# A Circular Grid-Based Feature Extraction Approach for Off-line Signature Verification

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**Abstract.** In this paper, a new feature extraction approach based on a circular grid is proposed for off-line signature verification. Graphometric features used in the rectangular grid segmentation approach are adapted to this new grid geometry. A Support Vector Machine (SVM) based classifier scheme is used for classification tasks and a comparison between the rectangular and the circular grid approaches is performed. The results obtained during the experimental phase have shown improvements when using the proposed features with respect to the case of using the features extracted from rectangular grids, specially, in discriminating simple and skilled forgeries.

**Key words:** Off-line Signature Verification, Support Vector Machines, Feature Extraction.

## 1 Introduction

Today's society need for personal authentication has made automatic personal verification to be considered as a fundamental task in many daily applications. Signature verification plays an important role in the field of personal authentication, being the most popular method of identity verification. For example, financial and administrative institutions recognize signatures as a legal means of verifying an individual's identity. In addition, no invasive methods of collecting the signature are needed and the use of signatures is familiar to people in their everyday's life. Two different categories of signature verification systems can be distinguished: off-line and on-line systems [1]. For off-line systems the acquisition process takes place once the writing process has finished, and the information acquired is a static image of the signature. For on-line systems, instead, the acquisition is performed during the writing process, thus dynamic information is available.

The aim of the signature verification system is to accurately distinguish between two categories of signatures, namely, genuine and forged signatures. Different types of classifiers have been applied to solve this classification problem, being those based on Hidden Markov Models (HMM) [2], [3], [4], [5], and Support Vector Machines (SVM) [4], [5], [6], among the most frequently used. To evaluate system performance, two types of errors, namely, the False Rejection Rate (FRR) and the False Acceptance Rate (FAR) have been defined. The False Rejection Rate concerns the rejection of genuine signatures, while the False Acceptance Rate concerns the acceptance of forged signatures. Additionally, it can be defined the Equal Error Rate (EER), which is the system error rate when FRR=FAR, and it is usually considered as a measure of the overall error of the verification system. Three types of forgeries are usually considered to compute the FAR, namely, random forgeries, simple forgeries and skilled forgeries. Random forgeries refer to signatures that belong to anyone else but the writer under consideration. Simple forgeries are signatures that the forger tries to make up without any previous knowledge of the original one, while skilled forgeries refer to the case when the forger tries to imitate the signature from an image of the original one. Fig. 1 shows an instance of an original signature (a), a random forgery of it (b), a simple forgery (c) and a skilled forgery (d).

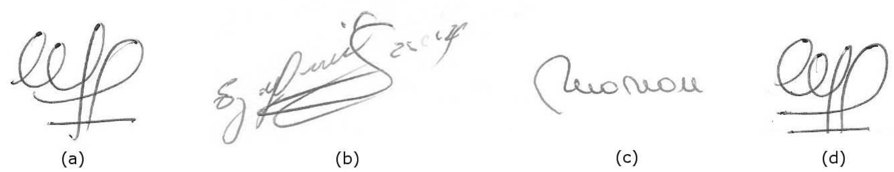


**Fig. 1.** An original signature instance and its different types of forgeries. (a) Original signature; (b) Random forgery; (c) Simple forgery; (d) Skilled forgery.

A fundamental step in a signature verification process is the feature selection. Different methods have been proposed in the off-line signature verification literature to perform the selection and the extraction of the features from the signature image. Generally, the features can be classified into two categories, namely, global features and local features. Global features refer to features that are representative of the whole signature image, while local features are those extracted from particular parts of the signature image. Grid segmentation schemes have been frequently used to compute local features. In addition, features used in graphology have been adapted to compute them resorting to grid schemes. Such features are called graphometric features as it is discussed in [7]. In [8], [2], [3], [5] and [7], graphometric features are computed resorting to a rectangular grid scheme.

In this paper, a new feature extraction approach based on a circular grid is proposed and graphometric features used in the rectangular grid approach are adapted to this new grid geometry. An SVM-based classifier is used to perform the verification process and the system is tested on a database containing genuine as well as forged signatures.

The paper is organized as follows. The proposed circular grid approach and the feature extraction are described in Section 2. Section 3 is devoted to the fundamentals of SVM-based classifiers and its application to signature verification. Experimental results are reported in Section 4. Finally, some concluding remarks are given in Section 5.

## 2 Feature Extraction

Different grid-segmentation schemes have been used in Off-line Signature Verification Systems for the purposes of graphometric feature extraction. As it was mentioned, in [2], [3], [5], [7] and [8], graphometric features are computed from a rectangular grid of the signature image. In this paper, a segmentation of the signature image using a circular grid is proposed. One of the motivations for using a circular grid is to avoid the problem of having empty sectors present when rectangular grid is employed. The ideal gridding technique would be to compute a bounding ellipsoid of the signature and to divide it into sectors, but then no regular sectors could be computed in this case. The circular grid, instead, allows the division in regular sectors.

In the proposed feature extraction approach, a circular chart enclosing the signature is divided in $N$ identical sectors, and graphometric features are computed for each sector. The circular grid is placed so that the center of the grid matches the center of mass (geometric center) of the binary image of the signature as shown in Fig. 2a. Such a choice for the center of the grid is made in order to avoid having empty grid divisions as much as possible. The center of mass of the signature image has already been used as a reference point in the literature. In [4], a polar representation of the contour of the signature is performed and the origin of the polar space is placed at the geometric center of the signature. For the rectangular grid approach, many ways of placing the center of the rectangle have been proposed. In [8], for example, the image is moved to the left before gridding in order to absorb horizontal variability. In the rectangular approach performed here, the rectangle is chosen as the bounding box of the signature, in order to reduce the number of empty rectangular cells of the grid.

In the method proposed in this paper, some of the graphometric features used in rectangular grid segmentation are adapted to the new grid structure. Three static graphometric features are considered: pixel density distribution $x_{PD}$, gravity center distance $x_{DGC}$ and gravity center angle $x_{AGC}$. The pixel density distribution is calculated as the number of black pixels inside each sector normalized by the total number of pixels inside the sector, as it is shown in (1). The gravity center distance is the distance between the gravity center of each sector (point $A$ in Fig. (c)-(d)) and the center of the circular grid ($d_{GC}$), normalized by the radius of the grid ($R$), as it is shown in (2); and the gravity center angle is the angle of the gravity center of each sector ($\alpha_{CG}$), normalized by the total angle of the sector ($\alpha_{max}$), as it is shown in (3). As usual, the gravity center of each sector is computed as the center of mass of the pixels inside the sector. In Fig. 2 it is shown how these features are obtained from one of the $N$ sectors of the

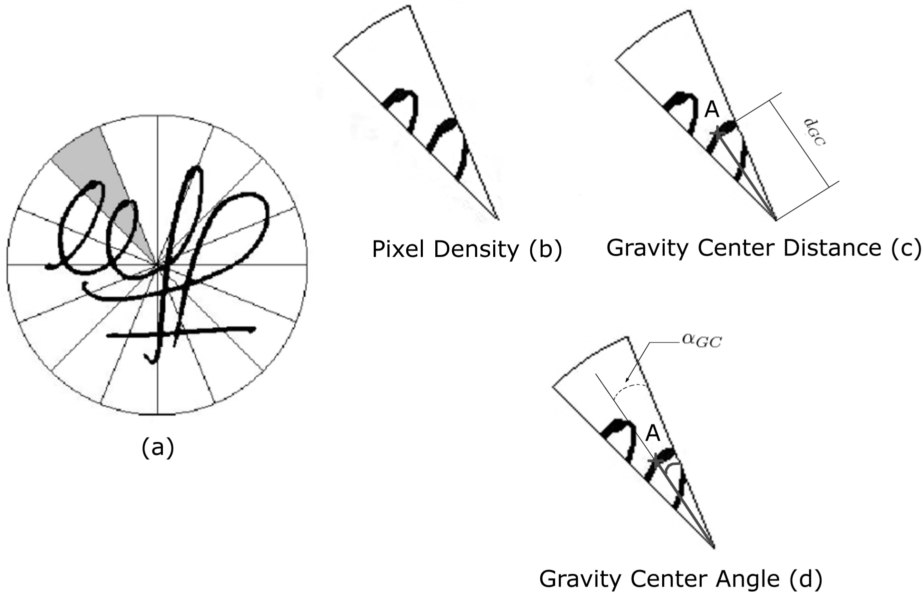


**Fig. 2.** Features extracted from segmented sectors with the circular grid approach: (a) Segmented sector being analyzed; (b) Pixel Density Distribution; (c) Gravity Center Distance; (d) Gravity Center Angle.

signature image.

$$x_{PD_i} = \frac{\text{number of black pixels inside the sector}}{\text{total number of pixels inside the sector}} \quad (1)$$

$$x_{DGC_i} = \frac{d_{GC_i}}{R} \quad (2)$$

$$x_{AGC_i} = \frac{\alpha_{GC_i}}{\alpha_{max}}, \quad \text{being } \alpha_{max} = \frac{2\pi}{N} \quad (3)$$

with $i = 1, ..., N$.

Finally, the feature vector $x_{sign}$ is composed of the features calculated for each of the $N$ angular sectors in which the signature image is divided, *i.e.*

$$x_{sign} = [x_{PD}^T, x_{DGC}^T, x_{AGC}^T]^T, \quad (4)$$

where

$$x_{PD} = [x_{PD_1}, x_{PD_2}, \cdots, x_{PD_N}]^T, \quad (5)$$

$$x_{DGC} = [x_{DGC_1}, x_{DGC_2}, \cdots, x_{DGC_N}]^T, \quad (6)$$

$$x_{AGC} = [x_{AGC_1}, x_{AGC_2}, \cdots, x_{AGC_N}]^T. \quad (7)$$

## 3 Support Vector Machines Classifier

### 3.1 Fundamentals of Support Vector Machines

Support Vector Machines is a quite recent technique of statistical learning theory developed by Vapnik ([9], [10]). In recent years, SVM has been successfully applied to a large number of estimation and classification problems. Particularly, SVM-based classifiers have shown a promising performance in Automatic Signature Verification as demonstrated in [5] and [6].

As an introductory example, suppose a separable classification problem in a two-dimensional input space. There are several separating hyperplanes that can separate the two data classes (Fig. 3a). Nevertheless, a unique separating hyperplane has to be chosen.

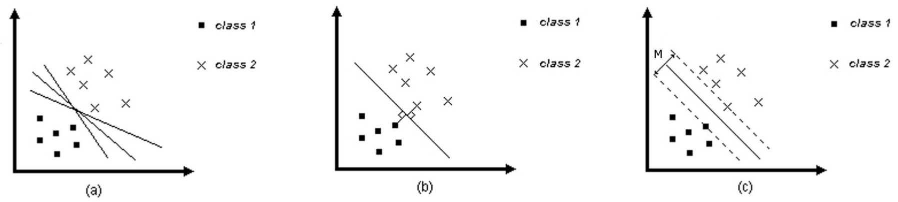


**Fig. 3.** Separable classification problem example: (a) Possible separating hyperplanes; (b) Selection of a unique hyperplane maximizing the distance between the nearest point of each class; (c) Optimal separating hyperplane that maximizes the margin.

Suppose there is a separating hyperplane such that the points $x$ which lie in the hyperplane satisfy $\omega^T x + b = 0$, where $\omega$ is normal to the hyperplane, $|b|/\|\omega\|_2$ is the perpendicular distance from the hyperplane to the origin and $\|\omega\|_2$ is the Euclidean norm of $\omega$. Defining the "margin" (M) as the sum of the distances between this hyperplane and the closest point of each class (Fig. 3b), the separating hyperplane will be optimal if it maximizes this margin (Fig. 3c).

In a separable case, the problem can be formally presented as follows. Consider a given training set $\{x_k, y_k\}_{k=1}^n$, with input data $x_k \in \mathbb{R}^d$, output data $y_k \in \{-1, +1\}$, and suppose that all the training data satisfy the following constraints:

$$\omega^T x_k + b \geq +1, \quad \text{for } y_k = +1 \tag{8}$$

$$\omega^T x_k + b \leq -1, \quad \text{for } y_k = -1 \tag{9}$$

This can be combined into one set of inequalities:

$$y_k[\omega^T x_k + b] - 1 \geq 0, \quad k = 1, ..., n. \tag{10}$$

Consider the points for which Eq.(8) holds, note that the requirement of such points to exist is equivalent to choosing a scale for $\omega$ and $b$. These points lie in the hyperplane $\omega^T x_k + b = 1$ (dotted line in the space of class 1 in Fig. 3c) with normal $\omega$ and distance from the origin $|1-b|/\|\omega\|_2$. Similarly, the points for which Eq.(9) holds, lie in the hyperplane $\omega^T x_k + b = -1$ (dotted line in the space of class 2 in Fig. 3c) with normal $\omega$ and distance from the origin $|-b-1|/\|\omega\|_2$. Then the margin M equals $2/\|\omega\|_2$ and the problem is solved by minimizing $\|\omega\|_2$ subject to the restrictions imposed by the data in Eq.(10). Those training points for which the equality in Eq.(10) holds, are called support vectors. The ones for which Eq.(8) holds, will be support vectors for class 1 and the ones for which Eq.(9) holds, will be support vectors for class 2.

It often occurs that relevant inputs are missing in the training database, data are incomplete, unreliable or noisy. These conditions results in a more general case of non-separable data where one cannot avoid misclassifications (Fig. 4).

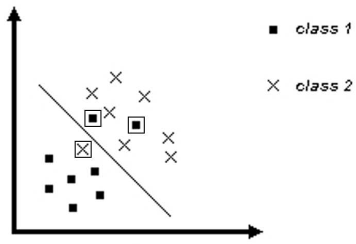


**Fig. 4.** Non-separable classification problem example.

In this case, additional slack variables ($\xi_k$) are introduced in the formulation of the problem in order to relax the constrains in (8) and (9) only when necessary. Then, the set of inequalities takes the following form

$$y_k[\omega^T x_k + b] \geq 1 - \xi_k, \quad k = 1, ..., n. \tag{11}$$

For an error to occur, the corresponding $\xi_k$ must exceed the unity, so $\sum_{k=1}^{n} \xi_k$ is an upper bound of the number of training errors. Then, changing the objective function to be minimized from $\|\omega\|_2$ to $\|\omega\|_2 + c\sum_{k=1}^{n} \xi_k$ will assign an extra cost for errors. The parameter $c$ is a penalization term where larger values of $c$ corresponds to higher penalty to errors.

The extension from the linear to the nonlinear case is straightforward. The linear separating hyperplane is calculated in a higher dimensional feature space where the input data lie after being mapped by a nonlinear mapping $\varphi(x)$. Then, the classifier in the case of nonlinear data can be written as

$$y_k[\omega^T \varphi(x_k) + b] \geq 1 - \xi_k, \quad k = 1, ..., n. \tag{12}$$

Taking into account the Lagrangian formulation of the problem, the training data only appear in the form of dot products between their nonlinear mapping, *i.e.*, they appear as $\varphi(x_k)^T\varphi(x_\ell)\ \forall k,\ell$. Then, no explicit construction of the nonlinear mapping $\varphi(x)$ is needed, by applying the so-called kernel trick. That is, by defining a Kernel as $K(x_k, x_\ell) = \varphi(x_k)^T\varphi(x_\ell)$ for $k,\ell = 1, ..., n$.

Finally, the SVM solution can be found by solving the following optimization problem

$$\begin{aligned} \min_{\omega,b,\xi} \; & J_P(\omega,\xi) = \tfrac{1}{2}\omega^T\omega + c\textstyle\sum_{k=1}^{n}\xi_k \\ \text{s.t.} \quad & y_k[\omega^T\varphi(x_k) + b] \geq 1 - \xi_k, \quad k = 1, ..., n \\ & \xi_k \geq 0, \qquad\qquad\qquad k = 1, ..., n. \end{aligned} \tag{13}$$

Resorting to the dual of problem (13), the solution of the Quadratic Programming (QP) problem is the set of the real positive constants $\alpha_k$, and the SVM classifier takes the following form

$$y(x) = sign[\sum_{k=1}^{n} \alpha_k y_k K(x, x_k) + b]. \tag{14}$$

Different Kernels have been used in the literature to solve pattern recognition problems. Linear, Polynomial and Radial Basis Functions (RBF) Kernels are among the most popular in the bibliography and they are respectively defined as follows

$$\begin{aligned} K_{linear}(x_k, x_\ell) &= x_k^T x_\ell, \\ K_{polynomial}(x_k, x_\ell) &= (1 + x_k^T x_\ell)^d, \\ K_{RBF}(x_k, x_\ell) &= exp(- \parallel x_k - x_\ell \parallel_2^2 / \sigma^2). \end{aligned}$$

### 3.2 SVM-based Classifier Applied to Signature Verification

The database further described in Subsection 4.1 includes genuine and forged signatures. For the latter, random, simple and skilled forgeries are available.

An SVM model was trained for each writer using a training set composed of genuine and false samples. The genuine samples were chosen as a subset of the available writer's genuine signatures. The corresponding false samples, were chosen as a subset of the genuine signatures (the ones separated for training purposes) of the remainder writers in the database. This set of signatures can be interpreted as random forgeries for the writer under consideration. Neither simple nor skilled forgeries were include in the training subset of false samples. For a real application, those types of forgeries are not available during the training phase. Then, avoiding their use for training results in a more realistic model.

To verify a signature, that is to verify the identity claimed by a writer, the feature vector (which is calculated as described in Section 2) is used as the input of an SVM classifier trained for the writer under consideration. The SVM classification process will determine whether the signature belongs to the genuine class or to the false class. Then, the signature will be assumed as genuine and the writer's claimed identity will be true if it belongs to the first class, otherwise the signature will be considered as a forgery assuming the claimed identity to be false. The signature verification experiments were performed resorting to the SVM toolbox for Matlab described in [12].

## 4 Evaluation Protocol

### 4.1 Signature Database

The database used is the GPDS300Signature CORPUS described in [1] [11]. This is a freely distributed version of the database described in [4]. There are 160 writers enrolled in the database. For each writer, there are 24 genuine signatures and 30 forged signatures, taking into account simple and skilled forgeries. That is, a total of $160 \times 24 = 3840$ genuine and $160 \times 30 = 4800$ forged signatures. For a writer in the database, genuine signatures of all the other enrolled writers were used as random forgeries.

### 4.2 Experiments and Results

The database was organized as follows: the 30 forged signatures available per writer were used exclusively for testing, while the 24 genuine signatures available per writer were randomly divided into two groups. The first one, containing 14 signatures, was used for training purposes. The second one, consisting of 10 signatures, was used for testing. For each writer, the set of training samples was composed of 14 genuine signatures and 795 random forgeries (5 genuine signatures randomly chosen from the 14 available for each of the 159 remainder writers). The set of testing sampleswas composed of 10 genuine signatures used to calculate the FRR and the FAR for random forgeries, and of 30 forged signatures used to calculate the FAR for simple and skilled forgeries.

The principal aim of the testing phase is to evaluate the circular grid performance. For this purpose, experiments with different number of grid divisions $N = 8$, $N = 16$, $N = 32$, $N = 64$ and $N = 128$, were carried out. $N$ is also the number of rectangular cells considered in the rectangular grid approach. Hence, $N$ could be any number but a prime number. Choosing the value of $N$ as a power of 2 makes the circular grid have a particular symmetry that makes computation of the proposed graphometric features easier. In order to compare both feature extraction techniques, experiments with the rectangular approach where carried out with the same number of divisions. Fig. 5 shows the FRR for the circular grid approach (top), and for the rectangular grid approach (bottom), for different number of divisions and three different kernels, namely, Linear, RBF, and Polynomial kernels. The FAR for simple and skilled forgeries for the same number of divisions, and the same kernels is shown in Fig. 6, for the circular (top)

[1] Even though in the title of the reference [11] the database has a different name (GPDS-960 CORPUS), the authors required the database to be named as GPDS300Signature CORPUS in the License Agreement for non-commercial research use of the database.

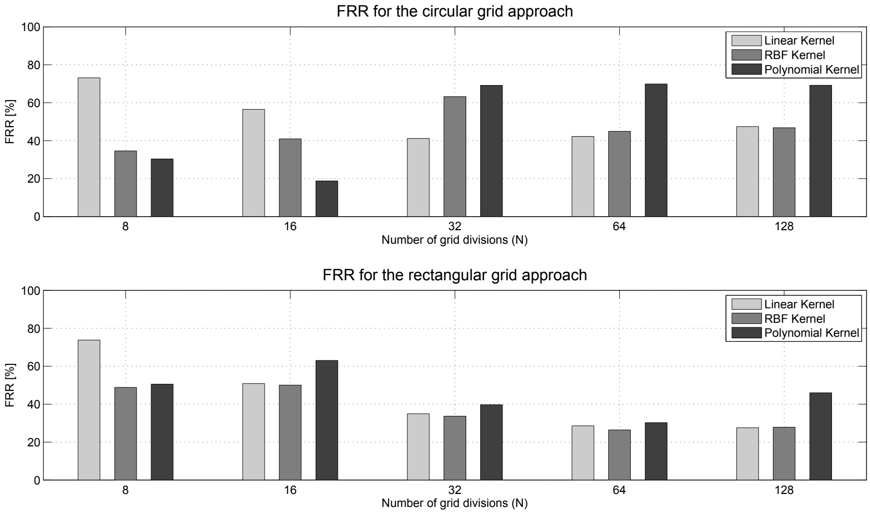


**Fig. 5.** FRR for different number of divisions and kernels, for the circular (top) and the rectangular (bottom) grid approaches

and the rectangular (bottom) gridding. Similarly the FAR for random forgeries is shown in Fig. 7. Polynomial kernels with different degrees and RBF kernels with different values of $\sigma^2$ were tested. The results shown in Fig. 5, Fig. 6 and Fig. 7 are the best results over all the results obtained with the tested kernel's parameters. The regularization parameter $c$ was also tested at several values. Results showed that $c$ has not a major influence on the classifier's performance. Then, $c$ was chosen to be $c = 1000$ (an intermediate value in order not to weakly penalize the training errors and not to highly penalize them either).

The proposed approach shows the best results when the number of divisions of the grid becomes smaller. For $N = 8$ and $N = 16$ the results obtained with the Polynomial Kernel are promising, specially in the case of the FAR for simple and skilled forgeries, showing the system's capability to highlight the interpersonal variability. For the FRR, results are not that good, but they still are acceptable and comparable with the methods of the state of the art. Particularly, the best result in the sense of the Equal Error Rate (EER) is obtained with 16 divisions of the grid ($N = 16$) and a Polynomial Kernel of degree 3, being the False Rejection Error Rate $FRR = 18.75\%$, the False Acceptance Error Rate for simple and skilled forgeries $FAR = 2.125\%$ and for random forgeries $FAR = 0.0727\%$.

For the conventional approach, instead, the best results are reached conforming the number of divisions of the grid is increased. Hence, the proposed approach has the advantage of getting good results while dealing with feature vectors in a lower dimensional space. This particularity can be related with the geometric structure of the proposed grid. As the number of divisions is increased,

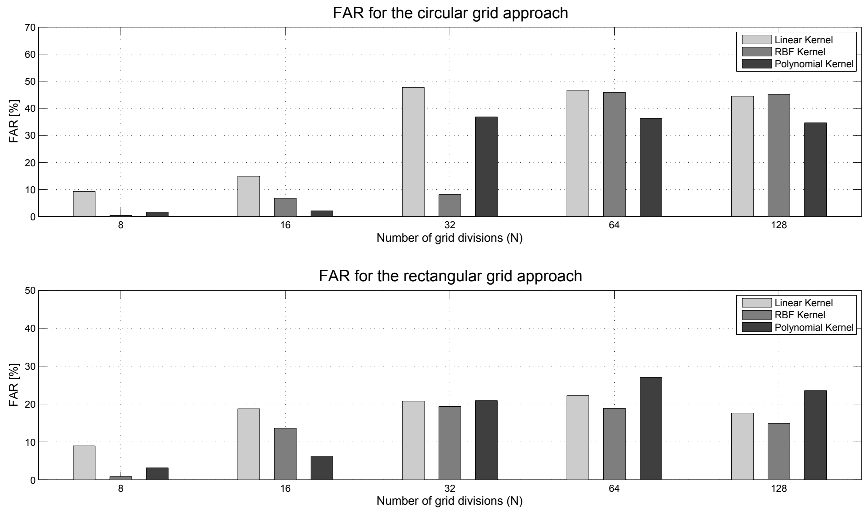


**Fig. 6.** FAR (simple and skilled forgeries) for different number of divisions and kernels, for the circular (top) and the rectangular (bottom) grid approaches

the size of the sectors is decreased. While not modifying the radius of the grid, only the angular amplitude of each sector is reduced. Then, the rate between the radius and the angular amplitude of a sector is not preserved. That makes the area near the center of the grid suffer a higher relative reduction than the area near the contour, resulting in a geometric structure more sensitive to changes in the number of divisions. When the number of divisions increases the area of the sectors is so small that it does not make sense to compute the features inside them. Computing the features inside such small sectors would introduce important errors.

It can be noticed that the best results obtained with the proposed method (corresponding to $N = 16$ and the third degree polynomial kernel) show improvements with respect to the best results achieved with the rectangular grid approach (corresponding to $N = 128$ and RBF kernel with ($\sigma^2 = 100$) ). This is summarized in Table 1.

The presented results show that the FRR needs to be reduced. This means incrementing the verification process capability to absorb the intrapersonal variability. Efforts have to be done in this direction, while keeping the capability to highlight the interpersonal variability.

To address the problem, a possible strategy is to introduce new graphometric features (specially dynamic ones) and choose a suitable combination to achieve a better model of the signature. Also modifying the number of samples used for training can be tested. Using much more signatures to train the false class than the genuine class, may result in a system more adapted to interpersonal

**Table 1.** Best results for circular and rectangular grid approaches

| | Circular Grid<br>N=16, Polynomial kernel<br>Feature Vect. Dim.=**48** | Rectangular Grid<br>N=128, RBF kernel<br>Feature Vect. Dim.=384 |
|---|---|---|
| FRR | **18.75**% | 27.875% |
| FAR (simple and skilled forgeries) | **2.125**% | 14.9167% |
| FAR (random forgeries) | 0.0727% | **0.0106**% |

variability than to intrapersonal variability, and that is the case of the model used. It is likely that reducing the number of random forgeries used to train the false class, will result in an improvement in the FRR value.

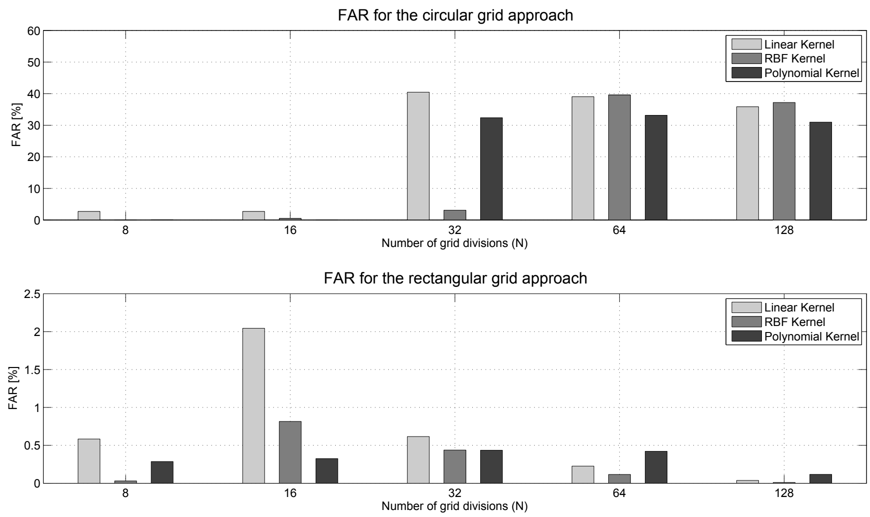


**Fig. 7.** FAR (random forgeries) for different number of divisions and kernels, for the circular (top) and the rectangular (bottom) grid approaches

## 5 Conclusions

In this paper, a new feature extraction approach based on a circular grid has been proposed for off-line signature verification. A comparison between the circular and the rectangular grid based feature extraction approaches has been performed over a SVM-based classification scheme. The classification results, quantified by the FRR and the FAR for simple and skilled, and random forgeries, using the proposed features have shown improvements with respect to the ones based on features extracted from rectangular grids. The low FAR obtained indicates an improvement in the capability of the system to highlight the interpersonal variability.

## References


1. Impedovo, D., Pirlo, G.: Automatic Signature Verification: The State of the Art. IEEE Transactions on Systems, Man and Cybernetics-part C: Applications and Reviews. vol. 38., no. 5. 609–635 (2008)
2. Justino, E., El Yacoubi, A., Bortolozzi, F., Sabourin, R.: An Off-Line Signature Verification System Using HMM and Graphometric Features. In: $4_{th}$ IAPR International Workshop on Document Analysis Systems, Rio de Janeiro, Brazil, 211–222 (2000)
3. Justino, Edson J. R., Bortolozzi, F., Sabourin, R.: Off-line Signature Verification Using HMM for Random, Simple and Skilled Forgeries. In: Internat. Conf. on Document Analysis and Recognition. vol. 1. Seattle, USA, 105–110 (2001)
4. Vargas, F., Ferrer, M.A., Alonso, J.B., Travieso, C.M.: Offline geometric parameters for automatic signature verification using fixed-point arithmetic. IEEE Transactions on Pattern Analysis and Machine Intelligence. vol. 27. 993–997 (2005)
5. Justino, Edson J.R., Bortolozzi, F., Sabourin, R.: A Comparison of SVM and HMM Classifiers in the Offline Signature Verification. Pattern Recognition Letters 26, 1377–1385 (2005)
6. Özgündüz, E., Sentürk T., Karsligil, M. E.: Off-line Signature Verification and Recognition by Support Vector Machine. In: European Signal Processing Conference. Turkey (2005)
7. Oliveira L. S., Justino, E., Freitas, C., and Sabourin, R.: The Graphology Applied to Signature Verification. In: 12th Conference of the International Graphonomics Society, 286–290 (2005)
8. Santos, C., Justino, E., Bortolozzi, F., Sabourin, R.: An Off-Line Signature Verification Method based on the Questioned Document Experts Approach and a Neural Network Classifier. In: Proceedings of the 9th Intl Workshop on Frontiers in Handwriting Recognition (IWFHR-9) (2004)
9. Vapnik, V.: The Nature of Statistical Learning Theory. Springer-Verlag, NY (1995)
10. Vapnik, V.: Statistical Learning Theory. Wiley, NY (1998)
11. Vargas, F., Ferrer, M.A., Travieso, C.M., Alonso, J.B.: Off-line Handwritten Signature GPDS-960 Corpus. In: IAPR 9th International Conference on Document Analysis and Recognition, Curitiba, Brazil, 764–768 (2007)
12. Canu, S., Grandvalet, Y., Guigue, V., Rakotomamonjy, A.: SVM and Kernel Methods Matlab Toolbox. Perception Systmes et Information, INSA de Rouen, Rouen, France (2005)