A PC-BASED PROGRAM FOR THE INTERACTIVE DESIGN OF CAUER FILTERS

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ABSTRACT

A program for the interactive design of elliptic approximation functions on personal computers is presented. The use of non-classical elliptic functions and arithmetic-geometric iterations results in a substantial reduction in computational costs when compared to classical numerical methods. In addition, since no compromise approximations are made, unusually accurate results are obtained. Moreover, since the elliptic function formulation is not avoided, a close parallelism between this formulation and the classical one is possible.

INTRODUCTION

The elliptic characteristic has enjoyed great popularity among filter designers because of its steep rolloff in the transition band for a given order and amplitude specifications. However, their design classically involved the computation of Jacobian elliptic functions and elliptic integrals of the first kind. This complicated task has been avoided by using tabulated information, or by computing approximations of limited accuracy.

In this paper, a user friendly PC program along with its algorithms is presented. No compromise to common filter specifications has been made and, in speed, accuracy, and flexibility, the program displays excellent performance.

THE BASIC PROBLEM OF ELLIPTIC APPROXIMATION

A. SPECIFICATIONS AND NOTATION

In the design of elliptic lowpass filters, the specifications are usually expressed in terms of: the filter order (N), the angular frequencies at the edge of the passband (ω_p) and at the edge of the stopband (ω_s), the maximum acceptable passband attenuation in dB (Amax), and the minimum stopband attenuation in dB (Amin). Sometimes the modular angle θ or the shape factor SF is used instead of ω_p and ω_s, and/or the reflection coefficient ρ replaces Amax. Their relationship is given by:

$$\omega_p/\omega_s = \sin\theta = 1/SF \ , \ Amax = -10\log_{10}(1-\rho^2) \qquad (1)$$

30th Midwest Symposium on Circuits and Systems
G. Glasford and K. Jabbour, eds.

The following parameters are then defined:

$$k = \omega_p/\omega_s \ , \ k' = (1-k^2)^{1/2}$$

$$k_1 = k_p/k_s = ((10^{Amax/10}-1)/(10^{Amin/10}-1))^{1/2}$$

$$k'_1 = (1-k_1^2)^{1/2}$$

$$\varepsilon^2 = k_p k_s = ((10^{Amax/10}-1)(10^{Amin/10}-1))^{1/2} \qquad (2)$$

We have normalized such that $\omega_p\omega_s=1$.

B. ELLIPTIC APPROXIMATION FORMULATION

The minimum required order to satisfy the filter specifications is given by:

$$N \geq \frac{K'(k')K_1(k_1)}{K(k)K'_1(k'_1)} \qquad (3)$$

where K, K', K_1, and K'_1 are the complete elliptic integrals of the first kind of parameters k, k', k_1, k'_1, respectively, given by:

$$K(k) = \int_0^{\pi/2} \frac{dr}{(1-k^2\sin^2 r)^{1/2}} \qquad (4)$$

After choosing an integer value for N, the corresponding transfer function of the lowpass elliptic filter becomes:

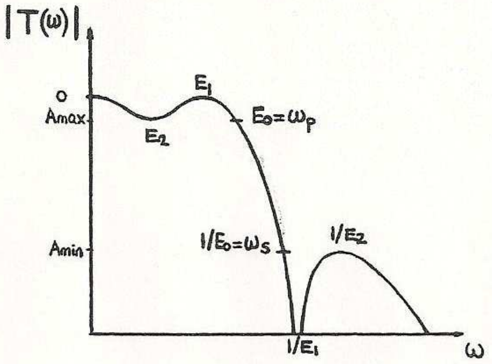
$$|T(\omega)|^2 = \frac{1}{1+\varepsilon^2 C_N^2(\omega)} \qquad (5)$$

where $C_N(\omega)$ is the Zolotarev rational function [1]. Depending on the order N, it is given by:

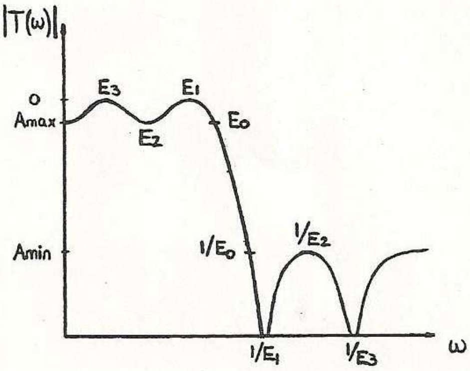
$$C_{Nodd} = \omega \prod_{\ell=1}^{(N-1)/2} \frac{E_{2\ell-1}^2-\omega^2}{1-E_{2\ell-1}^2\omega^2}$$

$$C_{Neven} = \prod_{\ell=1}^{N/2} \frac{E_{2\ell-1}^2-\omega^2}{1-E_{2\ell-1}^2\omega^2} \qquad (6)$$

Fig. 1 shows the resulting amplitude characteristics for N odd and N even (the so called type A). As we can see in the figure, the E's are the frequencies where the attenuation in the passband reaches either the value zero (odd subindices) or Amax (even subindices). Due to the frequency normalization, the inverses of the E's correspond to transmission zeroes (odd indices) or Amin attenuation (even indices) in the stopband.



a. N odd (N=3)



b. N even type A (N=4)

Figure 1. Elliptic filter characteristics

C. ELLIPTIC SINE FUNCTIONS

It is well known [2] that all the E's frequencies can be expressed in terms of the Jacobi elliptic sine function:

$$\int_0^x \frac{dr}{((1-r^2)(1-k^2r^2))^{1/2}} = SNJ^{-1}(x,k) = u(x,k) \tag{7}$$

then SNJ(u,k) = x (i.e. the inverse function for the incomplete elliptic integral of the first kind). This is a doubly periodic function of periods 4K and i2K' on the real and imaginary axis respectively. As the imaginary (real) period becomes arbitrarily large, this function degenerates into a trigonometric (hyperbolic) function. In terms of SNJ:

$$E_j = \sqrt{k}\ SNJ\left(\frac{N-j}{N}K, k\right) \tag{8}$$

Reference [3] describes a procedure based on the arithmetic-geometric mean that successfully computes SNJ(u,k). It requires square root, sine, and arcsine functions and the major computational cost is the evaluation of one sine-arcsine pair at each step of the iteration, for each value of u.

In reference [4], a different doubly periodic elliptic function is defined, more suitable to this formulation and simpler to compute:

$$\begin{aligned} SN(\mu, z) &= \tanh(z) \prod_{j=1}^{\infty} \tanh(j\mu - z)\tanh(j\mu + z) \\ &= \tanh(z) \prod_{j=1}^{\infty} \frac{\tanh^2(j\mu) - \tanh^2(z)}{1 - \tanh^2(j\mu)\tanh^2(z)} \end{aligned} \tag{9}$$

This function has a real period 2μ and an imaginary period iπ. Note that exp(-x) is the only transcendental function required (integer powers of exp(-x) can be produced just by simple products). It can be shown that there exists a direct relationship between the two elliptic sine functions. In particular:

$$SN\left(N\mu, \frac{j}{N}\frac{N\mu}{2}\right) = \sqrt{k}\ SNJ\left(\frac{j}{N}K, k\right) \tag{10}$$

$j = 0,1,\ldots N$ (μ and k are dependent)

The E's frequencies then become:

$$E_j = SN\left(N\mu, \left(\frac{N-j}{N}\right)\frac{N\mu}{2}\right) \tag{11}$$

By appropriate programming, the whole set of E's can be computed in a single loop (once the first term in the iteration has been computed, the remaining terms require only the four arithmetic operations). To have about the same accuracy, the computation of the set of Jacobi's elliptic sine takes much longer than the computation of the set of alternative elliptic sine functions. In addition, the use of the alternative elliptic sine function permits a close parallelism to the classical formulation. This is hard to find in some reported solutions to this approximation problem of comparable accuracy and computational cost [5,6].

D. THE NATURAL FREQUENCIES

The transmission poles of an elliptic filter lie on an ellipse in the s plane. In order to compute them, an extra parameter a_o is required, given by [4]:

$$ia_o = SN(N\mu, iW)\ ,\ i\varepsilon^{-1} = SN(\mu, iW) \tag{12}$$

Geometrically, a_o represents the smaller radius of that ellipse (equals the magnitude of the real pole for N odd). In terms of a_o and the E's, the poles $(R_j + iS_j)$ become:

$$R_j = E_{N+1-2j} \frac{E_0 E_{2j-1} + (E_0 E_{2j-1})^{-1}}{a_0 E_{2j-1} + (a_0 E_{2j-1})^{-1}},$$

$$S_j = \frac{(a_0^2 + a_0^{-2} + (E_0^2 + E_0^{-2}))^{1/2}}{a_0 E_{2j-1} + (a_0 E_{2j-1})^{-1}} \qquad (13)$$

E. EVEN ORDER TYPE B & C CHARACTERISTICS

To implement an even order elliptic ladder LC filter, the ideal type A even order characteristic must be modified. The type B characteristic has a double zero at infinite frequency, while the type C characteristic adds to that the equal terminations feature. Both characteristics can be obtained from an ideal elliptic filter of order N/2 having the same amplitude specifications by using the frequency transformations [4]:

$$f^2 = (\omega - E_{2N})/(1 - E_{2N-1}\,\omega) \qquad \text{TYPE B}$$

$$f^2 = (\omega - E_{2N-1})/(1 - E_{2N-1}\,\omega) \qquad \text{TYPE C} \qquad (14)$$

Even the type A characteristic can be derived from an ideal half order filter by the transformation:

$$f^2 = (\omega - E_{2N})/(1 - E_{2N}\,\omega) \qquad \text{TYPE A}$$

These three transformations are used to compute all relevant frequencies and shape factors when dealing with even order filters.

SOME COMPUTATIONAL ASPECTS

This program accepts any three of the four design parameters (N, Amin, Amax or ρ, ω_s/ω_p or θ) computes and displays the remaining one, and allows the user to perform modifications in the resulting parameter set. Once N is fixed, all the relevant frequencies are determined by μ and a_0.

The algorithms adopted depend upon which of the four parameters is not defined by the user. Thus:

a. N not given [3]: From equation (3), and defining the arithmetic-geometric mean AGM(m) by:

$$A_0 = 1, B_0 = m; A_1 = \frac{A_0 + B_0}{2}, B_1 = \sqrt{A_0 B_0}$$

$$A_{j+1} = \frac{A_j + B_j}{2}, B_{j+1} = \sqrt{A_j B_j} \quad \text{until} \qquad (15)$$

$A_w = B_w$ (within the desired accuracy), then

$$AGM(m) = Aw$$

since $K = \pi/(2AGM(k))$, this results in:

$$N \geq [AGM(k)AGM(k'_1)]/[AGM(k')AGM(k_1)]$$

In general, this is a real number.

b. Shape factor (of the ideal filter) not given [4]:

$$\mu = \frac{\pi}{2} \frac{AGM(k_1)}{AGM \frac{1-k_1}{1+k_1}} (1 + k_1)^{-1} \qquad (16)$$

c. Amax or Amin not given [4]:

$$\mu = \frac{\pi}{2N} \frac{AGM(k)}{AGM \frac{1-k}{1+k}} (1 + k)^{-1}$$

$$(k_1)^{1/2} = SN(\mu, \mu/2) \qquad (17)$$

k_1 is computed by using eq. (9)

d. Computation of a_0 [4]: From eq. (12), we need to compute W first. Applying the Landen transformation iteratively, defining

$$Q_0 = 1/\varepsilon,\ SN_0 = (k_1)^{1/2},\ SN_{j+1} = SN_j^2/(1 + \sqrt{1 - SN_j^4})$$

$$V_j = (Q_j SN_j)^{-1},\ Q_{j+1} = (V_j + \sqrt{V_j^2 + 1})^{-1}$$

we get

$$W = \frac{\mu}{\pi} \cdot \ln(\alpha + \sqrt{\alpha^2 + 1})$$

where

$$\alpha = \lim_{j \to \infty} Q_j / SN_j$$

In practice, 4 or 5 iterations will suffice for double precision accuracy. Once W is obtained, a_0 is computed by using eq. (9). At this point, the shape factor(s) and all the relevant frequencies are computed by using eqs. (11),(13), and (14).

PROGRAM FEATURES AND EXAMPLE

The program is written in double precision MS-Fortran and runs on IBM PC or compatibles. It accepts designs up to 20th. order (at least doubles the order available in the classical tables [1,2,7]). At the end, the frequency axis is normalized to have unity value at the passband edge frequency. The pole and zero locations are correct to about ten digits after the decimal point. When using a PC with the math chip, the program responds instantly (a few seconds are required for high orders with no math chip).

The user provides any three of the design parameters and, after the fourth parameter is computed, has the choice to either modify the parameter set or print the pole-zero locations list.

Example: For N=10, ω_s/ω_p=1.05, and Amax=0.1 dB:

First transmission zero:

PC: 1.0534095891

Ref.[6]: 1.053409590

Ref.[8]: 1.053409702

Pole with smallest real part:

PC: -0.0097036022 + i1.0058625503

Ref.[6]: -0.009703602 + i1.00586255

Ref.[8]: -0.009704 + i1.005863

Minimum sopband loss Amin:

PC: 55.681105

Ref.[6]: 55.6811

Ref.[8]: 55.681

CONCLUSIONS

An interactive program for elliptic filter design on personal computers is presented. Even though an alternative elliptic function is used, the formulation closely resembles the classical one. This simplifies computations, while preserving the usefulness of the existing theoretical work. In addition, no compromise approximations to common filter specifications are made in the computations.

As a design tool, it represents a big step forward over standard approaches. The user benefits from higher order designs, higher accuracy and speed, and ease of operation on small inexpensive computers. As a result, the design-modification cycle time is dramatically reduced, lowering costs and making more complicated designs possible.

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