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Simple modelling of time-temperature profiles in food during baking

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A R T I C L E I N F O

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A B S T R A C T

Time-temperature profiles (TTP) in food during baking and other heat treatments are essential to understand and characterise a series of changes related to product quality, and for design, control and optimisation of the process. Experimental determination of TTP is not always practical, so mathematical modelling has been applied to characterise and predict TTP. Although physics-based models may be available, their implementation can result quite complex at industrial level or for use in applications outside the field of process modelling and simulation, e.g., kinetic modelling of quality changes. Therefore, the objective of this work is to develop and test simple and effective equations to characterise TTP at core and surface of the product during baking, since these positions determine the most important quality changes. For the core position, a modified Gompertz equation type is proposed, while an adapted Page model is used for the surface position; both models have only two fitting parameters and are easily implementable tools. Models were tested with data generated by a baking numerical model and also experimental TTP. In addition, capability of simple equations was evaluated with frying and oven roasting data, since all three processes can be considered as moving boundary problems with a water vaporisation front. Overall, a good fitting performance was obtained: mean absolute percentage error is less than 5% in most of cases.

1. Introduction

Baking is an important process in food industry; it allows manufacturing staple foods like bread by application of a significant amount of thermal energy, so it can be classified as a heat transfer unit operation, even if mass (mainly water) transfer plays a major role in the process (Purlis, 2016). In this regard, time-temperature profiles (TTP) or thermal histories at different positions in the product are essential to understand and characterise a series of changes related to product quality. In particular, bread baking is a complex process where several physical and bio-chemical phenomena take place in food simultaneously and are indeed interrelated or coupled, e.g., multiphase heat and mass transport with phase change (vaporisation of water), volume expansion and formation/setting of a porous structure (solid foam), starch gelatinisation, crust development, and browning reactions (Mondal & Datta, 2008; Nicolas et al., 2014; Purlis & Salvadori, 2009a). For instance, the development of colour or browning at surface due to Maillard reaction and caramelisation of sugars depends on TTP (and water activity) at crust (Purlis & Salvadori, 2009c). On the other hand, TTP at the slowest heating point (core) determines the transformation of dough into crumb via starch gelatinisation kinetics (Zanoni et al., 1995a, 1995b). TTP have also been used to develop kinetic models to study and evaluate the thermal inactivation of enzymes (Zhang et al., 2017) and probiotics (Zhang et al., 2019), and thermal degradation of anthocyanins (Sui et al., 2015) in bread during baking. Similarly, the formation of acrylamide and 5-hydroxymethylfurfural (HMF) depends on thermal history, among other factors (Nguyen et al., 2016; Purlis, 2010; Schouten et al., 2022). All these changes (and thus TTP) are related to sensory and nutritional quality of products, so they are used for design, control and optimisation of the baking process (Purlis, 2012, 2014).

Experimental determination of TTP in the product is always desirable, but not possible or practical in all cases, besides it is time-consuming. Temperature measurement during baking presents some difficulties due to soft consistency at beginning and volume change of dough/bread during the process, hindering the precise positioning of sensors. Additionally, determination of surface temperature of food is often a difficult task. Moreover, industrial ovens are usually continuous or may have moving supporting devices, adding another complication for obtaining TTP in the product. Therefore, mathematical modelling has been applied to characterise and predict TTP, and thus for design, control and optimise processes, in order to find systematic solutions and avoid pure empirical or trial-and-error procedures. Different types of models have been proposed for baking, from phenomenological models to physics-based formulations including mechanical modelling of dough deformation (Purlis et al., 2021). The characteristic physical phenomenon of baking is the formation of a moving front inside the product, where vaporisation of water mainly occurs, dividing moist crumb from dry crust (Purlis & Salvadori, 2009a). This phenomenon makes this process especially complex to model since the solution of a multi-

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physics moving boundary problem is not straightforward, or the resulting model is not easy to simplify or reduce, in contrast with processes where only heat conduction or mass diffusion takes place and vaporisation occurs mainly at surface (Farid, 2002). That is, there is a lack of easy-to-use or simple models for baking. This aspect represents a significant bottleneck if we consider the implementation at industrial level (Djekic et al., 2019; Kansou et al., 2022), or applications outside the research field of process modelling and simulation, e.g., kinetic modelling of temperature-dependent chemical reactions and/or quality changes. In this sense, simple and effective, accurate and easy-implementable models can be very useful for product/process design, performance assessment, troubleshooting, evaluation of product quality and model-based control (Kondakci & Zhou, 2017; Putranto et al., 2011).

Some authors have proposed simplified models and/or solutions for certain aspects of moving boundary problems like baking, where phase change occurs, but not completely throughout the product like in freezing, thawing, or freeze-drying. For instance, Farid (2001) reported an approximate analytical solution to predict the evolution of crust thickness and weight loss, and also Farid (2002) developed a simple model for a moving boundary problem, but it requires a numerical solution based on finite difference or finite element methods, which may be not straightforward or require using advanced commercial software. On the other hand, Therdthai et al. (2002) utilised empirical linear regression models to correlate specific (discrete) values (e.g., black-box model) and optimise the bread baking process; this may be a practical solution but does not provide thermal histories. Putranto et al. (2011) applied the lumped reaction engineering approach (L-REA) to model the baking of thin layer of cake (ca. 3 mm); although this is a simple approach and presented good results, the system is different from an actual product regarding the size and thus involved physics. More recently, we have proposed a series of simple methods to predict the minimum baking time (Purlis, 2019) and the evolution of water loss of bread during the process (Purlis, 2020), based on understanding of the underlying physics of baking. Although these are simple methods, they are not focused on describing thermal histories. In summary, there is a lack of simple models or equations to describe TTP in food during baking and other similar processes involving a moving boundary problem with a water vaporisation front.

Therefore, the objective of this work is to develop and test simple equations to characterise time-temperature profiles in the product during baking, avoiding complex modelling or numerical solutions. That is, to provide simple models for heating kinetics during baking. We mainly aim at providing easily implementable or user-friendly tools, based on fundamental knowledge of the process, to help building systematic solutions at industrial scale and improving kinetic modelling of quality changes for product/process design.

2. Methodology

Briefly, the proposed methodology consists in first obtaining and/or collecting TTP data and then testing two simple equations, one for core position and one for surface position. The output of this study will be easy-to-use tools to characterise TTP or heating kinetics for further applications, like kinetic modelling of quality changes or product design and process optimisation, not a ready-to-use prediction tool.

2.1. Data from baking model

A first data set of TTP was obtained from simulation of a previously developed and validated heat and mass transfer model of bread baking, e.g., Purlis and Salvadori (2009a, 2009b). Main assumptions and (common practice) operating conditions of the simulated process are the following (Purlis, 2019, 2020):

- Conventional baking of French bread (*baguette* type) at constant oven temperature; an apparent heat transfer coefficient is used to lump all contributions (e.g., convection and radiation from oven ambient to food).
- Bread is considered as an infinite cylinder of constant radius equal to 3 cm, thus the domain is reduced to a one-dimension geometry via the axisymmetric assumption. The model does not consider volume variation; this simplification will be discussed later.
- Initial uniform temperature (25 °C) and water content (0.65 kg/kg, dry basis; 39.4%, wet basis).
- Oven temperature (T_m): 180, 200, 220, 240 °C.
- Apparent heat transfer coefficient (h): 20, 30, 40 W/(m^2 K).
- Relative humidity in oven ambient is assumed to be negligible (no steam injection and good air circulation within oven ambient).

For a detailed description of the model, including governing equations, boundary conditions, properties and experimental validation, the reader is referred to Purlis and Salvadori (2009a, 2009b). Time-temperature profiles at core and surface were obtained for a 30 min baking process. Simulations were performed by using the finite element method in COMSOL Multiphysics 3.4 (COMSOL AB, Burlington, MA, USA) coupled with MATLAB R2007b (The MathWorks Inc., USA). The 1D mesh consisted of 368 elements, where the maximum element size at the open boundary (surface) was set to 1×10^{-4} m (default values were used for the rest of parameters). Time step was set to 0.01 min. These conditions ensured convergence and quality of results.

2.2. Data from literature

A second data set of TTP was obtained from literature and contains published experimental data of bread baking (Table 1). References were selected to ensure a wide range of data regarding heating temperature, heat transfer coefficient, geometry of samples, processes and sources (i.e., different research groups), in order to perform a robust and objective analysis of the proposed simple equations. For details about each experimental set-up, the reader is referred to corresponding reference. It is worth noting that all experiments are quite standard for the food science and engineering audience.

An additional data set of potato frying and meat oven cooking/roasting was also collected from literature (Table 1). Although these are quite different processes from baking, all of them are characterised by the development of an evaporation moving front. So, using these data will help us to understand if a general underlying behaviour regarding TTP or heating kinetics can be captured by the proposed simple equations, without involving complex modelling.

2.3. Simple models

The development of simple models was based on a preliminary analysis of shapes of typical TTP in food during baking and similar processes like frying and oven roasting, e.g., moving boundary problems with heat and mass transfer and water vaporisation front forming a limited crust zone (phase change is not extended throughout the material). The main premise was to find simple equations with a low number of fitting parameters and, if possible, well-known formulations in the food engineering field in order to elaborate useful analogies and facilitate their implementation.

For the case of core position (slowest heating point), typical TTP during baking resembles a sigmoid growth curve or S-curve, with the maximum asymptotic value corresponding to phase change temperature (water vaporisation front), e.g., ca. 100 °C at normal pressure. That is, inner zone (crumb) of bread does not exceed 100 °C during baking (Purlis & Salvadori, 2009a). There are several sigmoid functions in the literature; in this work, we test a modified Gompertz equation, which is commonly used to model the bacterial growth curve (Zwietering et al., 1990):

$$y(t) = A \exp\left\{-\exp\left[\frac{\mu_m e}{A}(\lambda - t) + 1\right]\right\} \tag{1}$$

Table 1
Basic information about experimental data set obtained from literature; specific details can be found in references.

Process	*Food*	*Heating temperature (°C)*	*Position*	*Reference*
Baking	Bread	230	Core, surface	Nicolas et al. (2014)
Baking	Bread	175, 205, 235	Core, surface	Zhang et al. (2018)
Baking	Bread	160, 190, 220	Core	Bredariol et al. (2019)
Baking	Bread	142, 185, 210	Core, surface	Ureta et al. (2019)
Baking	Bread	175, 205, 235	Core	Zhang et al. (2019)
Baking	Bread	180, 200, 220	Core, surface	Silva et al. (2022)
Frying	Potato	160, 180	Core, surface	Farkas et al. (1996)
Frying	Potato	160	Core, surface	van Koerten et al. (2017)
Oven roasting	Meat	175	Core, surface	Feyissa et al. (2013)

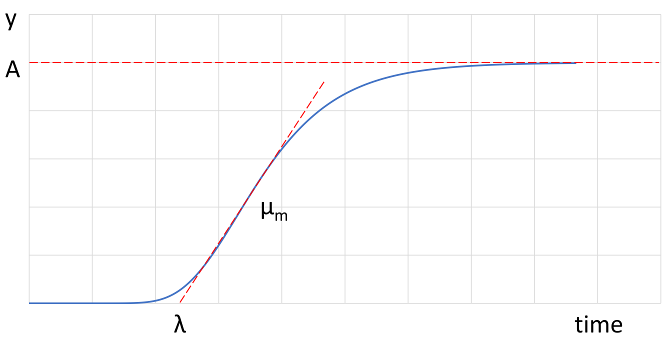

Fig. 1. Representative curve (in blue) of modified Gompertz equation given by Eq. (1).

Fig. 1 illustrates a representative curve given by Eq. (1): y is the dependent variable (e.g., logarithm of the relative population size), t is time, A is the asymptote or maximum value that can be reached, μ_m is the maximum specific growth rate (or tangent in the inflection point), λ is the lag time (or x-axis intercept of this tangent), and $e = \exp(1)$ or Euler's number.

To describe the increase of temperature at the core of food during baking, we propose the following adapted version of Eq. (1):

$$T_c^*(t) = \frac{T_c(t) - T_0}{T_f - T_0} = \exp\{-\exp[\mu_m^*(\lambda - t) + 1]\} \tag{2}$$

Where T_c^* is the dimensionless temperature at core, T_c is the core temperature (°C), T_0 is the initial temperature (°C), T_f is the temperature at vaporisation front (100 °C, for simplicity), and μ_m^* is the maximum dimensionless heating rate (1/time unit). Since the dependent variable T_c^* can take values between 0 and 1, the parameter A of Eq. (1) is equal to 1, and it does not appear in Eq. (2). Thus, a two-parameter (μ_m^*, λ) equation is finally obtained to describe TTP or heating kinetics at core.

It is worth noting that temperature of water vaporisation front in the product may be slightly different than 100 °C (at normal pressure), but this value is taken here for sake of simplicity. In any case, the user can easily correct or modify the model with the actual experimental value of T_f to improve accuracy.

By working with Eq. (2), we can establish a relationship between the parameter μ_m^* and the maximum heating rate (MHR, e.g., in °C/min), which is a useful value to characterise the heating curve:

$$\text{MHR} = \left(\frac{dT_c}{dt}\right)_{\max} = \left(\frac{T_f - T_0}{e}\right)\mu_m^* \tag{3}$$

For the case of surface position, typical TTP shows a concave down, increasing shape, tending to heating medium temperature at long times (Purlis & Salvadori, 2009a). After performing the preliminary analysis, we chose to test the Page model, which is an empirical modification of the so-called Lewis or exponential model, a semi-theoretical thin-layer drying model (Onwude et al., 2016):

$$y(t) = \exp(-kt^n) \tag{4}$$

Where y is usually the moisture ratio (dimensionless), k is the drying rate constant (1/time unit^n) and n is a dimensionless (empirical) parameter or model constant. The Page model is widely used to describe and characterise drying kinetics of various fruits and vegetables. If $n = 1$, Eq. (4) turns into the Lewis model, which is based on a lumped system approach. In our case, this assumption would not be valid for the entire system (although may be for a thin layer of crust), but Eq. (4) is proposed for data fitting and simplicity reasons.

Another result of the preliminary analysis was that a better fitting performance can be obtained if the TTP at surface is divided into two periods, e.g., before and after reaching the phase change temperature. Therefore, for the first period, we propose the following model, based on the Page equation:

$$T_{s,1}^*(t) = \frac{T_s(t) - T_m}{T_0 - T_m} = \exp\left(-k_1 t^{n_1}\right), T_s < 100\ ^\circ\text{C} \tag{5}$$

Where $T_{s,1}^*$ is the dimensionless surface temperature for the first period, T_s is the temperature at surface of food (°C), T_0 is the initial temperature of food (°C), T_m is the temperature of heating medium (°C), k_1 and n_1 are the heating rate constant and model constant, respectively, for the first period. For the second period, some parameters are redefined to use the same equation form:

$$T_{s,2}^*(t) = \frac{T_s(t) - T_m}{T_{0,2} - T_m} = \exp\left(-k_2 {t_2}^{n_2}\right), T_s \geq 100\ ^\circ\text{C} \tag{6}$$

Where $T_{s,2}^*$ is the dimensionless surface temperature, $T_{0,2}$ is the initial temperature of food (°C), k_2 and n_2 are the heating rate constant and model constant, respectively; all values for the second period. Note that time is redefined (t_2) so zero time for the second period corresponds to first value of surface temperature equal or above 100 °C (i.e., $T_{0,2}$). That is, both Eqs. (5) and (6) start with value 1 at initial zero time, e.g., $y(0) = 1$. Afterwards, the complete TTP at surface can be built upon both equations by using four parameters (e.g., two for each period).

2.4. Performance of simple models

The capability of the proposed equations to characterise TTP was assessed by means of a curve fitting procedure, via the method of least squares implemented in a MS Excel spreadsheet using the Solver tool (e.g., minimisation of the sum of the squares of the residuals). In all cases, different initial search values for parameters were tested to ensure convergence and independence of results. Fitting performance of the models was assessed through the mean absolute percentage error (MAPE, %):

$$\text{MAPE} = \frac{100}{N}\sum_{i=1}^{N}\left(\frac{|T_{\text{data}} - T_{\text{model}}|}{T_{\text{data}}}\right)_i \tag{7}$$

Where N is the number of i points or temperature values of a TTP, T_{data} is the temperature value from data set and T_{model} is the corresponding value obtained from simple model, at a given time.

It is worth recalling that the objective of the work is to propose and test simple mathematical tools for characterising TTP and not to report

Table 2
Estimated parameters and fitting performance of Eq. (2) for TTP at core position from baking model data set. T_m refers to heating temperature (°C) and h to apparent heat transfer coefficient (W/(m^2 K)). MAPE is defined in Eq. (7).

	μ_m^* (1/min)			λ (min)			MAPE (%)		
T_m / h	20	30	40	20	30	40	20	30	40
180	0.5813	0.6565	0.6834	4.8871	4.1083	3.7334	1.6763	1.1933	1.0208
200	0.6277	0.6773	0.7010	4.5151	3.8496	3.5631	1.4230	1.0586	0.9727
220	0.6536	0.6911	0.7160	4.2273	3.6824	3.4519	1.2333	1.0022	0.9603
240	0.6700	0.7061	0.7312	4.0119	3.5634	3.3815	1.1151	0.9676	0.9590

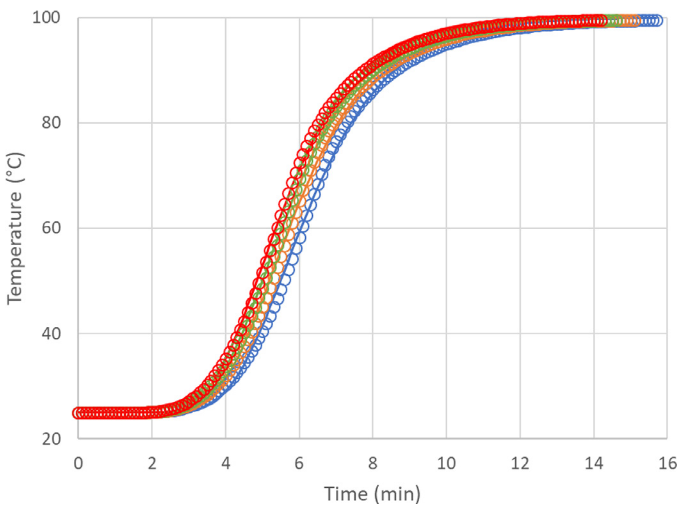

Fig. 2. Fitting results for TTP at core from baking model data set. Symbols (circles) correspond to data and lines to Eq. (2). Colours refer to heating (oven) temperature (°C): 180, blue; 200, orange; 220, green; 240, red. Results for $h = 30$ W/(m^2 K).

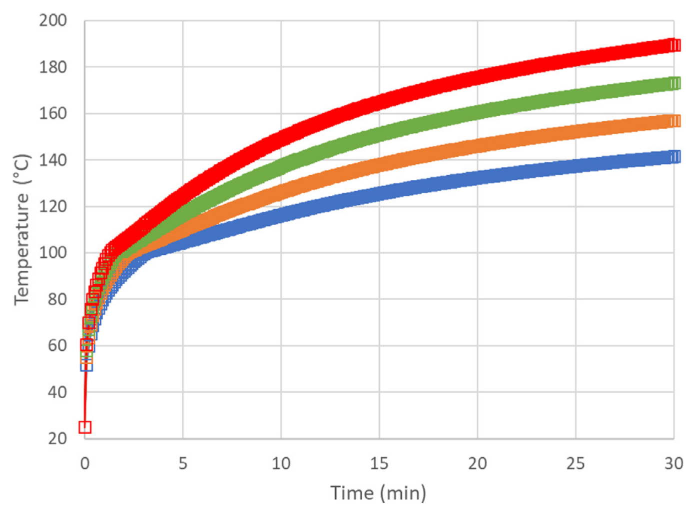

Fig. 3. Fitting results for TTP at surface from baking model data set. Symbols (squares) correspond to data and lines to Eqs. (5) and (6). Colours refer to heating (oven) temperature (°C): 180, blue; 200, orange; 220, green; 240, red. Results for $h = 30$ W/(m^2 K).

ready-to-use equations or parameter values for prediction (i.e., curve fitting is used for performance evaluation). That is, it is expected to provide simple models for heating kinetics during baking, in the same way that simple equations have been proposed and applied to describe other food processes, such as drying kinetics (e.g. Lewis model, Page model, etc.).

3. Results and discussion

3.1. Baking

Firstly, we consider the results obtained from the data set generated by simulation of the baking model. For the case of core position, Table 2 contains the values of estimated parameters of Eq. (2) for all conditions of this first data set, and Fig. 2 shows the results for all heating temperatures and only one value of apparent heat transfer coefficient, for simplicity (e.g., similar trends were found in all cases). Overall performance of data fitting is good, based on the low values of MAPE (< 1.7%) and also visual inspection of curves (indeed, it is hard to differentiate between data points and model outputs). That is, the proposed two-parameter model based on a modified Gompertz equation well describes the TTP at core position of bread during baking. Values of parameters can be interpreted in the following way: higher heating temperature and heat transfer coefficient, i.e., driving force for heat transfer, produce higher heating rates and maximum values are achieved earlier. That is, it is expected and physically consistent that μ_m^* increases and λ decreases with increasing heating temperature and heat transferred from oven ambient.

For the case of surface position, also for the first data set, Tables 3 and 4 present the estimated parameters of Eqs. (5) and (6), corresponding to the so-called first and second periods (e.g., below and above the temperature of water vaporisation), respectively, for all simulated conditions. Fig. 3 shows the results of data fitting for all heating temperatures and only one value of apparent heat transfer coefficient (for simplicity). Overall, fitting performance is also good, and even better than in the case of core position, considering the very low values of MAPE (< 1.1%, all cases), especially for the so-called first period of baking (< 0.5%). Thus, the proposed simple model based on the Page equation is able to characterise the TTP at surface position with good performance. Regarding interpretation of found values of parameters, let us consider both periods separately. For the first period, Table 3 shows that heating rate constant (k_1) increases with value of h, for a fixed heating temperature, which is expected. Conversely, for a fixed value of h, the heating rate constant decreases with heating temperature. This seems to be physically inconsistent, but actually the model constant (n_1) regulates or corrects the curve, e.g., for lower values of n, a steepest change is achieved at the beginning of the curve, so higher heating temperature produces more rapid and pronounced changes in TTP at surface, as expected. For the second period (Table 4), it can be seen that heating rate constant shows the expected trend: higher values of heating temperature and/or heat transfer coefficient, generate higher values of heating rate constant. The opposite trend is observed for the model constant (n_2), since higher driving force produces more rapid changes of temperature (lower values of n). The difference in these trends between the so-called first and second periods may be the presence/absence of latent heat. That is, heating periods are divided by water vaporisation temperature. In any case, it is worth noting that only one parameter does not explain the behaviour of TTP (at core or surface), but it is necessary to perform a comprehensive analysis based on physical concepts for a correct interpretation of parameters and understanding of the process.

Table 3
Estimated parameters and fitting performance of Eq. (5) for TTP at surface position (first period) from baking model data set. T_m refers to heating temperature (°C) and h to apparent heat transfer coefficient (W/(m^2 K)). MAPE is defined in Eq. (7).

	k_1 (1/min^n)			n_1 (-)			MAPE (%)		
T_m / h	20	30	40	20	30	40	20	30	40
180	0.3455	0.4498	0.5355	0.3695	0.3461	0.3312	0.4709	0.3682	0.2740
200	0.3330	0.4309	0.5118	0.3605	0.3394	0.3282	0.4555	0.3311	0.2251
220	0.3214	0.4137	0.4903	0.3539	0.3345	0.3235	0.4401	0.3036	0.1774
240	0.3107	0.3984	0.4716	0.3490	0.3319	0.3220	0.4471	0.2899	0.1311

Table 4
Estimated parameters of Eq. (6) for TTP at surface position (second period) from baking model data set. T_m refers to heating temperature (°C) and h to apparent heat transfer coefficient (W/(m^2 K)). MAPE is defined in Eq. (7).

	k_2 (1/min^n)			n_2 (-)			MAPE (%)		
T_m / h	20	30	40	20	30	40	20	30	40
180	0.0190	0.0420	0.0754	0.9678	0.8803	0.7924	0.1677	0.4691	0.6626
200	0.0225	0.0529	0.0932	0.9614	0.8474	0.7597	0.3147	0.6601	0.8549
220	0.0267	0.0660	0.1142	0.9442	0.8087	0.7232	0.4719	0.7807	0.9903
240	0.0315	0.0751	0.1312	0.9226	0.7921	0.7004	0.6180	0.9760	1.1166

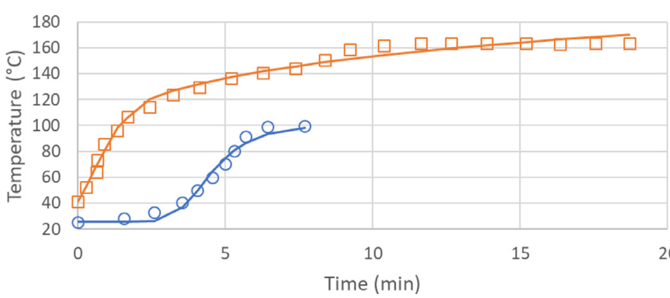

Fig. 4. Fitting results for TTP data (baking) from Nicolas et al. (2014). Symbols correspond to data and lines to simple models. References: circles/blue, core; squares/orange, surface.

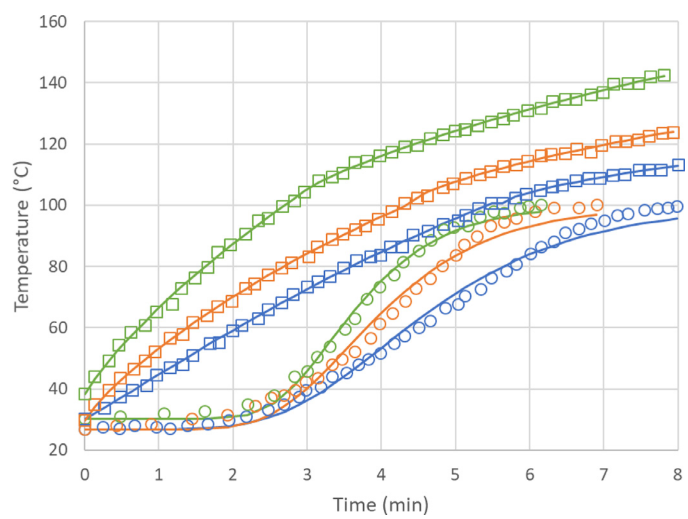

Fig. 5. Fitting results for TTP data (baking) from Zhang et al. (2018). Symbols correspond to data and lines to simple models. Circles refer to core and squares to surface position. Colours refer to heating temperature (°C): 175, blue; 205, orange; 235, green.

Secondly, Figs. 4–9 and Tables A.1–A.6 (see Appendix) present the results of data fitting considering the data set obtained from literature regarding baking (Table 1). In general terms, fitting performance is acceptable: only two conditions provided MAPE values between 5 and 6% (Tables A.1 and A.2). This confirms the capability of the proposed simple equations to describe and characterise TTP at core and surface during baking. Without discussing details about each data sub-set, it is worth noting that different conditions were used to obtain experimental thermal histories, e.g., product formulation, geometry and size/mass of sample, oven characteristics, heating temperature, etc. Moreover, data was obtained from different sources, providing a more robust and objective assessment of models. Interpretation of values of parameters can be done in a similar way as before, considering the trends of both parameters, for each simple model.

So far, we have demonstrated that the proposed models are able to describe and characterise the time-temperature profiles or heating kinetics at core and surface of bread during baking, in a simple and efficient way. Furthermore, these models can be used for diverse applications in product/process design, control and optimisation. For example, from results of core position of the first data set (Table 2), we calculated the maximum heating rate (MHR) given by Eq. (3), which is a valuable parameter to characterise a heating curve. Fig. 10 shows the MHR as a function of heating temperature, for different values of apparent heat transfer coefficient (e.g., all simulated conditions). In addition, we performed a linear regression to establish a simple quantitative relationship between MHR and heating temperature, for each value of h, which represents the oven characteristics in terms of heat and mass transfer. This is of particular interest for product/process design, since the heating rate is associated with starch gelatinisation kinetics and texture development of bread (Patel et al., 2005). For instance, it has been shown that control of baking time and baking kinetics is important to controlling staling kinetics, e.g., staling rate is faster when faster baking kinetics is established (Besbes et al., 2014; Bou Orm et al., 2021; Le Bail et al., 2009). In this sense, simple models of TTP can be used to reproduce experimental baking conditions in controlled tests to better understand different aspects of baking. For example, DSC (differential scanning calorimetry) simulation of baking allows studying the influence of heating rates on starch gelatinisation kinetics (Bredariol et al., 2019), and how the use of different flours in dough formulation affects heating rate and related changes (Grassi de Alcântara et al., 2020). Similarly, the formation of bread crust can be simulated in a DMTA (dynamic mechanical thermal analysis) rheometer by reproducing TTP at surface (Vanin et al., 2010). Finally, simple models of TTP can be used for design and optimisation of the baking process, together with kinetic models of quality changes (Purlis, 2012, 2014).

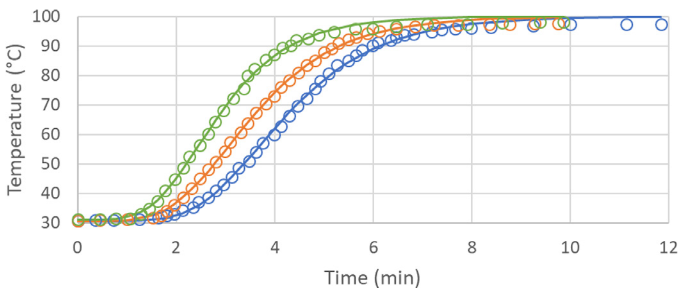

Fig. 6. Fitting results for TTP data (baking) from Bredariol et al. (2019). Symbols correspond to data (core) and lines to simple models. Colours refer to heating temperature (°C): 160, blue; 190, orange; 220, green.

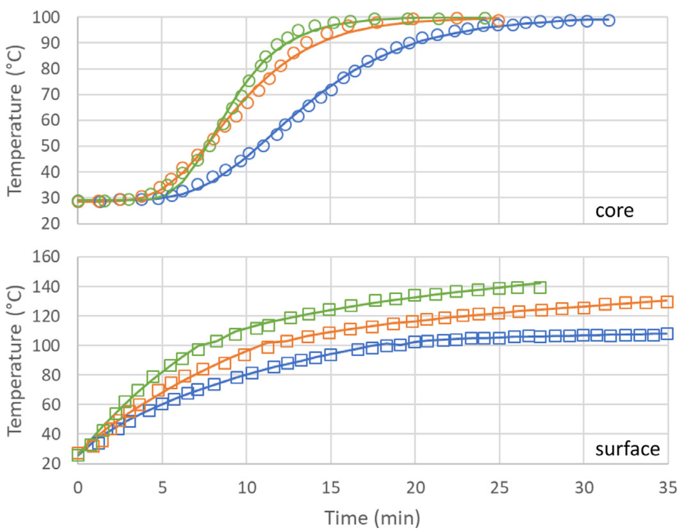

Fig. 7. Fitting results for TTP data (baking) from Ureta et al. (2019). Symbols correspond to data and lines to simple models. Circles refer to core (top) and squares to surface position (bottom). Colours refer to heating temperature (°C): 142, blue; 185, orange; 210, green.

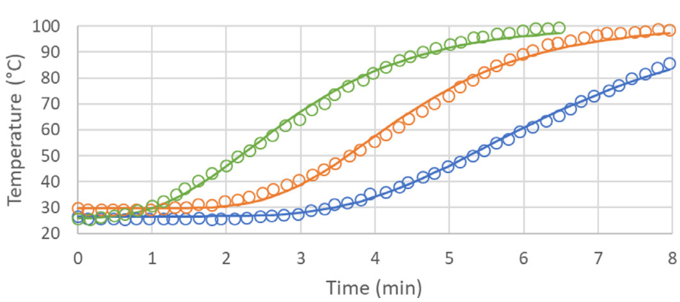

Fig. 8. Fitting results for TTP data (baking) from Zhang et al. (2019). Symbols correspond to data (core) and lines to simple models. Colours refer to heating temperature (°C): 175, blue; 205, orange; 235, green.

3.2. Frying and oven roasting

It was discussed in the Introduction that baking is considered a moving boundary problem (MBP), where basically a receding water vaporisation front divides the product into two zones, e.g., the outer dry crust and the inner moist crumb. There are other food processes that can be categorised as MBP (Farid, 2002). Given the characteristics of baking, and with the aim of further testing the proposed simple models, we analysed experimental data from two similar food processes, in the discussed terms: (potato) frying and (meat) oven roasting (Table 1). There are several (obvious) differences between baking, frying and oven roasting, e.g., typical processed food, transport mechanisms inside product, involved phases, and heating medium, but all three processes could be thought as MBP with a water vaporisation front. Similar to baking, different frying and roasting/cooking models can be found in the literature (e.g., Farkas et al.,1996; Feyissa et al., 2013; van Koerten et al., 2017), but there is a lack of simple equations to characterise time-temperature profiles for further applications.

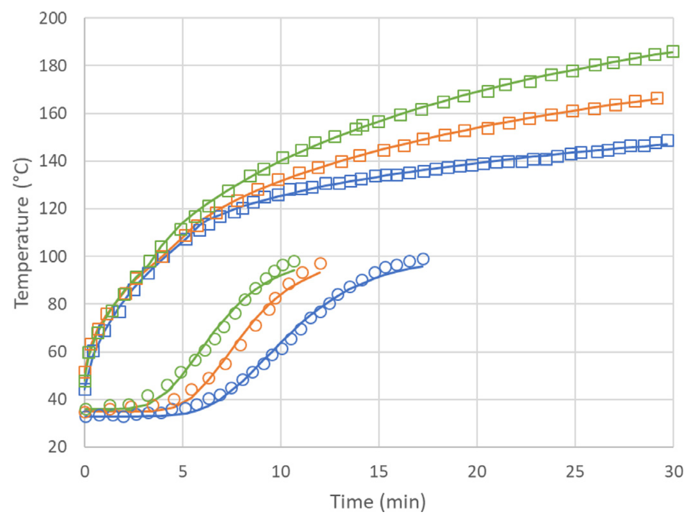

Fig. 9. Fitting results for TTP data (baking) from Silva et al. (2022). Symbols correspond to data and lines to simple models. Circles refer to core and squares to surface position. Colours refer to heating temperature (°C): 180, blue; 200, orange; 220, green.

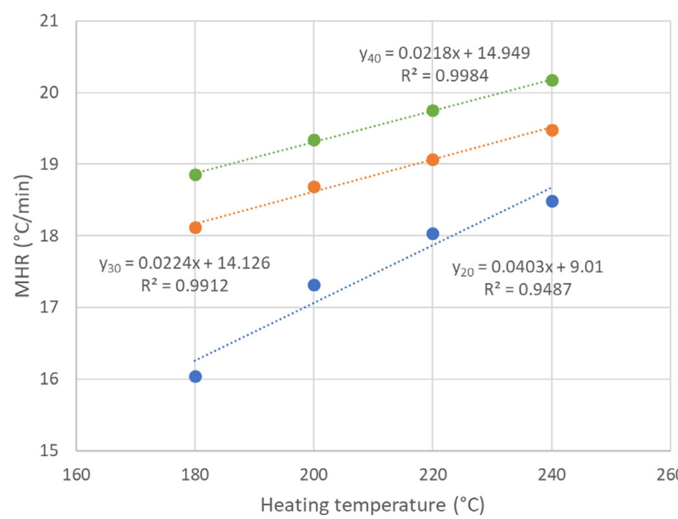

Fig. 10. Maximum heating rate (MHR) calculated from estimated parameters for TTP at core (Table 2) and Eq. (3), from baking model data set. Colours refer to different values of apparent heat transfer coefficient h (W/(m^2 K)): 20, blue; 30, orange; 40, green. Regression lines are included with corresponding equations and R^2 values (obtained by Excel application).

Figs. 11a and 11b, and Tables A.7 and A.8 present the results of data fitting for frying, and Fig. 11c and Table A.9 for oven roasting. Overall, fitting performance is again acceptable: only one condition presented MAPE > 5% (7.3%, Table A.7), showing that the proposed simple models would also be able to describe and characterise TTP in food during frying and oven roasting. These models could further be used to study temperature-dependent quality aspects of products. For instance, acrylamide formation is temperature-dependent and of major importance in frying processes (and baking also), and its kinetic modelling requires of accurate TTP (Carrieri et al., 2009). In the case of oven roasting,

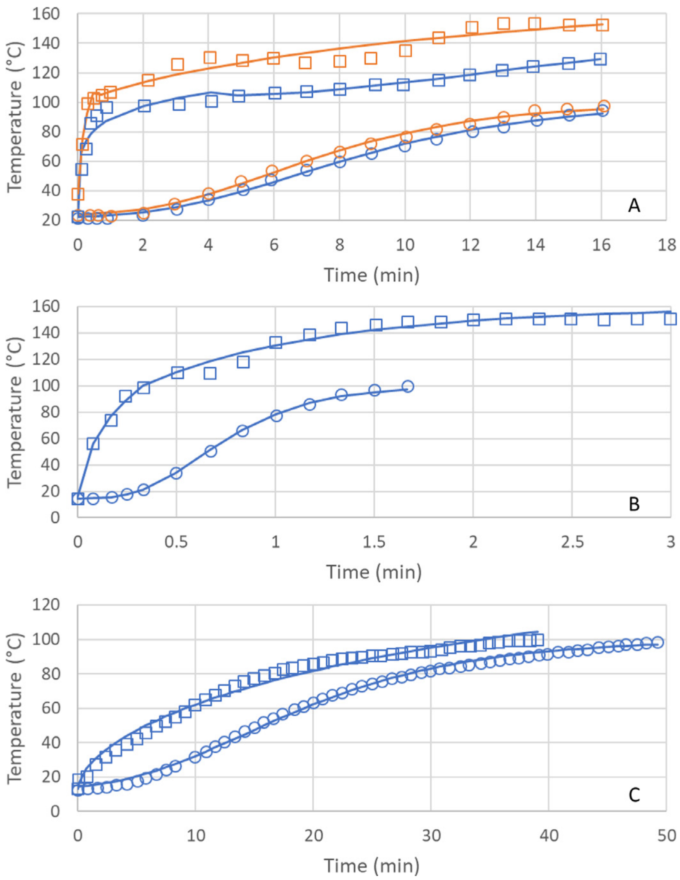

Fig. 11. Fitting results for TTP data from: (A, frying) Farkas et al. (1996), (B, frying) van Koerten et al. (2017), (C, oven roasting) Feyissa et al. (2013). Symbols correspond to data and lines to simple models. Circles refer to core and squares to surface position. In (A), colours refer to heating temperature (°C): 160, blue; 180, orange.

in particular of meat, TTP at core are essential to determine cooking time, but also changes of texture (Rabeler & Feyissa, 2018) and colour (Rabeler et al., 2019), which depend on time-temperature profiles in the product.

3.3. Final remarks

It has been shown that baking, frying and oven roasting/cooking certainly present a common behaviour regarding thermal histories or heating kinetics, driven by the moving water vaporisation front, which is effectively captured or represented by the proposed simple models. It is worth noting that all three processes are quite different in terms of heating rate, being frying the most rapid or intensive process, given the high value of heat transfer coefficient of hot/boiling oil. Another important aspect of these processes is deformation and volume change of products, especially for baking (expansion) and meat cooking (shrinkage), which can modify thermal gradients. These features should be considered when physics-based or mechanistic models are developed, besides specific transport mechanisms. However, the simple models developed in this work are focused on characterisation of heating kinetics, in a similar way that Lewis model, Page model and other semi-theoretical and empirical models are used to describe drying kinetics of food materials (Onwude et al., 2016).

4. Conclusions

The proposed models represent a simple and effective tool for parameterisation of time-temperature profiles or heating kinetics at core and surface of food during baking and similar processes like frying and oven roasting. The proposed two-parameter equations are continuous differentiable mathematical functions that can be integrated into other kinetic models to describe temperature-dependent changes, e.g., formation of chemical compounds, colour and texture development, phase transition kinetics. In general terms, modelling or parameterisation offers the possibility of reducing experimentation to characterise quantitatively a certain product/process. Furthermore, parameterisation of TTP can help in developing machine learning-based models. For example, parameters of TTP models can be used as inputs, together with formulation/product and process characteristics, for optimisation of certain outputs, e.g., achieving target values of colour and texture. In this sense, it would be possible to perform an optimisation procedure without using transport and kinetic models, and related optimisation routines.

Finally, it is expected that the proposed simple models can be used in industrial applications, especially when implementation of so-called complex models is not possible or may represent a barrier towards finding systematic solutions. Besides, they can be utilised in related studies of baking, frying, oven roasting, and other similar processes in the discussed terms, outside the specific domain of process modelling and simulation, e.g. for studying temperature-dependent kinetics of quality changes and relevant reactions in food under non-isothermal and actual operating conditions.

Ethical statement

This manuscript did not involve any studies in humans and animals

Declaration of Competing Interest

The authors declare that they have no known competing financial interests or personal relationships that could have appeared to influence the work reported in this paper.

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Appendix

Results of data fitting for experimental data set obtained from literature (see Table 1). In all cases, T_m refers to heating temperature (°C) and MAPE to mean absolute percentage error (Eq. (7)). Details about models and parameters can be found in Section 2.3.

Table A.1
Results for Nicolas et al. (2014).

T_m (°C)	μ_m^* (1/min)	λ (min)	MAPE (%)	k_1 (1/min^n)	n_1 (-)	MAPE (%)	k_2 (1/min^n)	n_2 (-)	MAPE (%)
230	1.0395	3.1942	6.0252	0.2617	1.0842	3.5889	0.1391	0.5836	2.3161

Table A.2
Results for Zhang et al. (2018).

T_m (°C)	μ_m^* (1/min)	λ (min)	MAPE (%)	k_1 (1/min^n)	n_1 (-)	MAPE (%)	k_2 (1/min^n)	n_2 (-)	MAPE (%)
175	0.7034	2.5959	3.8070	0.1052	1.0839	0.8611	0.0989	0.6944	0.2281
205	0.9464	2.5005	5.1576	0.1434	0.8670	0.7890	0.0898	0.7848	0.3113
235	1.2323	2.5154	2.7431	0.1537	0.8999	1.1859	0.0895	0.8480	0.3522

Table A.3
Results for Bredariol et al. (2019).

T_m (°C)	μ_m^* (1/min)	λ (min)	MAPE (%)
160	0.7620	2.3800	1.3764
190	0.8262	1.8611	0.9908
220	1.0033	1.4681	1.2350

Table A.4
Results for Ureta et al. (2019).

T_m (°C)	μ_m^* (1/min)	λ (min)	MAPE (%)	k_1 (1/min^n)	n_1 (-)	MAPE (%)	k_2 (1/min^n)	n_2 (-)	MAPE (%)
142	0.2237	7.1647	1.4365	0.0901	0.8474	1.2730	0.0584	0.4466	0.2699
185	0.3100	4.9814	2.2518	0.0671	0.9342	4.1531	0.0398	0.7476	0.3115
210	0.4388	5.8389	2.5045	0.0764	0.9702	3.0252	0.0575	0.7023	0.8253

Table A.5
Results for Zhang et al. (2019).

T_m (°C)	μ_m^* (1/min)	λ (min)	MAPE (%)
175	0.5466	3.6746	2.1030
205	0.7918	2.6410	2.7425
235	0.7941	1.0663	2.0081

Table A.6
Results for Silva et al. (2022).

T_m (°C)	μ_m^* (1/min)	λ (min)	MAPE (%)	k_1 (1/min^n)	n_1 (-)	MAPE (%)	k_2 (1/min^n)	n_2 (-)	MAPE (%)
180	0.3454	6.3447	3.1131	0.1994	0.6839	1.0598	0.1065	0.6262	0.5568
200	0.4724	5.1932	3.9127	0.1651	0.6340	0.4639	0.0835	0.7778	0.2885
220	0.4878	3.7671	3.5345	0.1610	0.6110	1.1552	0.0829	0.8245	0.3990

Table A.7
Results for Farkas et al. (1996).

T_m (°C)	μ_m^* (1/min)	λ (min)	MAPE (%)	k_1 (1/min^n)	n_1 (-)	MAPE (%)	k_2 (1/min^n)	n_2 (-)	MAPE (%)
160	0.2406	2.4856	2.8769	0.6622	0.2523	7.2852	0.0132	1.5838	0.4742
180	0.2775	2.2936	2.7941	1.0651	0.6446	3.9914	0.0984	0.8546	2.8423

Table A.8
Results for van Koerten et al. (2017).

T_m (°C)	μ_m^* (1/min)	λ (min)	MAPE (%)	k_1 (1/min^n)	n_1 (-)	MAPE (%)	k_2 (1/min^n)	n_2 (-)	MAPE (%)
160	3.2451	0.3100	1.3848	1.8703	0.6772	1.9821	1.0366	0.9810	2.5180

Table A.9
Results for Feyissa et al. (2013).

T_m (°C)	μ_m^* (1/min)	λ (min)	MAPE (%)	k_1 (1/min^n)	n_1 (-)	MAPE (%)
175	0.0969	3.7946	3.5892	0.0875	0.6138	4.7266

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