of the translational and Galilean invariance of the nuclear hamiltonians used in the calculation of $I^\pi = 1^-$ states. These invariances are required because the nuclear response to multipole excitations is sensitive to the spurious center of mass (c.m.) motion. This is the case, for instance, of μ–electron lepton-flavor violation processes [4]. Previous attempts, to remove states in which the c.m. is not at rest, include shell model treatments of two-body interactions in harmonic oscillator basis [5], the diagonalization of a modified Hamiltonian which includes a harmonic term in the variables corresponding to the c.m. [6], and the use of effective interactions [7]. The strength of effective, separable, two-body interactions, derived from geometrical arguments involving the self-consistency of either surface and volume fields, has been discussed in [8]. Symmetry restoring interactions, to be added to the symmetry violating total Hamiltonian, have been introduced in [9]. Another method has been proposed in the work [10]. It is based on the identification and subsequent elimination of the coupling between c.m. and intrinsic degrees of freedom. This formal decoupling leads to random phase approximation (RPA) equations which include also a finite

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frequency mode for the c.m. motion. In all three cases [8–10], the treatments are extended to restore also the Galilean invariance of the interaction. Their validity is always restricted to the RPA or equivalent linearized approximations. Another treatments are described in Refs. [11–14].

Our formalism deals with the construction of the Hamiltonian and with the specific application of the collective treatment (see the previous contribution by the same authors). In the first part we have followed the conceptual lines of Refs. [8–10], i.e., we have introduced counter-terms in the Hamiltonian, which goes beyond the RPA. Thus, the new interactions may influence not only the strength of transitions to $I^\pi = 1^-$ states but also other nuclear properties as well. Next, we have applied the method of [15] to eliminate infrared divergent terms due to a zero energy mode. Hereafter, we shall discuss the main structure of the method and avoid the details of the formalism, which can be found in [16].

2 Formalism

2.1 The counter-terms

Translational and Galilean invariances request that the Hamiltonian H should commute with the momentum component p_μ and with the coordinate r_μ. An ideal Hamiltonian, derived self-consistently, should satisfy these two conditions. However, this is not the case, in general, since single-particle energies are empirically obtained from single-particle nuclear states and the residual interaction is obtained either from schematic or realistic forces. As a consequence,

$$\pi_\mu \equiv [H, p_\mu]\,, \quad \rho_\mu \equiv [H, r_\mu] - \frac{\mathrm{i}}{m} p_\mu\,, \tag{1}$$

are non-vanishing operators.

Thus, we are forced to include counter-terms in the Hamiltonian, which we assume to be of the form

$$H_p = \vec{P}\,.\,\vec{r}\,, \quad H_r = \vec{R}\,.\,\vec{p}\,, \tag{2}$$

in order to uncouple the three degrees of freedom $(\vec{r}, \vec{p})$ from the remaining ones in the problem. Here the (spherical) components P_μ and R_μ are determined from the conditions

$$0 = \pi_\mu + \mathrm{i}AP_\mu + (-1)^\nu [P_{-\nu}, p_\mu] r_\nu + (-1)^\nu [R_{-\nu}, p_\mu] p_\nu\,, \tag{3}$$

$$0 = \rho_\mu - \mathrm{i}AR_\mu + (-1)^\nu [P_{-\nu}, r_\mu] r_\nu + (-1)^\nu [R_{-\nu}, r_\mu] p_\nu\,. \tag{4}$$

The n-body contributions to P, R yield $(n+1)$-body terms in the effective interactions (2). The one- and two-body terms of the solutions P_μ, R_μ to these equations are shown in Appendix A of [16], in terms of an expansion similar to the

one implicit in the RPA. In particular, the one-body terms are given by

$$\begin{aligned} P_{\mu(1v)} &= \frac{\mathrm{i}}{A}\pi_{\mu(1v)} + \frac{\sqrt{3}}{2A^2}[\pi, p]^0_{(00)} r_{\mu(1v)} \ , \\ R_{\mu(1v)} &= -\frac{\mathrm{i}}{A}\rho_{\mu(1v)} + \frac{\sqrt{3}}{2A^2}[\rho, r]^0_{(00)} p_{\mu(1v)} \ , \end{aligned} \tag{5}$$

where the notation (nm) in the subindices labels a n-body operator with m-bodies crossing the Fermi sea. The solution for the particle–hole equations ($v = 1$) coincides with the one found in [10] for the case in which the boson associated with the degree of freedom corresponding to the center of mass has zero energy. The two-body terms read (cf. Eqs. (A13) and (A16) of [16]):

$$\begin{aligned} P_{\mu(2v)} &= \frac{\mathrm{i}}{A}\tau_{\mu(2v)} + \frac{1}{2A^2}\sum_\alpha \sqrt{\frac{2\alpha+1}{3}} \\ &\quad \times \left([[\tau_{(2v)}, p_{(11)}]^\alpha_{(1(v-1))} r_{(11)}] - [[\eta_{(2v)}, p_{(11)}]^\alpha_{(1(v-1))} p_{(11)}] \right)^1_{\mu(2v)} \ , \\ R_{\mu(2v)} &= -\frac{\mathrm{i}}{A}\eta_{\mu(2v)} - \frac{1}{2A^2}\sum_\alpha \sqrt{\frac{2\alpha+1}{3}} \\ &\quad \times \left([[\tau_{(2v)}, r_{(11)}]^\alpha_{(1(v-1))} r_{(11)}] - [[\eta_{(2v)}, r_{(11)}]^\alpha_{(1(v-1))} p_{(11)}] \right)^1_{\mu(2v)} \ , \end{aligned} \tag{6}$$

where the two-body operators $\tau_{\mu(2v)}, \eta_{\mu(2v)}$ are derived from the known operators π_μ, ρ_μ according to [16].

2.2 Application to single-particle Hamiltonians

The systematic application of the previous procedure associates residual interactions (2) to any single-particle Hamiltonian. In the present section we study some applications to the most frequent single-particle contributions, namely the harmonic oscillator (ho) potential, the spin–orbit and the l^2 terms, and the empirical single-particle Hamiltonian. Details of the calculation are given in Appendix B of [16].

2.2.1 The harmonic oscillator case

We assume a single-particle Hamiltonian of the form

$$H^{\mathrm{ho}}_{\mathrm{sp}} = \frac{1}{2m}p^2 \ + \ \frac{m\omega^2}{2}r^2 \ . \tag{7}$$

The previously described procedure yields the interaction

$$\begin{aligned} H^{\mathrm{ho}}_p &= -\chi_{\mathrm{ho}}\, \vec{r}.\vec{r} \, , \\ \chi_{\mathrm{ho}} &= \frac{m\omega^2}{2A} \ , \end{aligned} \tag{8}$$

with the strength $\chi_{\rm ho}$ equal to the self-consistent value derived in [8]. The linear solutions (5) are exact for this case. The addition of the single-particle (7) and residual interaction (8) yields the well known two-body Hamiltonian

$$H^{\rm ho} = \frac{1}{2m}p^2 \, + \, \frac{\chi_{\rm ho}}{2}\sum_{ab}|\vec{r}_a - \vec{r}_b|^2 \, . \tag{9}$$

2.2.2 The spin–orbit and the l^2 terms

We start from a single-particle term

$$H^{\rm so}_{\rm sp} = -\chi_{\rm so}(\vec{l}.\vec{s})_{(1v)} \, , \tag{10}$$

which yields the two-body interactions

$$\begin{aligned} H^{\rm so}_p \, &= \, \frac{\chi_{\rm so}}{A}\sum_w (\vec{p}\times\vec{s})_{(1(v-w))}.\vec{r}_{(1w)} \, , \\ H^{\rm so}_r \, &= \, -\frac{\chi_{\rm so}}{A}\sum_w (\vec{r}\times\vec{s})_{(1(v-w))}.\vec{p}_{(1w)} \, . \end{aligned} \tag{11}$$

Therefore, the three spin-orbit contributions may be written as

$$\begin{aligned} H^{\rm so} \, &= \, H^{\rm so}_{\rm sp} + H^{\rm so}_p + H^{\rm so}_r \\ &= \, -\frac{\chi_{\rm so}}{2A}\sum_{ab}(\vec{r}_a - \vec{r}_b)\times(\vec{p}_a - \vec{p}_b)\,.(\vec{s}_a + \vec{s}_b) + \frac{\chi_{\rm so}}{2A}\,(\vec{l}.\vec{s})_{(2v)} \, , \end{aligned} \tag{12}$$

i.e., as a two-body spin orbit interaction which is obviously translational and Galilean invariant plus a term that, within the RPA, is only operative for spin-spin correlations ($I^\pi = 1^+$ resonances). The imposition of the form (2) to the counter-terms results in the existence of higher order terms which should compensate the $(\vec{l}.\vec{s})_{(2v)}$ interaction. Indeed, Eqs. (3) and (4) may be exactly solved also for the spin–orbit case. They yield (cf. Eq. (6))

$$\begin{aligned} P_\mu &= \frac{\chi_{\rm so}}{A}\left((\vec{p}\times\vec{s})_{\mu(1v)} - \frac{1}{2A}\sum_w \left(\vec{p}_{(1(v-w))}\times\vec{s}_{(1w)}\right)_{\mu(2v)}\right) \, , \\ R_\mu &= -\frac{\chi_{\rm so}}{A}\left((\vec{r}\times\vec{s})_{\mu(1v)} - \frac{1}{2A}\sum_w \left(\vec{r}_{(1(v-w))}\times\vec{s}_{(1w)}\right)_{\mu(2v)}\right) \, . \end{aligned} \tag{13}$$

Let us consider now the term

$$H^{ll}_{\rm sp} = -\chi_{ll}(\vec{l}.\vec{l}) \, . \tag{14}$$

The resultant interactions are

$$\begin{aligned} H^{ll}_p \, &= \, \frac{\chi_{ll}}{A}(\vec{p}\times\vec{l}).\vec{r} - \frac{\chi_{ll}}{A}(\vec{l}\times\vec{p}).\vec{r} - \frac{2\chi_{ll}}{3A^2}\langle p^2\rangle\vec{r}.\vec{r} \, , \\ H^{ll}_r \, &= \, \frac{\chi_{ll}}{A}(\vec{l}\times\vec{r}).\vec{p} - \frac{\chi_{ll}}{A}(\vec{r}\times\vec{l}).\vec{p} - \frac{2\chi_{ll}}{3A^2}\langle r^2\rangle\vec{p}.\vec{p} \, . \end{aligned} \tag{15}$$

The sum of the three contributions may be written as

$$H^{ll}_{\rm sp} + H^{ll}_{p} + H^{ll}_{r} = -\frac{\chi_{ll}}{2A}\sum_{ab}|(\vec{r}_a - \vec{r}_b)\times(\vec{p}_a - \vec{p}_b)|^2$$
$$-\frac{2\chi_{ll}}{3A}\langle p^2\rangle\left(\frac{1}{A}\vec{r}.\vec{r} - (\vec{r}.\vec{r})_{(11)+(10)}\right) - \frac{2\chi_{ll}}{3A}\langle r^2\rangle\left(\frac{1}{A}\vec{p}.\vec{p} - (\vec{p}.\vec{p})_{(11)+(10)}\right)$$
$$+\frac{\chi_{ll}}{A}\vec{l}.\vec{l} + \frac{\chi_{ll}}{A}\sum_{ab}\vec{r}_b\times\vec{p}_a.\vec{r}_a\times\vec{p}_b + \frac{\chi_{ll}}{A}(\vec{r}.\vec{r})_{(11)+(10)}(\vec{p}.\vec{p})_{(11)+(10)}$$
$$-\frac{\chi_{ll}}{A}\sum_{\mu,a,b}(-1)^{\mu}r_{a,\mu}p_{b,\mu}\left(r_{a,\mu+1}p_{b,\mu+1} + r_{a,\mu-1}p_{b,\mu-1}\right)\ . \qquad (16)$$

2.2.3 Empirical single-particle energies

We have performed three sets of RPA calculations using the empirical single-particle energies as in ^{208}Pb and introducing as interaction:

i. a $\vec{r}.\vec{r}$ term with the self-consistent strength (8);
ii. the same interaction as in i) with a strength such that there is an eigenvalue as close to zero as allowed by the computational facilities;
iii. the counter-terms with the values of $\vec{P}_{(11)}, \vec{R}_{(11)}$.

The results are given in Fig. 1, where the matrix elements of the operator $\vec{r}$, corresponding to transitions between the ground (g.s.) and excited states, are represented as a function of the excitation energy. Although the calculation i. with the self-consistent strength displays a prominent peak at low energies, the peak is finite and located at an energy significantly larger than zero. On the contrary, the results ii. and iii. show the peak at zero energy (within the numerical accuracy of the calculation). This similarity apparently supports the use of the procedure ii., appearing in the literature. Let us consider now the matrix elements to finite frequency modes, which are those that interest us from the physical point of view. In this case the calculations i. and ii. are those that yield very similar results, while the scale is smaller by two orders of magnitude for iii., although the excitation pattern is quite similar. We conclude that the admixture of the spurious to the finite frequency modes is not changed significantly by varying the strength of the $\vec{r}.\vec{r}$ interaction and thus the use of this interaction does not insure that we obtain correct matrix elements to excited states. On the contrary, the uncoupling of the spurious mode is accomplished through the counter-terms (2).

3 The collective formalism

The solution (5) guarantees that there is a zero-frequency RPA boson for each direction in space. This consequence of the homogeneity of space gives rise to infrared divergencies, which should be taken care of. One way to solve the problem is through the introduction of collective coordinates [15], which in the present case represent the coordinates $\mathcal{R}_\mu$ determining the position of a moving frame of reference relative to the laboratory frame. Within this description there is no way to

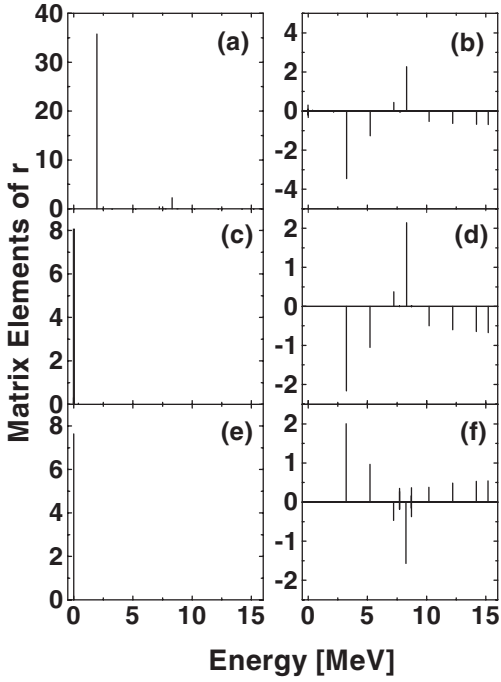

Fig. 1. The matrix elements of the coordinate operator, $\langle 1_n^- \parallel r \parallel g.s\rangle$, to the n-th one-phonon state, in units of fm. The results are scaled by a factor 10^{-2} in cases (c) and (e) and by a factor 10^2 in case (f). Cases (a) and (b) show the results of the RPA calculation performed with the $r.r$ interaction and using the harmonic oscillator coupling $\chi_{\rm ho}$, cases (c) and (d) correspond to the same interaction with a renormalized coupling which yields a solution at zero energy, cases (e) and (f) show the results obtained with the use of counter-terms. The single-particle basis used in the calculations is an empirical one which includes the $N_{\rm osc}$=5, 6 and 7 active shells, and $A = 126$ particles.

distinguish between the motion of the body in one direction and the displacement of the frame of reference in the opposite one. This gauge-type invariance is expressed by the constraint

$$p_\mu - \mathcal{P}_\mu = 0 \,, \tag{17}$$

where $\mathcal{P}_\mu$ is the generator of displacements of the moving frame, hereon the collective momentum ($[\mathcal{R}_{-\mu}, \mathcal{P}_\nu] = {\rm i}(-1)^\mu \delta_{\mu\nu}$). Physical states $|{\rm phys}\rangle$ are annihilated by the left hand side of Eq. (17) and physical operators $\mathcal{O}_{\rm phys}$ commute with it. As it is well known, the constraints may be taken into account by adding to the Hamiltonian terms proportional to Lagrange multipliers $\vec{\Omega}$

$$H \to H \,-\, \vec{\Omega}\,.\,(\vec{p} - \vec{\mathcal{P}}) \,, \tag{18}$$

and requiring the vanishing of $\vec{B}$, the momentum conjugate to $\vec{\Omega}$ ($[\Omega_{-\mu}, B_\nu] = {\rm i}(-1)^\mu \delta_{\mu\nu}$).

3.1 The BRST invariance

Following [15] [1]) the Becchi–Ronet–Stora–Tyutin (BRST) Hamiltonian reads

$$H_{\rm BRST} = H \,-\, \vec{\Omega}\,.\,\left(\vec{p} - \vec{\mathcal{P}}\right) \,+\, {\rm i}\vec{\pi}\,.\,\vec{\bar{\pi}} + \omega^2 \left(\frac{\vec{r}\,.\,\vec{B}}{A} - \frac{\vec{B}\,.\,\vec{B}}{2mA} - {\rm i}\vec{\bar{\eta}}\,.\,\vec{\eta}\right) \,. \tag{19}$$

[1]) See also [20].

3.2 The transformation to a moving system

In $H_{\rm BRST}$ (19), the term $\vec{\Omega}\,.\,\vec{\mathcal{P}}$ represents the coupling between the collective and the intrinsic motion. This coupling may be eliminated via a transformation setting in motion the intrinsic system, namely

$$T = \exp\left[\frac{\mathrm{i}}{A}\vec{\mathcal{P}}\,.\,\left(\frac{\vec{B}}{m} - \vec{r}\right)\right] , \tag{20}$$

$$T\,H_{\rm BRST}\,T^{+} = H'_{\rm BRST} + \frac{1}{2mA}\mathcal{P}^2 , \tag{21}$$

which explicitly displays the collective kinetic energy. Here $H'_{\rm BRST}$ is the BRST Hamiltonian without the coupling term $\vec{\Omega}\,.\,\vec{\mathcal{P}}$. Therefore, the translational collective sector has become totally uncoupled from the remaining degrees of freedom of the system.

We transform now the operator $F(\vec{r}_i + \vec{\mathcal{R}})$ according to (20), namely

$$T\,F(\vec{r}_i + \vec{\mathcal{R}})\,T^{-1} = F\left(\vec{r}_i + \frac{1}{A}\left(\frac{\vec{B}}{m} - \vec{r}\right) + \vec{\mathcal{R}}\right) \to F\left(\vec{r}_i - \frac{\vec{r}}{A} + \vec{\mathcal{R}}\right) , \tag{22}$$

since $\vec{B}$ is a nil operator. Therefore the transformation (20) replaces the collective coordinate $\vec{\mathcal{R}}$ by $\vec{\mathcal{R}} - \vec{r}/A$.

We may apply the formalism to the calculation of the coordinate operator $\vec{r}$. This operator appears in the excitation of $I^{\pi} = 1^{-}$ states. We recall that, within the RPA, this operator creates a phonon with frequency $\omega_{\rm g} = 0$, with an amplitude proportional to $1/\sqrt{\omega_{\rm g}}$ (a clear example of infrared problems). From the practical point of view, this behaviour leads to unphysical predictions whenever even small amplitudes of the spurious state are present in a finite frequency RPA mode.

The corresponding physical operator is $\vec{r} + A\vec{\mathcal{R}}$, and it reduces to $A\vec{\mathcal{R}}$ after being boosted: the vector $\vec{r}$ has disappeared from the calculation (and the associated infrared divergencies as well). The problem is reduced to the calculation of the well-behaved operator $\vec{\mathcal{R}}$ within the collective sector of the Hilbert space.

3.3 The Hilbert space

In the moving frame of reference the collective variables $\vec{\mathcal{R}}$ are considered to be real variables and thus, as a tradeoff, some original degrees of freedom must join the spurious sector. At the level of elementary modes of excitation these are given by the RPA zero-frequency modes. In addition to the spurious sector $|n_{0\mu}, n_{1\mu}\rangle$, the intrinsic sector displays elementary modes of excitation, which are represented by the finite-frequency RPA modes $|n_\nu\rangle$, $(\omega_\nu > 0)$.

4 Transition operators to be used in realistic calculations

The conversion of muons into electrons may proceed according to the lepton-flavor violating processes discussed in [4, 17, 18]. The main interest of such process

lies on the necessary mixing of muon and electron neutrinos. So far there are experimental upper limits for this process [4]. An example of the current calculations is given in Refs. [17] and [18]. Considering the dominance of the contributions due to the excitation of 1^- states, see Refs. [17, 18], in the context of the RPA diagonalization, it is obvious that, from the nuclear structure point of view, one has to produce an estimation as accurate as possible of the nuclear matrix elements involved in the transitions.

The vector operator exciting $I^\pi = 1^-$ states may be written

$$j_1(qr)Y_{1\mu}|_{(11)} = \sum_{n_\nu}\langle n_\nu|j_1Y_1|\rangle\left(\gamma^+_{n_\nu,\mu} - (-1)^\mu\gamma_{n_\nu,(-\mu)}\right) - \frac{\mathrm{i}}{A}(-1)^\mu\langle[j_1Y_{1\mu}, p_{-\mu}]\rangle r_{\mu(11)} \ , \tag{23}$$

where n_ν denotes a finite-frequency RPA mode and a similar expression should be used for the dipole axial-vector term of the weak current. Its contribution is not affected by the treatment of the spurious sector. The amplitude in the second line may be regularized as in Subsection 3.2. The results corresponding to the transition matrix elements of the shifted operator (23), obtained in the RPA diagonalization, are shown in Fig. 2. The similarity between the results obtained with the three different Hamiltonians supports the claim about the validity of the procedure. The same effects are expected to materialize in the case of realistic calculations and work is in progress to include the counter-terms obtained in the previous sections, starting from realistic two-body forces.

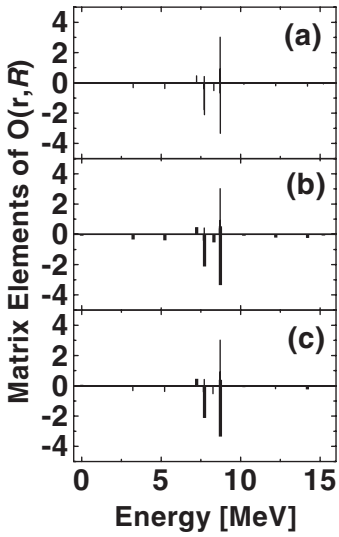

Fig. 2. The matrix elements of the vector operator $j_1(kr)Y_{1,\mu}$. Cases (a),(b) and (c) correspond to cases (b),(d) and (f) of Figure 1. All values are scaled up by a factor 10.

5 Conclusions

We have attacked the problem of the center of mass motion in nuclear spectroscopy calculations in two succesive steps, namely i) the reconstruction of the translational invariance of the Hamiltonian and ii) the inclusion of collective variables in order to eliminate infrared singularities. The solutions in both steps are exact, albeit perturbative. They share in common the same perturbation parameter, $A^{-\frac{1}{2}}$, which is a convenient feature if a given order of perturbation is envisioned.

Quite generally, the first step requires the introduction of counter-terms, which has been performed following specially Ref. [10]. We have tried the procedure for pure independent-particle Hamiltonians: in the case of harmonic oscillator potential, the counter-terms reproduce the dipole force with the self-consistent strength (see Ref. [8]); for spin–orbit and l^2 terms in the central potential the procedure leads to the introduction of two-body terms of similar character; finally, the application of the procedure to an empirical single-particle spectrum ensures the elimination of all the matrix elements of the coordinate operator to excited $I^\pi = 1^-$ RPA states, which is not the case for the dipole interaction, no matter how close to zero the lowest RPA energy is made by adjusting the strength of the interaction.

To conclude, we thus suggest the use of the counter-terms (2) and of the operators (23) in cases where, like in the (μ^-, e^-) conversion, the dominance of the $I^\pi = 1^-$ channels is apparent.

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