GEOACTA, Vol. 15, n. 2 (1988) pág. 45 a 60

# ON THE EXISTENCE OF DETERMINISTIC CHAOS IN THE SOLAR TERRESTRIAL ENVIRONMENT

L. Romanelli, S. Radicella, M. A. Figliola, and F. A. Hirsch
Centro Argentino de Estudios de Radiocomunicaciones y Compatibilidad Electromagnética
Julián Alvarez 1218 - (1414) Buenos Aires - Argentina

## ABSTRACT

Time series corresponding to F10.7 solar flux, AE index and the ionospheric critical frequency $f_0F_2$ are analyzed, for periods of low solar activity, in order to find whether they show either deterministic behavior or stochastic noise.
Both the correlation dimension and a lower bound of the Kolmogorov entropy are calculated.
For the time series analyzed, deterministic chaos is found and its implications are discussed.

## RESUMEN

Se analizan series temporales correspondientes a: frecuencia 10.7 cm del flujo solar, índice AE y la frecuencia ionosférica crítica del sondeo vertical $f_0F_2$ para baja actividad solar, con el objeto de determinar si las variaciones en ellas presentan comportamiento determinístico o corresponden a ruido estocástico.
Se calculan la dimensión de correlación y el límite inferior de la entropía de Kolmogorov.
Se encuentra que estas series temporales presentan caos determinístico y se discuten sus implicancias.

## 1. INTRODUCTION

A first approach to modelling natural phenomena is to assume that the system involved obeys deterministic laws and try to find a corresponding set of linear equations such as:

$$x_1(t+1) = F_1(x_1(t),x_2(t),\ldots,x_n(t)) \qquad (1)$$
$$x_2(t+1) = F_2((x_1(t),x_2(t),\ldots,x_n(t))$$
$$\ldots\ldots\ldots\ldots\ldots\ldots$$
$$x_n(t+1) = F_n((x_1(t),x_2(t),\ldots,x_n(t))$$

where the functions $F_1 \ldots\ldots F_n$ and their derivatives are taken as continuous. If an experimental time series of a variable related to the system is analyzed, the first step would be to search for periodicities or trends by using spectral analysis. Once they are found and reasonably described by a set of equations like (1), there is usually a certain amount of variability left, that can not be explained by those periodicities or trends and that is treated as "additive noise". This "noise" is usually considered the stochastic component of the time series analyzed.

A different approach to the problem is to look for a possible deterministic behavior intrinsic to the system that can not be described by equations (1). To do that, it is necessary to reconstruct the phase space in order to find out if the system evolves in a limited region of phase space. If such a region exists it is said that the solutions lie in an attractor, otherwise they are stochastic. The attractor may be a fixed point, a limit cycle or a chaotic attractor.

Then it is necessary to look if the chaotic attractors are of low dimension. In this case the dimension will give information on the minimum number of variables to be introduced in the description of the dynamical system.

As the next step, it is possible to find, from the experimental time series, the value of or at least an upper bound for the intrinsic turbulent or chaotic component of the time series. So what is generally considered as additive noise, actually supplies information about the dynamical system.

Previously Nicolis and Nicolis (1984) have analyzed the possible existence of a climatic attractor, Kurths and Herzel (1987) found chaotic behavior in solar radio pulsation data set and Fraedrich (1987) investigated the predictability in climatic variables.

The purpose of the present paper is to search for evidence of chaotic behavior in the magnetospheric-ionospheric medium under the influence of solar radiation. To do so, series of selected variables have been used.

In a previous work Romanelli et al (1987) have found a low dimensional chaotic attractor in the time series of solar flux at 10.7 cm. The present paper extends the investigation to time series of geomagnetic and ionospheric variables.

This paper is organized as follows: In section 2 a brief introduction of the concept of chaos is given, in section 3 the method of analysis is described, and in sections 4 and 5 a result of the analyzed data and the conclusions are discussed.

## 2. THE CONCEPTS OF CHAOS

### Attractors

As it is well known, anything that moves or changes governed by deterministic rules can be described by a system of N differential equations of first order:

$$dx_n/dt = F_n (x_1,..........,x_n) \qquad n= 1......N. \qquad (2)$$

The functions $F_n$ may be non linear functions of the $x_n$'s and there may be many different solutions. The coordinates $x_i$ represent the observables. The numbers $(x_1,..........,x_n)$ may be considered as a point in an N dimensional space (phase space), specifying the state of the system, and the rules $F_n$ determine its time evolution. N is the number of independent variables needed to specify an initial condition uniquely. If an initial condition is picked at random, and the system is allowed to evolve for a long time, it is necessary to analyze the nature of the motion when all the transients have died out. In dissipative systems the motion will be limited to a subset of phase space called *attractor*. The set of points (initial conditions) that are attracted form the *basin of attraction*. A dynamical system may have more than one attractor, each with its own basin. There is now strong evidence from a variety of experiments that chaos can be described in terms of low dimensional chaotic attractors (see Haken, 1982, Haken,1983, Schuster,1984 ). This implies that out of the infinite number of degrees of freedom in a large dissipative system only a few will be active. Chaotic solutions have most of the properties of random functions. It is remarkable that no randomness is ever explicitly added. The equations of motion are purely deterministic, and the random behavior emerges spontaneously. Whereas randomness usually implies ignorance, deterministic chaos is different, in that it arises from the geometry of the dynamical system and its attractor.

Geometrical objects such as points, lines or hypersurfaces (smooth topological manifolds) are characterized by integer dimensions. There are objects (like some chaotic attractors) which have a non-integer dimension and they are known as fractals (Mandelbrot,1977). They are important because they model irregular, time-dependent phenomena characterized by two facts: an extreme sensitivity to initial conditions, and the appearance of large variability similar to stochastic motion, although the dynamical system is deterministic.

## Characterization of the attractors

The theory of nonlinear dynamics and chaotic attractors has been helpful to understand the irregular behavior of complex systems. Chaotic (or turbulent) behavior can be assumed if a broad power spectrum is found in the time series, but this information is not enough to establish whether the chaos is either deterministic or stochastic. It is necessary to define a set of quantities which provides further information.

From the infinite set of dimensions found by Hentschel and Procaccia (1983), those providing a relevant information and are invariant under a smooth change of coordinates are:

*The correlation dimension* related as a lower bound of the fractal dimension, which provides some measure of the number of degrees of freedom involved in the dynamics of the system under consideration. Its knowledge is necessary for modelling the system (Mandelbrot, 1977).

*The Kolmogorov entropy,* that is a complementary measure based on information theory that describes the loss of information (bits by iteration) giving an insight about its predictability (Benettin et al. 1976).

The positive *Lyapunov exponents* measure the divergence of two nearby trajectories on the attractor, while the negatives refer to the convergence on the attractor (Farmer et al. 1983).

The first two quantities are discussed in this paper. The last one is not treated here because its determination requires more computer power than we presently have .

Grassberger and Procaccia (1983) have found that the correlation dimension is the lower bound of the fractal dimension. They have developed an algorithm by which it can be determined from experimental time series. They have also given, as will be discussed later, the lower bound of the Kolmogorov entropy, that indicates how chaotic the system is. This method is used in this paper.

## 3. METHOD OF ANALYSIS

When analyzing experimental time series most of the N variables of the system under study are usually unknown or unavailable. Therefore the question is whether and how it is possible to substitute the missing information. The system described by equation (2) can be reduced to a differential equation (generally nonlinear) of order N in the variable of interest.

Ruelle (1981) has found that instead of $X(t)$ and its derivatives it is easier to work with $X(t)$ and the set of variables obtained by shifting the original series by fixed lags or delay times $\tau$. This is enough to reconstruct, from one dimensional space, a multidimensional phase space of the dynamical system.
The nature of the attractors provides information on the time behavior of the variables and on the nature of their coupling.
Consider a set of N points on an attractor embedded in a phase space of d dimensions (where d is the embedding dimension), obtained from a time series:

$$
\begin{array}{l}
X(t_1),\ldots\ldots\ldots\ldots\ldots,X(t_N) \\
X(t_1+\tau),\ldots\ldots\ldots\ldots,X(t_N+\tau) \\
X(t_1+(d-1)\tau),\ldots\ldots,X(t_N+(d-1)\tau)
\end{array}
$$

Thus a time series for a single observable is used to reconstruct phase portraits of the attractor, as suggested by Packard et al. (1983), $\tau$ is the time delay, chosen to coincide with the first zero of the correlation function so that the variables will be linearly independent. No significant variation was observed, in our data analysis, over a wide range of $\tau$.
The structure of the attractor is inferred from the correlation dimension and the entropy $K_2$ (Grassberger and Procaccia, 1983) by the so called integral correlation function $C(r)$ given by:

$$C(r) = (1/N^2) \sum_{i \neq j} \theta(\, r - |x_i(t) - x_j(t)|\,) \qquad (3)$$

where $\theta$ is the Heaviside function and N is the total number of data.
For convenience, we use a vector notation $X_i$ for a point of phase space whose coordinates are $\{X(t_i),\ldots\ldots,X(t_i+(d-1)\tau)\}$.
A point $X_i$ is chosen from the data, as a reference point, and all the distances $|X_i-X_j|$ of the N-1 remaining points are computed, then the data points that lie

within a distance r from $X_i$ are counted. Repeating the process for all values of i one arrives at the quantity given by equation (3).

Grassberger and Procaccia (1983) have shown that for small r, the integral C(r), scales as $C(r) \sim r^{\upsilon}$.

From the slopes of the log-log plots of C(r) vs. r, values of $\upsilon$ as a function of d can be derived. The saturation value of the $\upsilon$ vs. d plot is the correlation dimension $D_c$ ( for practical purposes d should be at least twice as large as $D_c$ ).

The situation is analogous to having an N-dimensional object is projected in m-dimensional space. The dimension of the projection is equal to m unless m > n. In that case it remains constant, that is, there is saturation.

When random noise is present the correlation integral scales as

$C(r) \sim r^d$ and there is no saturation.

A lower bound of the Kolmogorov entropy ($K_2$) is found from:

$$K_2 = (1/\tau)\log(C_d(r)/C_{d+1}(r))$$

$K_2 > 0$ for deterministic chaos. If the system evolves periodically $K_2 = 0$ and for stochastic systems $K_2 = \infty$.

## 4. DATA ANALYSIS AND DISCUSSION

Romanelli et al (1987), have found that at least 1700 data points are necessary, in noisy time series, to calculate the correlation dimension of a chaotic attractor. It was also shown that there is evidence of a chaotic attractor of low correlation dimension.

In order to analyze daily values of solar flux at 10.7 cm corresponding to a period of low solar activity, January 1973 to December 1977 (mean values smaller than 80) were selected.

Figure 1 shows the variation of the correlation as a function of the embedding dimension and a characteristic saturation. The values found were $D_c$= 4.5 and

$K_2$ = 0.04, indicating the presence of a chaotic attractor.

In order to search for a similar behavior in a parameter reflecting the solar influence in the aeronomical environment, the auroral index AE, introduced by Davis and Sugiura (1966) was chosen.

This index, although obtained from geomagnetic data of selected sites of the Northern Hemisphere, may be considered to give reliable auroral activity information on a global scale, including the Southern Hemisphere (Mayaud,1980). To look for the existence of a chaotic behavior in this index, 2928 hourly values of AE from four months (September to December, 1983) were used, also during low solar activity. Phase space was reconstructed using a delay time $\tau$ = 8 hours, as discussed in the previous section.The results are not significantly affected for $\tau$ in the the range from 3 to16 hours.

Figure 2 shows the variation of the correlation dimension as a function of the embedding dimension. In this case the presence of a saturation value of $D_C$ = 3.3 is indicative of an attractor of low dimension and the value of $K_2$ = 0.08 shows that it is chaotic.

A wider search for chaotic behavior in an aeronomical parameter was made using hourly values of the ionospheric critical frequency $f_0F_2$ for a station in the Southern Hemisphere at middle to high geomagnetic latitude. The station chosen was Argentine Island (-65.25º , -64.27º) and the period covered was from June 20 to September 10 of 1977. In this year there was also low solar activity.

A set of random data was tested, showing that with 2000 points saturation is not reached. So, we are sure that the saturation of $f_0F_2$ data points is not an artificial effect due to the number of points used.

Figure 3 shows a plot similar to those of Figure 1 and Figure 2. Here the saturation value of $D_C$ = 7.4 is much higher than those found for 10.7 cm solar flux and auroral index and the value of $K_2$ = 0.04 found for the $f_0F_2$ data points.

The values of the correlation dimension obtained, higher than those corresponding to AE and solar flux can be attributed to the complexity introduced by local factors, or by the fact that $f_0F_2$ involves variable ionospheric heights as a function of time.

The results given above are summarized in Table 1.

## 5. CONCLUSIONS

From the results obtained it can be concluded that the solar-terrestrial environment appears to behave as a system exhibiting deterministic chaos with low dimensional attractors.

Larger dimensions should appear when local variables are considered. Only the presence of nonlinear terms in models can explain chaotic behavior and therefore they should be kept in all important stages of calculations.

The results of such theories will be "irregular" curves instead of the "smooth" ones obtained from linear analysis ( for which the irregularities are considered as noise). Still undetermined is the percentage of stochastic noise present, which will be the object of future work.

**Acknowledgements**

The authors wish to thank Ms. L. Troyano for help with the calculations and Dr. J. Scheer for helpful suggestions.

This work was supported by P.I.D. 9205-00 of CONICET and is part of the Programa Nacional de Radiopropagación (PRONARP).

**TABLE 1**

| Parameter | R | Sampling time | data size | $D_C$ | $K_2$ |
|---|---|---|---|---|---|
| F10.7 | 25.6 | daily | 1826 | 4.5 | 0.04 |
| AE | 43.2 | hourly | 2928 | 3.3 | 0.08 |
| $f_0F_2$ | 30.9 | hourly | 1968 | 7.4 | 0.04 |

Characteristics of the time series analyzed, the average sunspot numbers ($\bar{R}$) and the values of the dimensions found.

## REFERENCES

Benettin, G., Galgani, L. and Strelcyn, J. M., 1976 : *Kolmogorov Entropy and Numerical Experiments*, Phys. Rev. A **14** , 2338-2345.

Davis,T. N. and Sugiura, M., 1966 : *Auroral Electrojet Activity Index AE and its Universal Time Variation*, J. Geophys. Res. **71**, 785-792.

Farmer, J. D. , Ott, E. and Yorke, J. A.,1983 : *The Dimension of Chaotic Attractor*, Physica **7D**, 153-180.

Fraedrich, K., 1987: *Estimating Weather and Climate Predictability on Attractors*, J. Atmosph. Scies. **44**, Nº 4, 722-728.

Kurths, J. and Herzel,H.,1987: *An Attractor in a Solar Time Series*, Physica **25D** 165-172.

Grassberger, P., Procaccia, I.,1983 : *Characterization of Strange Attractors*, Phys. Rev. Lett. **50**, (5), 346-349.

Haken, H., (editor),1982 : *Evolution of Order and Chaos in Physics, Chemistry and Biology*, Springer Series in Synergetics **17**, Springer, Berlin.

Haken, H., 1983: *Advanced Synergetics*, Springer Series in Synergetics **20**, Springer, Berlin.

Hentschel, H.G.E. and Procaccia, I., 1983: *The Infinite Number of Generalized Dimensions of Fractals and Strange Attractors*, Physica **8D**, 435-444.

Mandelbrot, B., 1977: *Fractals, Form, Chance and Dimensions*, Freeman, San Francisco.

Mayaud, P.N.,1980 : *Derivation, Meaning, and Use of Geomagnetic Indices*, American Geophysical Union, Washington D.C..

Nicolis, C., and Nicolis,G. ,1984 : *Is there a Climatic Attractor?*, Nature,**311**, Nº 5986 529-532.

Packard, N. H., Crutchfield, J. P., Farmer, J. D. and Shaw, R. S., 1980: *Geometry from a Time Series*, Phys. Rev. Lett. **45**, 712 - 716.

Romanelli, L., Figliola, M.A., Hirsch, F.A. and Radicella, S. ,1987: *Chaotic Behavior of Solar Radio Flux*, Solar Physics **110**, 391-395.

Ruelle, D., 1981: *Nonlinear Phenomena in Chemical Dynamics*, ( eds. A. Pacault and C.Vidal), Springer, Berlin.

Schuster, H. G.,1984: *Deterministic Chaos-An Introduction*, Physik-Verlag, Weinheim.

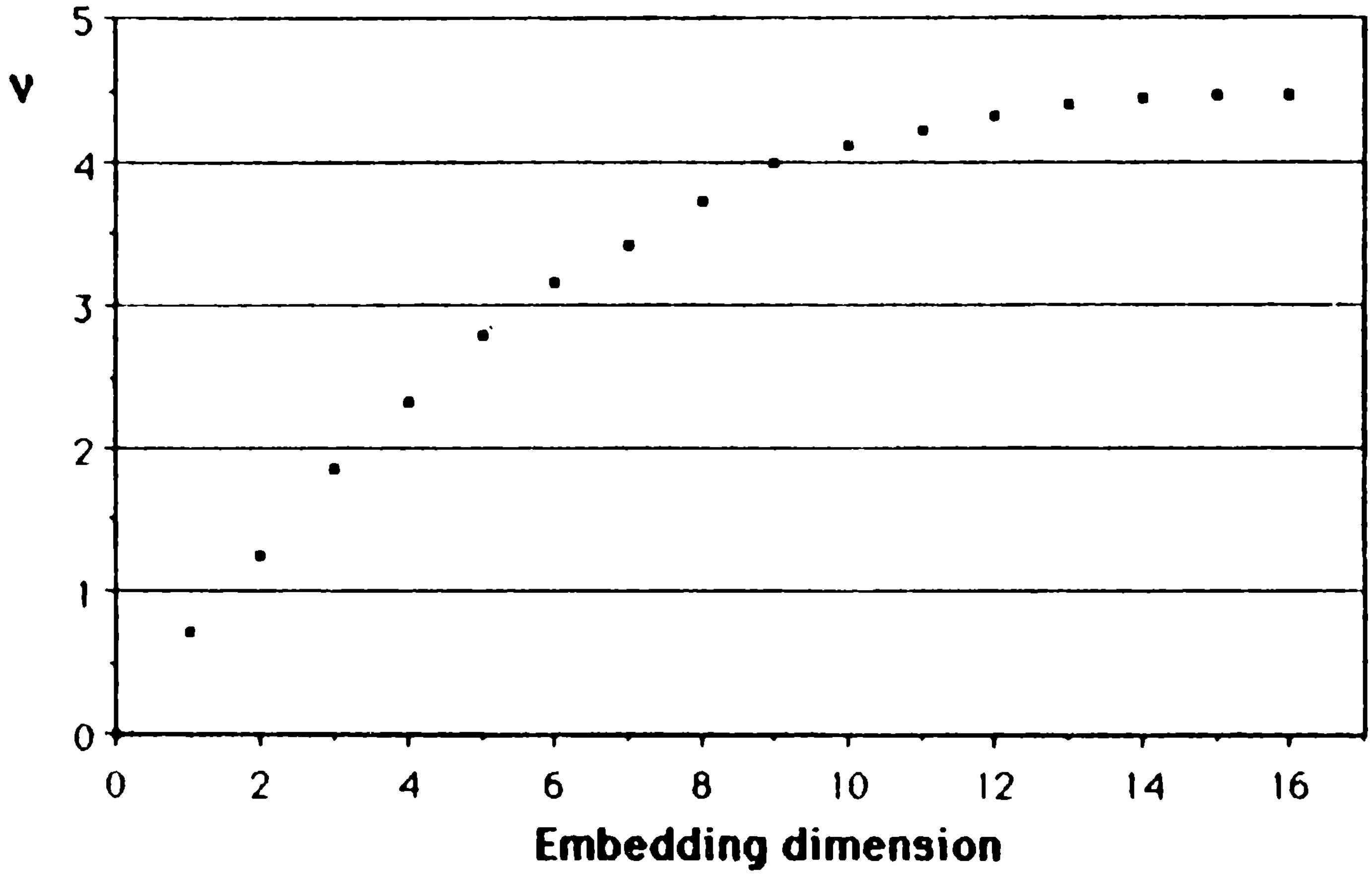


Fig. 1 Variation of the correlation integral ($\upsilon$) as a function of the embedding dimension (d) for the daily values of solar flux at 10.7 cm for the low activity period 1973-1977. The delay time used was $\tau$ = 3 days.

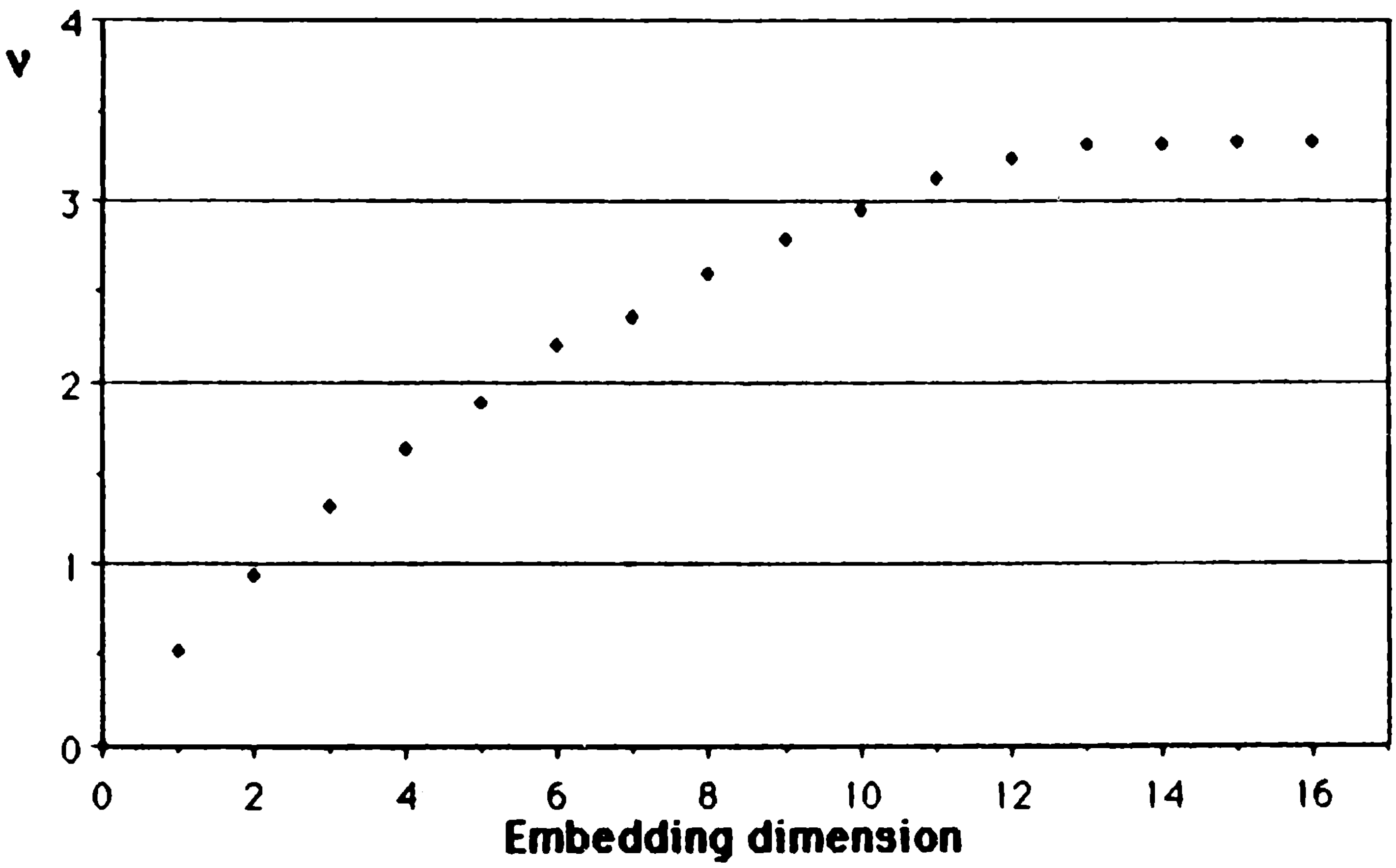


Fig. 2 Variation of the correlation integral ($\upsilon$) as a function of the embedding dimension (d) for hourly values of auroral index AE for the low activity period September to December 1983. The delay time used was $\tau$ = 8 hours.

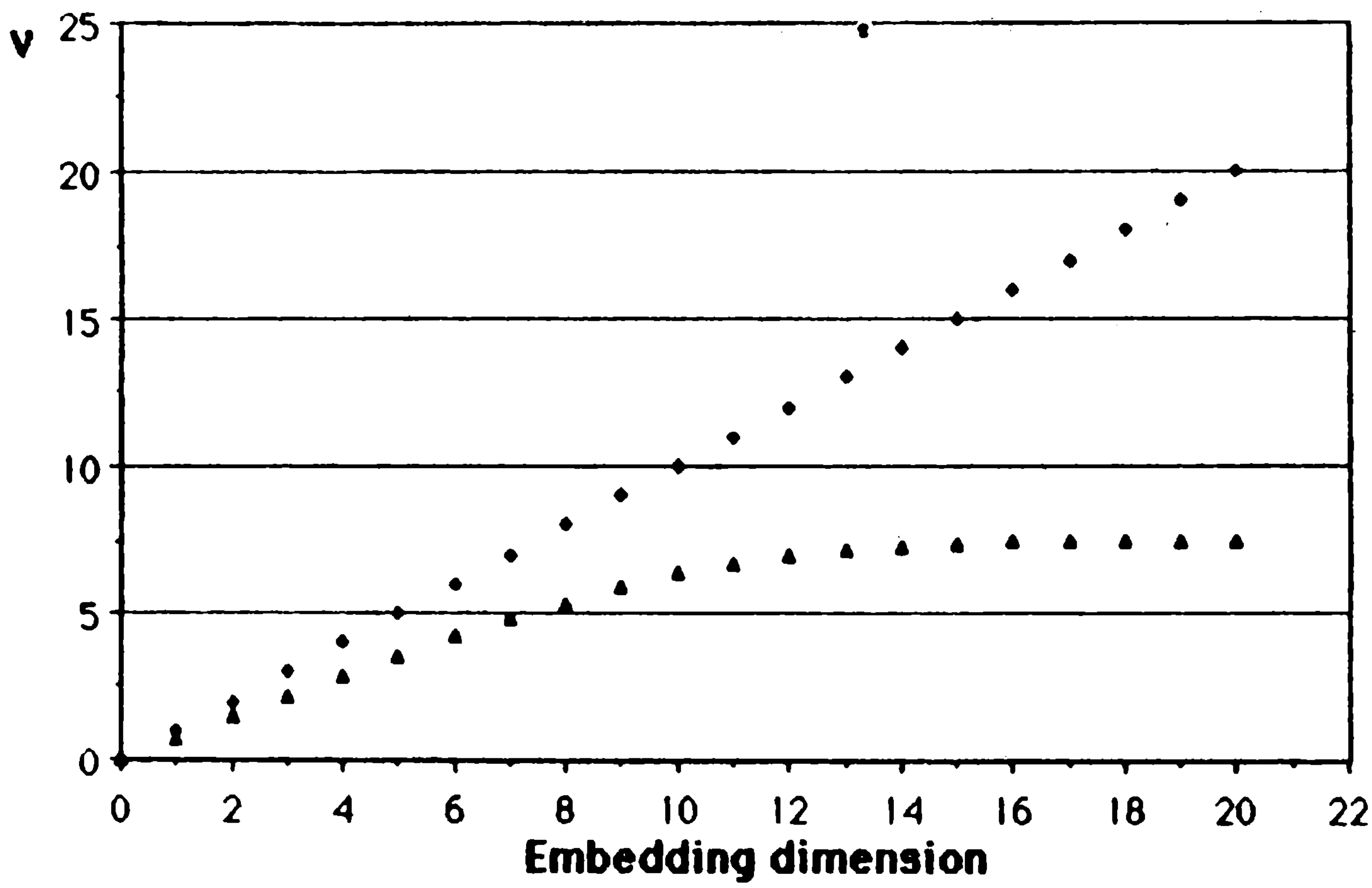


Fig. 3 Variation of the correlation integral ($\upsilon$) as a function of the embedding dimension (d) for hourly values of $f_0F_2$ (Argentine Island) for the low activity period from June 20 to September 10 1977. The delay time used was $\tau$ = 8 hours. Also shown the values found for a random function that exhibits not saturation with the same amount of data.