Admissible sets of arguments in a timed context

Maria Laura Cobo, Diego C. Martinez, and Guillermo R. Simari

Artificial Intelligence Research and Development Laboratory (LIDIA)
Department of Computer Science and Engineering, Universidad Nacional del Sur
Av. Alem 1253 - (8000) Bahía Blanca - Bs. As. - Argentina
{mlc,dcm,grs}@cs.uns.edu.ar
http://www.cs.uns.edu.ar/lidia

Abstract. Timed abstract frameworks (TAFs) are a novel formalism for argumentation where arguments are valid only during specific intervals of time, called availability intervals. Attacks between arguments are relevant only when both the attacker and the attacked arguments are available. Thus, the outcome of the framework may vary in time. Previous formalization of TAFs are based on discrete time, with single-interval argument validity. In this work we study argument admissibility notions in a timed abstract framework with intermittent arguments under a dense time representation.

1 Introduction

Abstract argumentation systems [9, 13, 2, 3] are formalisms for argumentation where some components remain unspecified, being the structure of an argument the main abstraction. This kind of formalism is used as a platform for the study of argumentation semantics, $i.e.$, rationally based positions for determining sets of collectively accepted arguments. Most of these systems are based on the single abstract concept of *attack* and extensions are defined as sets of possibly accepted arguments. The underlying nature of attack is also kept abstract. An argument $\mathcal{A}$ *attacks* another argument $\mathcal{B}$ if the acceptance of $\mathcal{B}$ is conditioned by the acceptance of $\mathcal{A}$, but not the other way around.

The minimal abstract framework is defined by Dung in [9], and it includes a set of abstract arguments and a binary relation of attack between arguments. Several semantics notions are defined and the Dung's argument extensions became the foundation of further research, either by the addition of new elements to the framework [2, 6, 12], or by the elaboration of new semantic notions [10, 5].

In [7, 8] a novel framework is proposed, called *timed abstract framework*, combining arguments and temporal reasoning. In this formalism, arguments are relevant only in a single period of time, called its *availability interval*. This kind of timed-argument has a limited influence in the system, given by the temporal context in which these arguments are taken into account. A skeptical, timed interval-based semantics is proposed, using admissibility notions.

For example, consider the following arguments

A1: *If we set sail westward from Spain, we will arrive to East Asia.*

A2: *There is a big mass of land between Spain and East Asia, and then circumnavigation of the Earth is no possible.*

A3: *There is a strait connecting the Atlantic Ocean and the Pacific Ocean.*

Argument A1 was starting to be considered during late pre-columbian period of history. Argument A2 was available just before the discovery of American continent. As a tentative piece of reasoning, it is still a valid argument. However, since year 1520 also argument A3 is available. Argument A2 attacks argument A1 since year 1492. Argument A3 attacks A2 since year 1520.

Timed abstract frameworks capture the previous argument model by assigning arguments to an availability interval of time. In this work we study argument admissibility notions in a timed abstract framework with intermittent arguments under a dense time representation, as an extended version of the TAFs presented in [7, 8]. Intermittent arguments are arguments that are available with (possibly) some repeated interruptions in time. Non-continuous, dense intervals of time require a more complex treatment and classical acceptability needs to be adapted. The main issue in this formalism is not to find if a set of arguments *is* admissible or not, but *when* this will happen as the frameworks evolves through time.

The paper is structured as follows, in section 2 we present the basics on classic abstract argumentation. In section 3 we introduce the time representation used on the new framework, TAF presented in the following section. Finally in section 5 we present the semantical aspects of the framework based on admissible sets of arguments in a timed context.

2 Classic abstract argumentation

Dung defines several argument extensions that are used as a reference for many authors. The formal definition of the classic argumentation framework follows.

Definition 1 [9] *An argumentation framework is a pair* $AF = \langle AR, attacks \rangle$ *where* AR *is a set of arguments, and* $attacks$ *is a binary relation on AR, i.e.* $attacks \subseteq AR \times AR$.

Arguments are denoted by labels starting with an uppercase letter, leaving the underlying logic unspecified. A set of accepted arguments is characterized in [9] using the concept of *acceptability*, which is a central notion in argumentation, formalized by Dung in the following definition.

Definition 2 [9] *An argument* $\mathcal{A} \in AR$ *is acceptable with respect to a set of arguments* S *if and only if every argument* $\mathcal{B}$ *attacking* $\mathcal{A}$ *is attacked by an argument in* S.

If an argument $\mathcal{A}$ is acceptable with respect to a set of arguments S then it is also said that S *defends* $\mathcal{A}$. Also, the attackers of the attackers of $\mathcal{A}$ are called *defenders* of $\mathcal{A}$. We will use these terms throughout this paper.

Acceptability is the main property of Dung's semantic notions, which are summarized in the following definition.

Definition 3 *A set of arguments* S *is said to be*
– *conflict-free if there are no arguments* $\mathcal{A}, \mathcal{B}$ *in* S *such that* $\mathcal{A}$ *attacks* $\mathcal{B}$.
– *admissible if it is conflict-free and defends all its elements.*

– a preferred extension if S is a maximal (for set inclusion) admissible set.
– a complete extension if S is admissible and it includes every acceptable argument w.r.t. S.
– a grounded extension if and only if it is the least (for set inclusion) complete extension.

The grounded extension is also the least fixpoint of a simple monotonic *characteristic* function:

$$F_{AF}(S) = \{\mathcal{A} : \mathcal{A} \text{ is acceptable with respect to } S\}.$$

In [9], theorems stating conditions of existence and equivalence between these extensions are also introduced.

Example 1 *Consider the argumentation framework $AF_1 = \langle AR, attacks\rangle$, where $AR = \{\mathcal{A}, \mathcal{B}, \mathcal{C}, \mathcal{D}, \mathcal{E}, \mathcal{F}, \mathcal{G}, \mathcal{H}\}$ and $attacks = \{(\mathcal{B}, \mathcal{A}),\ (\mathcal{C}, \mathcal{B}),\ (\mathcal{D}, \mathcal{A}),\ (\mathcal{E}, \mathcal{D}),\ (\mathcal{G}, \mathcal{H}),\ (\mathcal{H}, \mathcal{G})\}$. Then*
– $\{\mathcal{A}, \mathcal{C}, \mathcal{E}\}$ is an admissible set of arguments.
– $\{\mathcal{A}, \mathcal{C}, \mathcal{E}, \mathcal{F}, \mathcal{G}\}$ is a preferred extension. It is also a complete extension.
– $\{\mathcal{A}, \mathcal{C}, \mathcal{E}, \mathcal{F}\}$ is the grounded extension.

In the following section we prepare the road to timed argumentation by introducing several time-related concepts.

3 Time representation

Timed abstract frameworks associate arguments with availability intervals of time. In [7, 8] *temporal intervals of discrete time* are used as primitives for time representation. In this paper we consider an expanded version of TAFs, using *dense time* and arguments with intermittent availability.

Definition 4 *A temporal interval I represents a continuous period of dense time [1], identified by a pair of time-points. The initial time-point is called the startpoint of I, and the final time-point is called the endpoint of I. The intervals can be:*

- *closed: defines a period of time that includes the definition points (startpoint and endpoint). Closed intervals are noted as $[a, b]$.*
- *open: defines a period of time without the start and enpoint. Open intervals are noted as (a, b).*
- *semi-closed: the periods of time includes one of the definition points but not both. Depending wich one is included, they are noted as $(a, b]$ (includes the endpoint) or $[a, b)$ (includes the startpoint).*

If X is an interval then X^- denotes the startpoint of X and X^+ denotes the endpoint of X. It is important to remark that $-\infty < i$ and $i < \infty$ for any value i. Also that $\infty = \infty$ and $-\infty = -\infty$.

On Table 1 we present seven relations between intervals [1].

Relation	*Symb*	*Relation on Endpoints*	*Relation*	*Symb*	*Relation on Endpoints*
X Before Y	ⓑ	$X^+ < Y^-$	X Meets Y	ⓜ	$X^+ = Y^-$
X Overlaps Y	ⓞ	$X^- < Y^-, X^+ > Y^-$	X Starts Y	ⓢ	$X^- = Y^-, X^+ < Y^+$
X During Y	ⓓ	$X^- > Y^-, X^+ < Y^+$	X Finishes Y	ⓕ	$X^+ = Y^+, X^- > Y^-$
X Equal Y	ⓔ	$X^- = Y^-, X^+ = Y^+$			

Table 1. Qualitative relations between intervals [1]

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Since arguments are available during different intervals of time, we will usually work with sets of intervals. Several definitions and properties are needed for semantic elaborations.

Since we are dealing with time references, an interval of time may be *included* in a set of intervals, although not explicitly.

Definition 5 *Let I be interval and S a set of intervals. $I \Subset S$ if $\exists I_a \in S : I \subseteq I_a$*

Intersection is a commonly required operation on intervals. The intersection of two intervals is the interval formed by all the common points of both of them. Its startpoint and endpoint are the minimal and maximal time points in common, respectively.

Definition 6 *Let I_1 and I_2 be two intervals. The intersection is defined as: $I_1 \cap I_2 = [x, y]$ with $x, y \in I_1$ and $x, y \in I_2$ such that there are no $w, z : w, z \in I_1$ and $w, z \in I_2$ with $w < x$ or $y < z$.*

We will also need to intersect two sets of intervals. We are interested in common timepoints referenced by its intervals, instead of simply intervals in common. This is captured in the following definition.

Definition 7 *Let S_1 and S_2 be two sets of intervals. The intersection of these sets, noted as $S_1 \Cap S_2$, is: $S_1 \Cap S_2 = \{I : I = I_1 \cap I_2 \neq [\,], \forall I_1 \in S_1, I_2 \in S_2\}$*

We will often need to operate over sets of intervals. We are interested in a special difference of sets, called timed-difference Notice that intervals difference return a set of intervals. The set may be empty, unitary or may include two intervals. For example, $[35, 50] - [40, 45] = \{[35, 40), (45, 50]\}$, $[35, 50] - [20, 40] = \{(40, 50]\}$ and $[35, 50] - [10, 100] = \{\}$

Definition 8 *Let S_1 and S_2 be two sets of intervals. The* basic difference *of S_1 and S_2, noted as $S_1 \doteq S_2$ is defined as $S_1 \doteq S_2 = \{I : I \in I_1 - I_2, I \neq [\,], \forall I_1 \in S_1, I_2 \in S_2\}$*

Definition 9 *Let S_1 and S_2 be two sets of intervals. The timed-difference of these sets, noted as $S_1 \overset{I}{-} S_2$, is:*

$$S_1 \overset{I}{-} S_2 = S_1 \doteq S_2 \qquad \text{if } (S_1 \doteq S_2) \doteq S_2 = S_1 \doteq S_2$$
$$S_1 \overset{I}{-} S_2 = (S_1 \doteq S_2) \overset{I}{-} S_2 \qquad \text{otherwise}$$

In the following section we present Timed Abstract Argumentation Frameworks. In Section 5 the timed notions of argument defense are defined towards a suitable admissibility semantics.

4 Timed Argumentation Framework

In [7, 8] timed arguments are associated with a single interval of time. In this work we expand the formalism by modeling arguments that can be available over a set of non-contiguous intervals. The availability of an argument is determined by the following mapping from arguments to sets of intervals.

Definition 10 *The availability function $\mathcal{A}v$ is defined as $\mathcal{A}v : Args \rightarrow \wp(\Re)$, such that $\mathcal{A}v(\mathcal{A})$ is the set of availability intervals of an argument $\mathcal{A}$.*

The formal definition of our timed abstract argumentation framework follows.

Definition 11 *A timed abstract argumentation framework (or simply TAF) is a 3-tuple $\langle Args, Atts, \mathcal{A}v\rangle$ where $Args$ is a set of arguments, $Atts$ is a binary attack relation defined over $Args$ and $\mathcal{A}v$ is the availability function for timed arguments.*

Example 2 *The triplet $\langle Args, Atts, \mathcal{A}v\rangle$, where $Args = \{\mathcal{A}, \mathcal{B}, \mathcal{C}, \mathcal{D}, \mathcal{E}\}$, $Atts = \{(\mathcal{B}, \mathcal{A}), (\mathcal{C}, \mathcal{B}), (\mathcal{D}, \mathcal{A}),\}$ and the availability function is defined as*

$Args$	$\mathcal{A}v$	$Args$	$\mathcal{A}v$
$\mathcal{A}$	$\{[10, 40], [60, 75]\}$	$\mathcal{B}$	$\{[30, 50]\}$
$\mathcal{C}$	$\{[20, 40].[45, 55], [60, 70]\}$	$\mathcal{D}$	$\{[47, 65]\}$
$\mathcal{E}$	$\{[10, +\infty]\}$		

is a timed abstract argumentation framework.

The framework of Example 2 can be depicted as in Figure 1, using a digraph where nodes are arguments and arcs are attack relations. An arc from argument $\mathcal{X}$ to argument $\mathcal{Y}$ exists if $(\mathcal{X}, \mathcal{Y}) \in Atts$. Figure 1 also shows the time availability of every argument, as a graphical reference of the $\mathcal{A}v$ function. It is basically the framework's evolution in time. Startpoints and endpoints are marked with a vertical line, except for $-\infty$ and ∞.

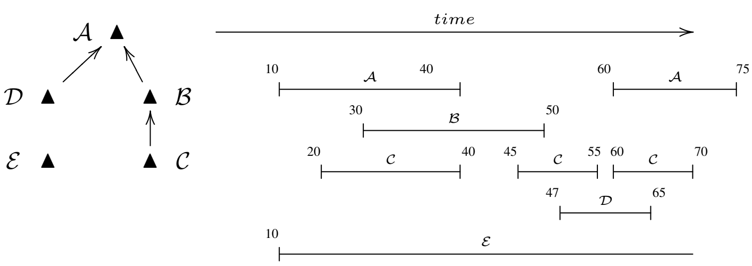

Fig. 1. Framework of Example 2

Definition 12 *Let $\Phi\langle Args, Atts, \mathcal{A}v\rangle$ be a TAF. A timed argument profile in Φ, or simply* t-profile*, is a pair $\langle \mathcal{A}, T\rangle$ where $\mathcal{A} \in Args$ and T is a set of time intervals. The t-profile $\langle \mathcal{A}, \mathcal{A}v(\mathcal{A})\rangle$ is called the basic* t-profile of $\mathcal{A}$.

An attack may actually occur if an availability interval of the attacker overlaps an availability interval of the attacked argument, $i.e.$ both intervals share time points. As in the present paper an argument may be available in several disjoint intervals of time, the main issue now is finding *when* an argument may be attacked or not. This is because the overlapping availability of both arguments may occur several times. The overlapping portion of availability intervals is called an *attainability interval* of the attack. This is captured by a timed argument profile.

Definition 13 *Let $\mathcal{A}, \mathcal{B} \in Args$ such that $(\mathcal{A}, \mathcal{B}) \in Atts$. The set of intervals in which the attack $(\mathcal{A}, \mathcal{B})$ is attainable, denoted $AttAtts_{\Phi}((\mathcal{A}, \mathcal{B}))$ is defined as $AttAtts_{\Phi}((\mathcal{A}, \mathcal{B})) = \mathcal{A}v(\mathcal{A}) \Cap \mathcal{A}v(\mathcal{B})$*

Definition 14 *Let $\langle Args, Atts, \mathcal{A}v \rangle$ be a TAF, I an interval and $S \subseteq Args$. The set S is said to be conflict-free at I if there are not two arguments $\mathcal{A}$ and $\mathcal{B}$ such that $(\mathcal{A}, \mathcal{B}) \in Atts : I \Subset AttAtts_{\Phi}(\mathcal{A}, \mathcal{B})$.*

Definition 15 *Let $\Phi = \langle Args, Atts, \mathcal{A}v \rangle$ be a TAF. The set of threat intervals for $\mathcal{A}$, denoted $\tau_{\Phi}(\mathcal{A})$, is: $\tau_{\Phi}(\mathcal{A}) = \bigcup_{\mathcal{X} \in Args} AttAtts_{\Phi}((\mathcal{X}, \mathcal{A}))$*

The overlapping of availability intervals implicitly defines several subintervals. These subintervals are determined by the startpoints and endpoints of such intervals. Figure 2 depicts two overlapping closed intervals with the induced subintervals. If argument $\mathcal{B}$ attacks argument $\mathcal{A}$, then this attack occurs in $[45, 50]$, which is the precise period of time in which both arguments are available. Argument $\mathcal{A}$ is not attacked by $\mathcal{B}$ in $[30, 45)$ nor in $(50, 65]$. Note that these subintervals are semi-closed, since moments 45 and 50 belong to both original intervals.

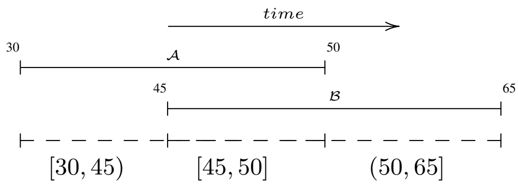

Fig. 2. Induced subintervals induced from overlapping

In the same figure, suppose arguments $\mathcal{A}$ and $\mathcal{B}$ are both attackers of an argument $\mathcal{C}$ such that $\mathcal{A}v(\mathcal{C}) = \{[30, 65)\}$. Then $\mathcal{C}$ requires a defender in $[30, 45)]$ against $\mathcal{A}$. It requires defenders against both $\mathcal{A}$ and $\mathcal{B}$ in $[45, 50]$, and it requires a defender against $\mathcal{B}$ in $(45, 65)$. It is important to identify these subintervals, since an argument may be well defended only in certain periods of time, depending in the overlapping portion of its attackers and defenders. In the framework of Figure 1, argument $\mathcal{A}$ is defended by $\mathcal{C}$ in $[30, 40]$.

Since attacks and defenses are going in and out while arguments become or cease to be available, it is possible to define the set of all the minimal subintervals induced by multiple overlappings. This is formalized in the following definition.

Definition 16 *Let S be a set of intervals. The partition of S, denoted* $\mathsf{Part(S)}$ *is defined as:*

- $\mathsf{Part(S)} = S$ *if* $\forall I_1, I_2 \in S, I_1 \cap I_2 = \emptyset$.
- $\mathsf{Part(S)} = \mathsf{Part(S} - \{\mathsf{I_1}, \mathsf{I_2}\} \cup \{\mathsf{I_1} - (\mathsf{I_1} \cap \mathsf{I_2}), \mathsf{I_2} - (\mathsf{I_1} \cap \mathsf{I_2}), \mathsf{I_1} \cap \mathsf{I_2}\})$*, with* $I_1, I_2 \in S$ *and* $I_1 \cap I_2 \neq \emptyset$

The partition of a set of arguments *brokes* overlapped intervals in smaller intervals. This notion simplifies semantic elaborations, since it discretizes the evolution of the framework according to moments where arguments start or cease to be available. As stated in the following proposition, there are no overlapping intervals in the partition. Proofs are omitted for space reasons.

Proposition 1 *Let S be a set of intervals. There are not two arguments $I_1, I_2 \in$ $\mathsf{Part(S)}$ such that $I_1 \cap I_2, I \neq [\,]$*

Partition also allows the characterization of difference between intervals, as shown in the following definition.

Definition 17 *Let I_1 and I_2 be two intervals. The difference, noted as $I_1 - I_2$, is defined as: $I_1 - I_2 = \{I : I \in \mathsf{Part}(\{\mathsf{I_1}, \mathsf{I_2}\}), I \subseteq I_1, I \not\subseteq I_2\}$*

5 Defense through time

An argument may be attacked in several intervals of time. It is defended in those threat intervals, only when another argument has an available attack to its attacker. Thus, unlike classic frameworks where an argument may be defended or not, in this timed formalism of argumentation an argument may be defended or not in some moments of time. It is not enough to establish if a defense condition is present by looking attacks. It is mandatory to find out *when* these defenses may occur.

Definition 18 *Let $\Phi = \langle Args, Atts, \mathcal{A}v\rangle$ be a TAF and S a set of arguments. The set of defense intervals for $\mathcal{A}$ against $\mathcal{B}$, denoted $\delta_{\mathcal{A}}^{\mathcal{B}}(S)$, is defined as:*
$\bigcup\{AttAtts_{\Phi}((\mathcal{X}, \mathcal{B})) \Cap AttAtts_{\Phi}((\mathcal{B}, \mathcal{A})) : \mathcal{X} \in S\}$

Given a timed-argument $\mathcal{A}$, if $\tau_{\Phi}(\mathcal{A}) \neq \emptyset$ then $\mathcal{A}$ needs a defense in certain periods of time. An argument $\mathcal{B}$ may attack $\mathcal{A}$ in different moments, and these moments must be properly captured.

Definition 19 *Let $\Phi = \langle Args, Atts, \mathcal{A}v\rangle$ be a TAF and $\mathcal{A} \in Args$. The set of all the attackers and their threats on $\mathcal{A}$, denoted $needsDefense(\mathcal{A})$ is formally defined as:*
$needsDefense(\mathcal{A}) = \{\langle \mathcal{X}, I\rangle : I = \mathsf{Part}(\tau_{\Phi}(\mathcal{A})) \Cap AttAtts_{\Phi}((\mathcal{X}, \mathcal{A}))\}$

The set $needsDefense(\mathcal{A})$ is a set of t-profiles denoting concrete threats on $\mathcal{A}$. It is based on the partition of threat intervals of $\mathcal{A}$. This discretization of threats is needed since the number of attackers remain the same within each of these intervals. It allows the individualization of intervals with multiple attackers.

Proposition 2 *If $\mathcal{A}v(\mathcal{X}) \Cap \mathcal{A}v(\mathcal{Y}) \neq \emptyset$ and $(\mathcal{X}, \mathcal{Z}), (\mathcal{Y}, \mathcal{Z}) \in Atts$ for some $\mathcal{Z} \in Args$, then there are profiles $\langle \mathcal{X}, I_X \rangle$, $\langle \mathcal{Y}, I_Y \rangle \in needDefense(\mathcal{Z})$ such that $I_X \Cap I_Y \neq \emptyset$*

We will analyze the behavior of the framework in a given interval. The attackers of an argument in a period of time I is obtained as follows.

Definition 20 *Let $\Phi = \langle Args, Atts, \mathcal{A}v \rangle$ be a TAF, $\mathcal{A} \in Args$ and $I \in \mathsf{Part}(\tau_\Phi(\mathcal{A}))$*

$$attackers(\mathcal{A}, I) = \{\mathcal{X} : \langle \mathcal{X}, set \rangle \in needsDefense(\mathcal{A}), I \in set\}$$

In particular we are interested, for an argument $\mathcal{A}$, in the smaller intervals induced by the partition of the set of threat intervals of $\mathcal{A}$. The following definitions capture the set of intervals in which an argument $\mathcal{A}$ is actually defended.

Definition 21 *Let $\Phi = \langle Args, Atts, \mathcal{A}v \rangle$ be a TAF, S a set of arguments, $\mathcal{A} \in Args$ and $I \in \mathsf{Part}(\tau_\Phi(\mathcal{A}))$. Let $\mathcal{X} \in attackers(\mathcal{A}, I)$. The set of intervals where S defends $\mathcal{A}$ from $\mathcal{X}$, noted as $defense(\mathcal{X}, \mathcal{A}, I)$, is defined as:*

$$defense(\mathcal{X}, \mathcal{A}, I) = \delta^{\mathcal{X}}_{\mathcal{A}}(S) \Cap \{I\}$$

Definition 22 *Let $\Phi = \langle Args, Atts, \mathcal{A}v \rangle$ be a TAF and S a set of arguments. Let $I \in \mathsf{Part}(\tau_\Phi(\mathcal{A}))$ The set of defended intervals for $\mathcal{A}$ at I, denoted $\Delta^S_{\mathcal{A}}(I)$, is defined as:*

$$\Delta^S_{\mathcal{A}}(I) = \Cap_{\mathcal{X} \in attackers(\mathcal{A}, I)} defense(\mathcal{X}, \mathcal{A}, I)$$

Note $attackers(\mathcal{A}, I)$ is a set that always has at least a member, since its definition estabish that I belongs to the partion of the threat intervals for $\mathcal{A}$. The interval belongs to that set only if there there is at least an active or attainable attack.

Definition 23 *Let $\Phi = \langle Args, Atts, \mathcal{A}v \rangle$ be a TAF and S a set of arguments. The set of defended intervals for $\mathcal{A}$, denoted $\Delta^S_{\mathcal{A}}$, is:*

$\Delta^{\emptyset}_{\mathcal{A}} = \mathcal{A}v(\mathcal{A}) \stackrel{I}{-} \tau_\Phi(\mathcal{A})$
$\Delta^{S}_{\mathcal{A}} = \Delta^{\emptyset}_{\mathcal{A}} \cup \bigcup_{I \in \mathsf{Part}(\tau_\Phi(\mathcal{A}))} \Delta^S_{\mathcal{A}}(I)$ *when* $S \neq \emptyset$.

In timed abstract frameworks, an argument $\mathcal{A}$ is acceptable with respect to a set S only during the time defined in $\Delta^S_{\mathcal{A}}$. The following example illustrates these notions.

Example 3 *Let $\Phi = \langle Args, Atts, \mathcal{A}v \rangle$ be a TAF where $Args = \{\mathcal{A}, \mathcal{B}, \mathcal{C}, \mathcal{D}, \mathcal{E}, \mathcal{F}\}$, $Atts = \{(\mathcal{B}, \mathcal{A}), (\mathcal{C}, \mathcal{A}), (\mathcal{D}, \mathcal{C}), (\mathcal{E}, \mathcal{B}), (\mathcal{F}, \mathcal{B}),\}$ and the availability function is defined as*

$Args$	$\mathcal{A}v$	$Args$	$\mathcal{A}v$
$\mathcal{A}$	$\{[0, 30], [60, 75], [80, 100]\}$	$\mathcal{B}$	$\{[0, 20], (85, 95)\}$
$\mathcal{C}$	$\{[10, 30]\}$	$\mathcal{D}$	$\{[0, 15], [25, 40]\}$
$\mathcal{E}$	$\{[12, 30]\}$	$\mathcal{F}$	$\{[80, 90]\}$

The set of intervals where attacks are attainable is the following.

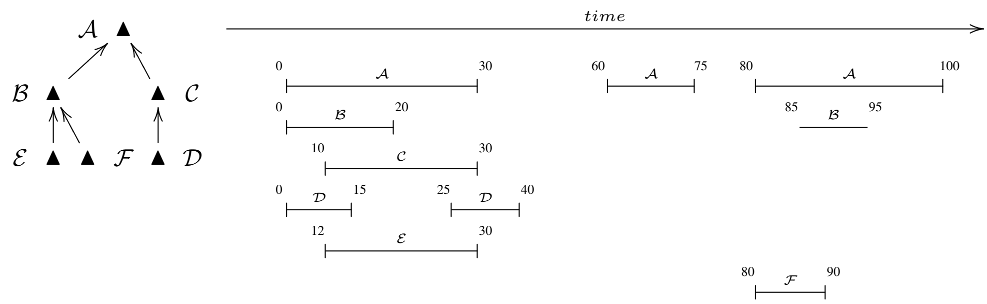

Fig. 3. Timed Framework of Example 3

$Atts$	$AttAtts_{\Phi}$	$Atts$	$AttAtts_{\Phi}$
$(\mathcal{B},\mathcal{A})$	$\{[0,20],(85,95)\}$	$(\mathcal{C},\mathcal{A})$	$\{[10,30]\}$
$(\mathcal{D},\mathcal{C})$	$\{[10,15],[25,30]\}$	$(\mathcal{E},\mathcal{B})$	$\{[12,20]\}$
$(\mathcal{F},\mathcal{B})$	$\{(85,90]\}$		

The attainability of $(\mathcal{B},\mathcal{A})$ *was obtained as follows:*

$$\begin{aligned} AttAtts_{\Phi}((\mathcal{B},\mathcal{A})) &= \mathcal{A}v(\mathcal{B}) \Cap \mathcal{A}v(\mathcal{A}) \\ &= \{[0,20],(85,95)\} \Cap \{[0,30],[60,75],[80,100]\} \\ &= \{[0,20] \cap [0,30],(85,95) \cap [80,100]\} \qquad [*] \\ &= \{[0,20],(85,95)\} \end{aligned}$$

$[*]$ *the other possible intersections lead to* $[\,]$*, i.e.*$[0,20]\cap[60,75] = [0,20]\cap[80,100] = (85,95) \cap [0,30] = (85,95) \cap [60,75] = [\,]$. *Remember that the empty interval is not included in the intersection.*

To determine where argument $\mathcal{A}$ *is defended, it is necessary to determine where it is threatened.*

$$\begin{aligned} \tau_{\Phi}(\mathcal{A}) &= AttAtts_{\Phi}((\mathcal{B},\mathcal{A})) \cup AttAtts_{\Phi}((\mathcal{C},\mathcal{A})) \\ &= \{[0,20],(85,95)\} \cup \{[10,30]\} \\ &= \{[0,20],(85,95),[10,30]\} \end{aligned}$$

In order to determine the attackers and its moments of threats we need to partitionate $\tau_{\Phi}(\mathcal{A})$.

$$\begin{aligned} \mathsf{Part}(\tau_{\Phi}(\mathcal{A})) &= \mathsf{Part}(\{[0,20],(85,95),[10,30]\}) \\ &= \mathsf{Part}(\{[0,20],(85,95),[10,30]\} - \{[0,20],[10,30]\} \cup \\ &\quad \{[0,20]-[10,20],[10,30]-[10,20],[10,20]\}) \qquad [1] \\ &= \mathsf{Part}(\{(85,95)\} \cap \{[0,10),(20,30],[10,20]\}) \\ &= \mathsf{Part}(\{(85,95),[0,10),(20,30],[10,20]\}) \\ &= \{\{(85,95),[0,10),(20,30],[10,20]\}\} \qquad [2] \end{aligned}$$

$[1]$ *the recursive part of the definition is applied since* $[0,20]\cap[10,30] = [10,20] \neq [\,]$.
$[2]$ *basic case applies, since all the possible intersections between* S *members are equal to* $[\,]$.

The set $needsDefense(\mathcal{A})$ contains all the attackers of $\mathcal{A}$ along with the set of intervals where they actually attack $\mathcal{A}$. These intervals differs from those in $AttAtts_{\Phi}$ since they are partitioned in order to consider the rest of the attacks.

$needsDefense(\mathcal{A}) = \{\langle\mathcal{B}, \{[0, 10), [10, 20], (85, 95)\}\rangle, \langle\mathcal{C}, \{(20, 30], [10, 20]\}\rangle\}$.

It can now be calculated who attacks $\mathcal{A}$ at each interval in $\mathsf{Part}(\tau_{\Phi}(\mathcal{A}))$ and the correspondig defenses.

$I \in \mathsf{Part}(\tau_{\Phi}(\mathcal{A}))$	$attackers(\mathcal{A}, I)$	$I \in \mathsf{Part}(\tau_{\Phi}(\mathcal{A}))$	$attackers(\mathcal{A}, I)$
$[0, 10)$	$\{\mathcal{B}\}$	$[10, 20]$	$\{\mathcal{B}, \mathcal{C}\}$
$(20, 30]$	$\{\mathcal{C}\}$	$(85, 95)$	$\{\mathcal{B}\}$

The defenses provided from set S against an attacker $\mathcal{X}$ of $\mathcal{A}$ happens during $\delta^{\mathcal{A}}_{\mathcal{X}}(S)$.

$$\delta^{\mathcal{B}}_{\mathcal{A}}(Args) = \{[12, 20], (85, 90]\} \qquad \delta^{\mathcal{C}}_{\mathcal{A}}(Args) = \{[10, 15], [25, 30]\}$$

Notice that, for example,

$$\begin{aligned}\delta^{\mathcal{B}}_{\mathcal{A}} = {} & AttAtts_{\Phi}((\mathcal{E}, \mathcal{B})) \Cap AttAtts_{\Phi}((\mathcal{B}, \mathcal{A})) \cup \\ & AttAtts_{\Phi}((\mathcal{F}, \mathcal{B})) \Cap AttAtts_{\Phi}((\mathcal{B}, \mathcal{A})) \\ & \{(12, 20]\} \Cap \{[0, 20], (85, 95)\} \cup \{(85, 90]\} \Cap \{[0, 20], (85, 95)\} \\ & \{[12, 20], (85, 90]\}\end{aligned}$$

The periods where $\mathcal{A}$ is actually defended from a particular attack over each interval in $\mathsf{Part}(\tau_{\Phi}(\mathcal{A}))$ is characterized as follows:

$$defense(\mathcal{B}, \mathcal{A}, [0, 10) = \delta^{\mathcal{B}}_{\mathcal{A}} \Cap \{[0, 10]\} = \{[12, 20], (85, 90]\} \Cap \{[0, 10]\} = \emptyset$$

the rest are:

$$\begin{aligned} & defense(\mathcal{B}, \mathcal{A}, [0, 10)) = \emptyset \\ & defense(\mathcal{B}, \mathcal{A}, [10, 20]) = \{[12, 20]\} \\ & defense(\mathcal{B}, \mathcal{A}, (85, 95)) = \{(85, 90]\}\end{aligned} \qquad \begin{aligned} & defense(\mathcal{C}, \mathcal{A}, [10, 20]) = \{[10, 15]\} \\ & defense(\mathcal{C}, \mathcal{A}, (20, 30]) = \{[25, 30]\}\end{aligned}$$

We can now determine defenses of all attacks over an interval.

$$\begin{aligned}\Delta^{Args}_{\mathcal{A}}([0, 10)) &= \emptyset \\ &= defense(\mathcal{B}, \mathcal{A}, [10, 20]) \\ \Delta^{Args}_{\mathcal{A}}([10, 20]) &= \{[12, 15]\} \\ &= defense(\mathcal{B}, \mathcal{A}, [10, 20) \Cap defense(\mathcal{C}, \mathcal{A}, [10, 20]) \\ &= \{[12, 20]\} \Cap \{[10, 15]\} \\ \Delta^{Args}_{\mathcal{A}}((20, 30]) &= \{[25, 30]\} \\ &= defense(\mathcal{C}, \mathcal{A}, (20, 30]) \\ \Delta^{Args}_{\mathcal{A}}((85, 95)) &= \{(85, 90]\} \\ &= defense(\mathcal{B}, \mathcal{A}, (85, 90))\end{aligned}$$

Finally the argument $\mathcal{A}$ is defended by $Args$ in:

$$
\begin{aligned}
\Delta_{\mathcal{A}}^{\emptyset} &= \mathcal{A}v(\mathcal{A}) \stackrel{I}{-} \tau_{\Phi}(\mathcal{A}) \\
&= \{[0,30],[60,75],[80,100]\}) \stackrel{I}{-} \{[0,20],(85,95),[10,30]\} \\
&= \{[60,75],[80,85),(95,100]\} \\
\Delta_{\mathcal{A}}^{Args} &= \Delta_{\mathcal{A}}^{\emptyset} \cup \Delta_{\mathcal{A}}^{Args}([0,10)) \cup \Delta_{\mathcal{A}}^{Args}([10,20]) \cup \Delta_{\mathcal{A}}^{Args}((20,30]) \cup \Delta_{\mathcal{A}}^{Args}((85,95)) \\
&= \{[60,75],[80,85),(95,100]\} \cup \emptyset \cup \{[12,15]\} \cup \{[25,30]\} \cup \{(85,90]\} \\
&= \{[60,75],[80,85),(95,100],[12,15],[25,30],(85,90] \\
&= \{[12,15],[25,30],[60,75],[80,85),(85,90],(95,100]
\end{aligned}
$$

Having the timed notion of defense, where the focus is put in the characterization of those intervals in which an argument is defended, leads to the adapted timed notion of acceptability of arguments and admissible sets.

Definition 24 *Let $\Phi = \langle Args, Atts, \mathcal{A}v\rangle$ be a TAF and S be a set of arguments. An argument $\mathcal{A} \in Args$ is acceptable with respect to S if $\Delta_{\mathcal{A}}^{S} \neq \emptyset$. If $\mathcal{A}$ is acceptable, then it is acceptable at $\Delta_{\mathcal{A}}^{S}$.*

Definition 25 *Let S be a set of arguments, let $AdProf(S) = \{\langle \mathcal{A}, \Delta_{\mathcal{A}}^{S}\rangle : \mathcal{A} \in S\}$ and $AdInterv = \{\Delta_{\mathcal{A}}^{S} : \mathcal{A} \in S\}$ The set S is admissible if: $\forall_{I \in \mathsf{Part}(\mathsf{AdInterv})}\{\mathcal{X} : \langle \mathcal{X}, I_X\rangle \in AdProf(S), I \Subset I_X\}$ is conflict-free at I*

6 Related Work

Argumentation and time is a recent line of research. In [7, 8] a novel abstract framework is proposed, where arguments are relevant only in a single period of time. A skeptical, timed interval-based semantics is proposed, using admissibility notions on discrete time representation. The present paper study intermittent arguments with dense time representation, as suggested by other authors. As related work in combining time and argumentation, in [11] a calculus for representing temporal knowledge is proposed, and defined in terms of propositional logic. This calculus is then considered with respect to argumentation, where an argument is defined in the standard way: an argument is a pair constituted by a minimally consistent subset of a database entailing its conclusion. This work is thus related to [4]. In contrast, here we maintain our development at the abstract level in an effort to capture intuitions related with the dynamic interplay of arguments as they become available and cease to be so.

7 Conclusions and future work

In this work we proposed a novel abstract argumentation framework where arguments are only valid for consideration in some periods of time, which is defined for every individual argument. Thus, the attainability of attacks and defenses is related to time. Since arguments are attacked and defended in different moments, the relevant semantic must inquiery *when* this conditions are met. We formally defined time-related defense conditions on dense time.

Future work has several directions. Argument semantics other than the admissibility are to be studied in the context of timed argumentation. In particular, we are interested in the treatment of cycles of timed arguments and its semantic consequences.

Also, for a given semantic notion $\mathcal{S}$, there may be intervals of time in which the extensions induced by $\mathcal{S}$ do not change, even when some arguments become or cease to be available during these intervals. These are called *steady periods* of the framework and are also an interesting topic. It may be used to model *eras of thinking* for a rational agent or a society, and the impact of including new arguments. New semantics elaborations based in this notion are being studied.

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