On the Regularization of Quantum Electrodynamics Induced by Nonpolynomial Lagrangians.

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Recently many investigations have been carried out in connection with a possible regularizing mechanism of quantum field theory provided by gravity (1). All approaches to this subject agree that the presence in the Lagrangian of the nonpolynomial multiplicative factor $\sqrt{-g}$, which is needed to preserve the invariance of the full action under generalized co-ordinate transformations, is responsible for the appearance of an infinity suppression mechanism. So far, it has been considered that the nonpolynomial factor gives origin to supergraphs which in turn provide the regularization of the usual ultraviolet infinities of the theory.

Our purpose in this paper is to analyse the coupling of a real massless scalar field with the interacting Dirac-Maxwell fields through a nonpolynomial factor which simulates the presence of $\sqrt{-g}$. We give a proof that no such a thing as a superpropagator appears and, as consequence, the removal of infinities does not follow, at least in a straightforward manner, from the nonpolynomial character of the Lagrangian. The implied suggestion is that, if gravity plays a regularizing role in the theory, either its tensor character is essential for that purpose or the regularization is provided by other coupling.

We start from the gauge-invariant Lagrangian density (***)

$$\mathscr{L}(x) = \mathscr{L}_\varphi(x) + \tfrac{1}{2}\{F(\varphi(x)), \mathscr{L}_{\text{ED}}(x)\} + \tfrac{1}{2}[\bar{\psi}, \eta] + \tfrac{1}{2}[\bar{\eta}, \psi] + J_\mu A_\mu + \xi\varphi\,, \tag{1}$$

where

$$\mathscr{L}_\varphi(x) = -\tfrac{1}{2}\partial_\mu\varphi\cdot\partial_\mu\varphi$$

is the free Lagrangian density of the real scalar field $\varphi(x)$,

$$\mathscr{L}_{\text{ED}}(x) = -\tfrac{1}{4}F_{\mu\nu}F_{\mu\nu} - \tfrac{1}{2}[\bar{\psi}\gamma_\mu\cdot(\partial_\mu + ieA_\mu)\psi + m\bar{\psi}\cdot\psi] + \text{h.c.}$$

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(***) We use $x_4 = ict$.
(1) See, for example, A. Salam and J. Strathdee: *Lett. Nuovo Cimento*, **4**, 101 (1970); A. Salam: I.C.T.P., Trieste, preprint IC/70/106; A. Salam: I.C.T.P., Trieste, preprint IC/71/3; C. J. Isham, A. Salam and J. Strathdee: I.C.T.P., Trieste, preprint IC/71/13.

is the Lagrangian density of the coupled Dirac and Maxwell fields, $F(\varphi(x))$ is an arbitrary nonpolynomial factor, although, in order to simplify the algebra, we will consider the specific case $F(\varphi)=\exp[k\varphi]$, k being the minor coupling constant, $\eta(x)$, $\bar{\eta}(x)$, $J_\mu(x)$ and $\xi(x)$ are the external sources of the Dirac, Maxwell and scalar field, respectively. The condition $\partial_\mu J_\mu=0$ guarantees the gauge invariance of the theory. The Hermitian character of $\mathscr{L}(x)$ follows from the Hermiticity properties of the field operators and the microcausality relations.

If the analysis of the problem is carried out within the framework of the interaction picture, we should split the Lagrangian density $F(\varphi)\,\mathscr{L}_{\text{ED}}$ into

$$\mathscr{L}_{\text{free}}+\left\{[F(\varphi)-1]\,\mathscr{L}_{\text{free}}+ie\,{:}F(\varphi)(\bar{\psi}\gamma_\mu\psi A_\mu){:}\right\}$$

and use as interaction Lagrangian the expression between curly brackets. It is true that if the term $[F(\varphi)-1]\,\mathscr{L}_{\text{free}}$ is dropped out from the interaction Lagrangian, for instance on the grounds of being of order k in comparison with $ie\,{:}F(\varphi)(\bar{\psi}\gamma_\mu\psi A_\mu){:}$, then a superporpagator appears and one obtains a finite value for the physical electron mass. However, if the interaction Lagrangian is taken in full the computation of the electron self-mass, up to the order e^2, involves the sum of an infinite number of terms each one of them being a n-point supergraph ([2]) and the result is no longer clear. So far, a proof of the convergence of this series has not been supplied.

To analyse the proposed problem we adopt the Schwinger formalism ([3]). Following along the lines indicated by VISCONTI ([4]) we obtain Schwinger-type equations for the normalized propagators (*) $G[s|x,x']$, $\mathscr{G}_\mu[s|z]$ and $g[s|y]$ of the electron, photon and scalar quantum, respectively. In the limit $\eta(x)=0$, $\bar{\eta}(x)=0$, these equations read (**)

$$\left\{\gamma_\mu\partial_\mu+m+e\gamma_\mu\frac{\delta}{\delta J_\mu(x)}+ie\gamma_\mu\mathscr{G}_\mu[s|x]-\frac{ik}{2}\gamma_\mu\left(\partial_\mu\frac{\delta}{\delta\xi(x)}\right)+\frac{k}{2}\gamma_\mu\big(\partial_\mu g[s|x]\big)\right\}G[s|x,x']=$$
$$=-i\,\delta(x-x')\sum_{n=0}^{\infty}\frac{(-k)^n}{n!}\,g^{(n)}[s|x,\dots,x]\,, \tag{2a}$$

$$\left\{\square_z-ik\left(\partial_\nu\frac{\delta}{\delta\xi(z)}\right)\partial+k\big(\partial_\nu g[s|z]\big)\,\partial_\nu\right\}\mathscr{G}_\mu[s|z]=$$
$$=-ie\,\mathrm{Tr}\left\{G[s|z,z]\,\gamma_\mu\right\}-\sum_{n=0}^{\infty}\frac{(-k)^n}{n!}\,g^{(n)}[s|z,\dots,z]\,J_\mu(z)\,, \tag{2b}$$

$$\square_y g[s|y]=\xi(y)+\frac{k}{u_0[s]}\exp\left[\frac{k}{i}\frac{\delta}{\delta\xi(y)}\right]\mathscr{L}_{\text{ED}}\left[\frac{1}{i}\frac{\delta}{\delta\bar{\eta}(y)},\frac{1}{i}\frac{\delta}{\delta\eta(y)},\frac{1}{i}\frac{\delta}{\delta J_\mu(y)}\right]u_0\bigg|_{\eta=0,\bar{\eta}=0}, \tag{2c}$$

where $u_0[s]\equiv\langle 0|U[\sigma_{\text{II}},\sigma_{\text{I}},s]|0\rangle$ is the propagator generating functional.

As is known, at the limit $k=0$ eqs. (2a) and (2b) provide a rigorous description of $G[s|x,x']$ and $\mathscr{G}_\mu[s|z]$ ([5]).

([2]) H. FANCHIOTTI, C. A. GARCÍA CANAL, H. GIROTTI and H. VUCETICH: University of La Plata, preprint, to be published in *Nucl. Phys.*
([3]) J. SCHWINGER: *Proc. Nat. Acad. Sci.*, **37**, 452 (1951).
([4]) A. VISCONTI: *Théorie quantique des champs*, Vol. **2** (Paris, 1965).
(*) In order to get a more compact writing we shall indicate the set of external sources $\{\eta(x),\bar{\eta}(x), J_\mu(x),\xi(x)\}$ by $\{s\}$.
(**) Our notation is that of ref. ([4]) unless otherwise specified.
([5]) See ref. ([3]).

From the system of equations (2) we derived the differential equations satisfied by the two-point photon and scalar-particle propagators $\overline{\mathscr{G}}_{\mu\nu}[s|z, z']$ and $\bar{g}[s|y, y']$, respectively. These two last equations and (2a) are then recast into a clearer form in terms of the vertex, mass and vacuum polarization functionals. Since we are mainly interested in the electron self-mass, we let all external sources go to zero. In doing so, we recover the translational and charge conjugation invariance of the theory. In the usual approximation of free propagating scalar particles, we obtain after some lengthy calculations

$$\{\gamma_\mu \partial_\mu + m\} G(x-x') + \int \mathrm{d}^4 u' \mathscr{M}(x-u') G(u'-x') = -i\,\delta(x-x') \sum_{n=0}^{\infty} \frac{(-k)^n}{n!} g^{(n)}(x, \dots, x)\,, \tag{3}$$

$$\Box_z \overline{\mathscr{G}}_{\mu\nu}(z-z') + \int \mathrm{d}^4 u' \{\mathscr{M}_{\mu\beta}(z-u') + \pi_{\mu\beta}(z-u')\} \overline{\mathscr{G}}_{\beta\nu}(u'-z') = i\delta_{\mu\nu}\,\delta(z-z') \sum_{n=0}^{\infty} \frac{(-k)^n}{n!} g^{(n)}(z, \dots, z)\,, \tag{4}$$

together with the gauge condition $\partial_\mu \overline{\mathscr{G}}_{\mu\nu} = 0$, and

$$\Box_y \bar{g}(y-y') = -i\,\delta(y-y')\,. \tag{5}$$

The electron and photon mass operators are given, respectively, by

$$\mathscr{M}(x-u') = -ie^2 \int \gamma_\mu G(x-u) \Gamma_\varrho(v, u, u') \overline{\mathscr{G}}_{\varrho\mu}(v-x) - \frac{k^2}{2} \gamma_\mu \frac{\partial}{\partial x_\mu} \int G(x-u) \Gamma(v, u, u')\, \bar{g}(v-x)\,, \tag{6a}$$

$$\mathscr{M}_{\mu\beta}(x-u') = k^2 \int \left(\frac{\partial}{\partial(z_\alpha - u_\alpha)} \overline{\mathscr{G}}_{\mu\sigma}(z-u) \right) \Gamma_{\sigma\beta}(v, u, u') \left(\frac{\partial}{\partial(v_\alpha - z_\alpha)} \bar{g}(v-z) \right), \tag{6b}$$

and the vacuum polarization functional $\pi_{\mu\beta}$ by

$$\pi_{\mu\beta}(z-u') = -ie^2\, \mathrm{Tr} \int \gamma_\mu G(z-v) \Gamma_\beta(u', v, v') G(v'-z)\,. \tag{7}$$

In eqs. (6a), (6b) and (7) integration over repeated variables must be understood. The vertex functionals Γ_ν, Γ and $\Gamma_{\sigma\beta}$ are defined as follows:

$$\Gamma_\nu[s|z, x, x'] \equiv \frac{\delta G^{-1}[s|x, x']}{e\,\delta \mathscr{G}_\nu[s|z]}\,,$$

$$\Gamma[s|z, x, x'] \equiv \frac{\delta G^{-1}[s|x, x']}{k\,\delta g[s|z]}\,,$$

$$\Gamma_{\sigma\beta}[s|z, x, x'] \equiv \frac{\delta \overline{\mathscr{G}}^{-1}_{\sigma\beta}[s|x, x']}{k\,\delta g[s|z]}\,.$$

It can be shown that the vertex operators are related to the mass operators through relations which, for the electron-photon vertex, read

$$\sum_{n=0}^{\infty} \frac{(-k)^n}{n!} g^{(n)}(x, \dots, x) \Gamma_\nu(z, x, u') = -\gamma_\nu\,\delta(x-z)\,\delta(x-u') + \frac{i}{e} \frac{\delta \mathscr{M}[s|x, u']}{\delta \mathscr{G}_\nu[s|z]}\bigg|_{s=0}\,. \tag{8}$$

Finally, the n-point functions g and $\bar{g}$ for the scalar quantum are defined by

$$g^{(n)}[s|x_1, \dots, x_n] = \frac{1}{i^n}\frac{1}{u_0[s]}\frac{\delta^n u_0[s]}{\delta\xi(x_1)\dots\delta\xi(x_n)},$$

$$\bar{g}^{(n)}[s|x_1, \dots, x_n] = \frac{1}{i^{n-1}}\frac{\delta^{n-1} g[s|x_1]}{\delta\xi(x_2)\dots\delta\xi(x_n)}.$$

The contribution to the self-mass of the electron from the photon and scalar fields can be clearly seen from eq. (6a). The zero-mass condition for the photon can be gotten through the combined use of the gauge condition $\partial_\mu \overline{\mathscr{G}}_{\mu\nu} = 0$ and eq. (4).

In the approximation in which the scalar quantum propagates freely, the ansatz of CAIANELLO [6,7], together with translational invariance allow us to compute explicitly the infinite series of scalar-particle tadpoles

$$\sum_{n=0}^{\infty}\frac{(-k)^n}{n!}\,g^{(n)}(x, \dots, x) = 1\,. \tag{9}$$

This result implies that in the lowest-order approximation Γ_ν tends to the bare electromagnetic vertex.

It is now an easy task to construct the electron mass operator $\mathscr{M}$ by successive approximations [8,9]. This iterative procedure, between eqs. (6a) and (8), yields up to order e^2

$$\mathscr{M}(x-x') = ie^2\gamma_\mu S_F(x-x')\gamma_\mu D_F(x'-x) + O(k^2)\,, \tag{10}$$

where $O(k^2)$ stands for all remaining terms coming from eq. (6a). From the fact that only one scalar quantum is absorbed or emitted at each vertex Γ, it follows that no powers of the propagator $\bar{g}$ appear in the different terms of the series giving $\mathscr{M}$. Therefore, we conclude that this series cannot be summed to give a superpropagator. Thus, the regularizing mechanism, supposedly provided by the nonpolynomial character of the Lagrangian is doubtful. The modifications to the electron and photon propagators produced by the scalar field are vertex and self-energy parts analogous to those of quantum electrodynamics.

On the other hand, it is easy to show that if the nonpolynomial factor only multiplies the Dirac-Maxwell interaction Lagrangian, a clearly non–gauge-invariant coupling, a superpropagator arises and one obtains a finite value for the electron self-mass.

We want to end remarking that our conclusions do not invalidate the arguments about the possible regularizing effects of quantum gravity. However, what they suggest is that if an infinity suppression mechanism is induced by gravity it would not come from the common factor $\sqrt{-g}$.

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(6) See ref. (4).
(7) E. CAIANIELLO: *Nuovo Cimento*, **18**, 504 (1960).
(8) See ref. (3).
(9) See ref. (4).