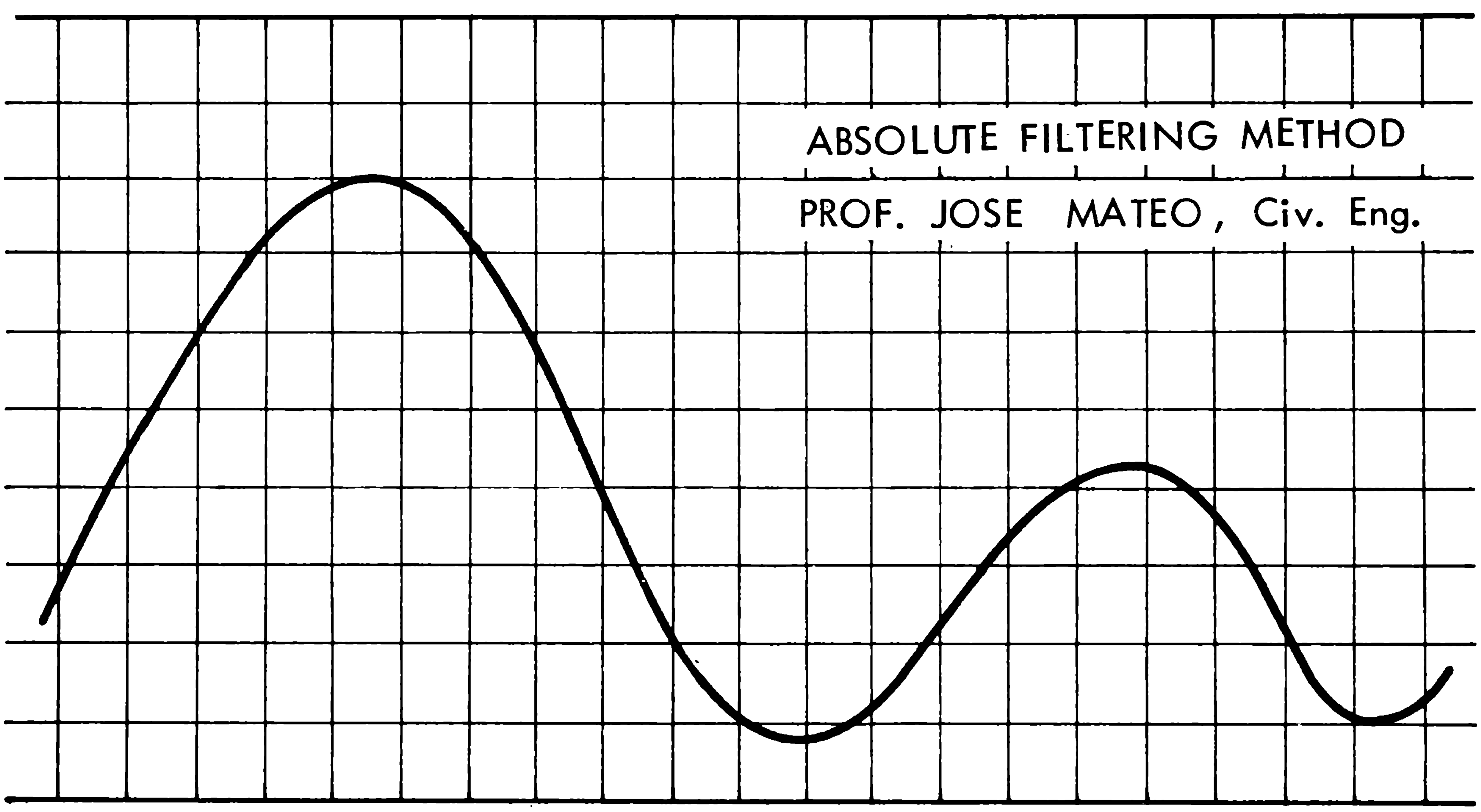
OBSERVATORIO ASTRONOMICO
UNIVERSIDAD NACIONAL DE LA PLATA

SERIE GEOFISICA - VOL. X

ANALYSIS OF TIDAL RECORDS



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CONTENT

FOREWORD

In the present work and starting from numerical or graphical tidal records, a new method for the analysis of tidal waves is developed.

Owing to the modern technology facilities, continuous and authomatic tidal records are available all the time.

The filtering method proposed permits eliminate tidal components one by one, in an absolute form; thereof the name of "absolute filtering".

From the complete theory developed and the numerical example of its application given in this work, three important facts can be drawn out:

a) Short-period tidal records can be used; b) the tidal record can have discontinuities, and c) a linear drift is authomatically eliminated throughout computations.

To apply the method we have worked on a theoretical tidal wave specially computed for La Plata latitude, at the place where our Astronomical Observatory is located.

It might be argued that the numerical computation has been made on a noiseless tidal wave; but this is justly necessary if we want to prove the method and bring out its possibilities.

The number of tidal components which is possible to separate from the tidal record, can be as many as the analyst wants to. The limit is given by the lenght of the tidal record; while the number (n) of tidal components to separate increases arithmetically, the number of the positions to read on the record increases geometrically as 2^n.

In the BLOCK DIAGRAM we assume to have four tidal constituents, p_1, p_2, p_3 and p_4, whose periods T_1, T_2, T_3 and T_4 are known in advance.

For the sake of our explanation, we have represented every wave by means of an ellipse. The size of the ellipse has nothing to do neither with the semi-amplitude nor with the period of the wave. It is just a sketch to show a process. Every pair of like ellipses is separated by a full period T. The written positions from [1] to [8] are those where the reading of ordinates have been performed on the tidal record.

For the wave p_1 there are eight ellipses; for the wave p_2 there are four ellipses, every one involving two positions of p_1. For the wave p_3 there are two ellipses, involving each one two ellipses of p_2 and in turn four positions of p_1. For the wave p_4 we have only one ellipse and its filtering is not possible, so that p_4 is an independent tidal component which can be solved.

Observe the BLOCK DIAGRAM and pay attention to the right side of it. If we subtract the positions ([2] - [1]), ([4] - [3]), ([6] - [5]), and ([8] - [7]), the wave p_1 is filtered because every pair of the positions above are separated by a full period T_1. If we subtracted either

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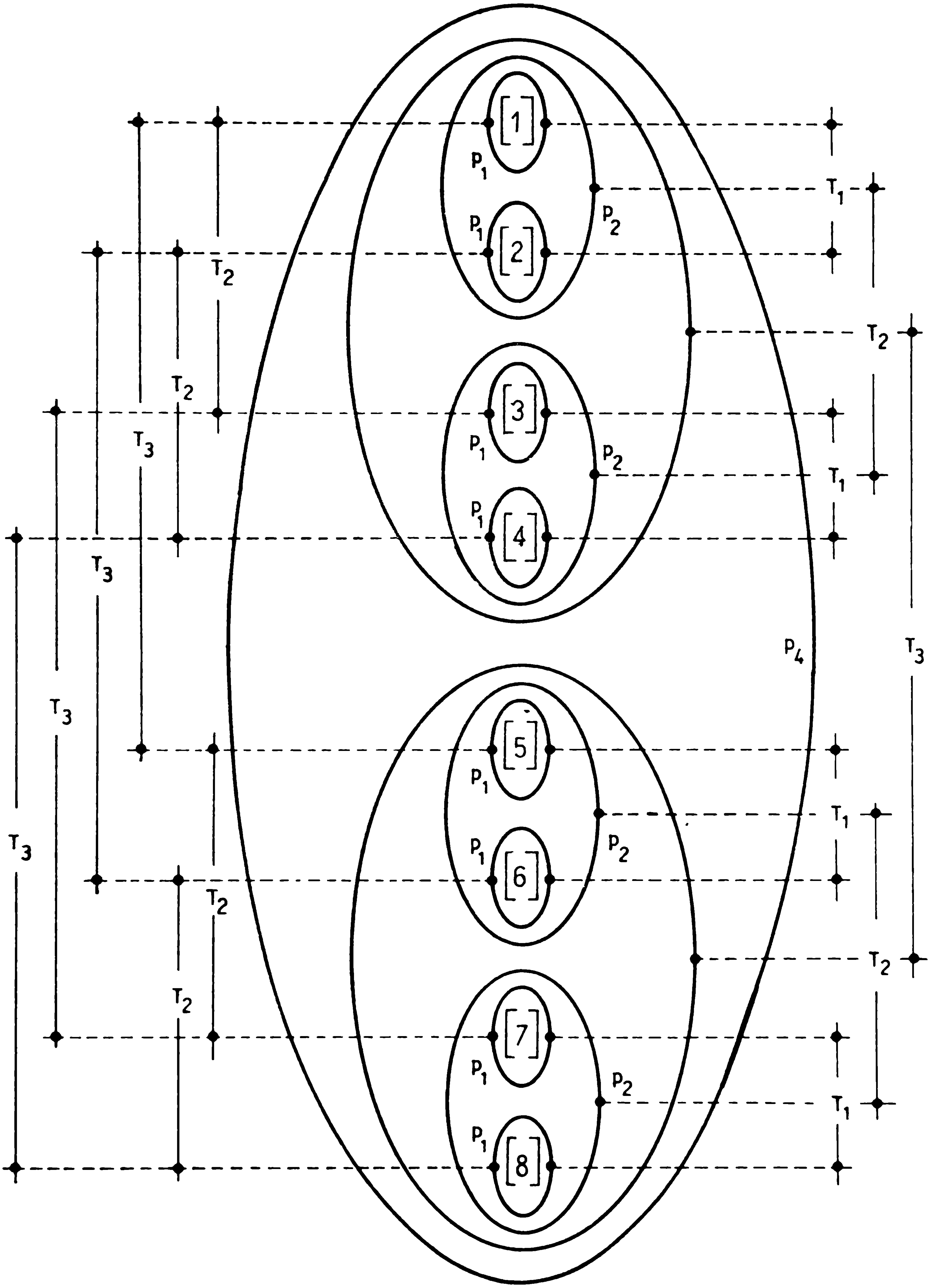

BLOCK DIAGRAM

([4]-[3]) - ([2]-[1]) = [4] - [3] - [2] + [1], or
([8]-[7]) - ([6]-[5]) = [8] - [7] - [6] + [5] now the wave p_2 is filtered because the separations are exactly T_2. Next, if we subtracted ([8] - [7] - [6] + [5]) - ([4] - [3] - [2] + [1]) = [8] - [7] - [6] + [5] - [4] + [3] + [2] - [1] the wave p_3 is now filtered, because the position of the former two block positions is exactly T_3.

The wave p_4 remains alone, and can be solved by using the positions in the following manner:

$$[8] - [7] - [6] + [5] - [4] + [3] + [2] - [1] \qquad (a)$$

The solution of p_4 is one and only one, whatever the order of filtering we have chosen.

In fact: suppose that we want to perform the filtering of the three former tidal waves, but this time in the order p_3, p_2, p_1. Now pay attention to the left side of the BLOCK DIAGRAM. It is easy to see that if we subtract the following pair of positions, ([8] - [4]), ([7] - [3]) ([6] - [2]) and ([5] - [1]), the wave p_3 is fully filtered because each pair of positions, as shown above, is separated by a full period T_3 Next, if we subtract either the positions ([8] - [4]) - ([6] - [2]) = [8] - [4] - [6] + [2] or ([7] - [3]) - ([5] - [1]) = [7] - [3] - [5] + [1] we have two blocks separated, in both cases, by a full period T_2.

This means that if we subtracted both blocks of positions, the wave p_2 is fully filtered. Then

([8] - [4] - [6] + [2]) - ([7] - [3] - [5] + [1]) =
[8] - [4] - [6] + [2] - [7] + [3] + [5] - [1]

which can be ordered as follows:

$$[8] - [7] - [6] + [5] - [4] + [3] + [2] - [1] \qquad (b)$$

This ordainment to solve the wave p_4 is exactly the same as the former one (a). Of course the wave p_1 has been absolutely filtered, because each pair of positions shown in (b), say (8-7), - (6-5), - (4-3) and (2-1) is separated precisely by T_1. By the text the reader can learn the easy way of using all the posibilities.

In the treatment of the problem, we have shown that the same ordainment is repeated once and again. The BLOCK DIAGRAM looks like a VENN diagram within the Groups' Theory, and at some extent it is an application of it.

To perform the computations to obtain the quick results here presented, we have used a small programming desk computer Hewlet-Packard 9820, with mathematical and trigonometric functions and a memory storage capacity up to 450 entrances.

The author owes gratitude and recognition to the geophysicist Lic. Graciela Font de Affolter for her help in preparing this publication and for the many previous computations performed on different type of schemes until the understandable application of the system was catched on.

I am also indebted to our draftman Mr. Alejandro José Mateo who wrote by hand all the formulas and drawings of this book with a patience proper of a Lamaist monk.

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Working Group

BASIC NOTATIONS

a	$\cos \omega t$ or $\sum_0^n \cos \omega \left[t+n\Delta t\right]$, (Sec. 51).
$a^{\bullet}$	a in a reiterated filtering scheme.
a_i	General form of a
a'	Either $a \cos^2 \delta$ or a sen 2δ
$a'^{\bullet}$	a' in a reiterated filtering scheme.
a'_i	General form of a'.
A	$R \cos \varphi$
A_i	General form of A
b	sen ωt or $\sum_0^n$ sen $\omega\left[t+n\Delta t\right]$, (Sec. 51).
$b^{\bullet}$	b in a reiterated filtering scheme.
b_i	General form of b
b'	Either $b \cos^2 \delta$ or b sen 2δ
$b'^{\bullet}$	b' in a reiterated filtering scheme.
b'_i	General form of b'.
B	R sen φ
B_i	General form of B.
C	Mean coefficient of a tidal component wave.
$C^{\bullet}$	Actual coefficient of a tidal component wave.
f	Node factor.
h_i	Ordinate from the u-u axis to any particular wave p_i
H_i	$\sum h_i$

H_T	Height of the tide, Eq. 53.
p_i	Any particular tidal component wave.
P_i	Real ordinate from the v-v axis to any wave p_i
q_u	Ordinate from the u-u axis to the total tidal wave Q_T
q_v	Real ordinate from the v-v axis to the total tidal wave Q_T
Q	Either q_u or $\sum_1^n q_u$
$Q^{\bullet}$	Q in a reiterated filtering scheme.
$\bar{Q}$	A computed value, equal to $Q + \sum P_i$, (Eq.65).
$\bar{Q}^{\bullet}$	$\bar{Q}$ in a reiterated or different filtering scheme.
Q_T	The total tidal wave.
r	Ratio $R_{Sun}/R_{Moon} = 0.46$
R	Actual semiamplitude of a tidal component wave $= (A^2 + B^2)^{1/2}$
R_i	General form of R
$\mathcal{R}$	Mean semiamplitude of a tidal component wave $= f \cdot R$
t	Time reckoned on a record from anarbitrary origin.
t_o	Time span where equally separeted ordinates are read.
t_h	Hour angle
(T)	A semidiurnal tidal wave which cannot be separated from a tidal wave, unless used particular method. (Sec. 52).
T_o	Time separation between the original and reiterated filtering scheme
T_i	Period of a tidal wave p_i
T_S	Period of a solar tidal wave.
T_L	Period of a lunar tidal wave.

u-u	Arbitrary axis on a record, supposed parallel to the v-v axis.
v-v	Real axis of all the tidal waves.
u'-u'	An u-u axis following a linear instrumental drift. (Fig. 6)
v'-v'	The v-v axis following a linear instrumental drift. (Fig.6)
W	A fictitious wave, not used in this work (Sec.52)
z	Distance between u-u and v-v axes.
Z	$\sum z$
α	Right ascention.
δ	Declination.
δ_S	Sun declination.
δ_L	Moon declination.
ρ_a	Real distance of the celestial body (moon, sun).
ρ_m	Mean distance of the celestial body.
$\rho_m^{\bullet}$	"Actual" mean distance of the celestial body.
$\sum$	Summation sign.
φ	Initial phase angle, corresponding at t=0
φ_i	General form of φ
ϕ	Station latitude.
ω	Hourly speed of a tidal wave.
ω_i	General form of ω.
ω_S	Hourly speed of the Sun.
ω_L	Hourly speed of the Moon.
Δ	Difference.

Δa	$a - a'$
Δb	$b - b'$
ΔQ	$Q - \bar{Q}$
ΔP	$\Delta a_i A_i - \Delta b_i B_i$ (See Sec. 66, Eq. 64)
Δt	Time space between two consecutive ordinates, whithin the interval t_0, in every single position, say [1], [2]. .. . . .

A) BASIS

1. Let p_i generically be one of the many tidal components of the tidal wave Q_T, whose elongations P_i in function of the time starting at 0, (an arbitrary origin), can be expressed as: (Fig. 1)

$$P_i = R_i \cos(\omega_i t + \varphi_i) \qquad (1)$$

wherein

R_i = Semiamplitude of p_i

$\omega_i = \dfrac{2\pi}{T_i}$ = Pulsation of p_i

T_i = Period of the pulsation

t = Time reckoned from the origin 0.

φ_i = Initial phase angle at 0.

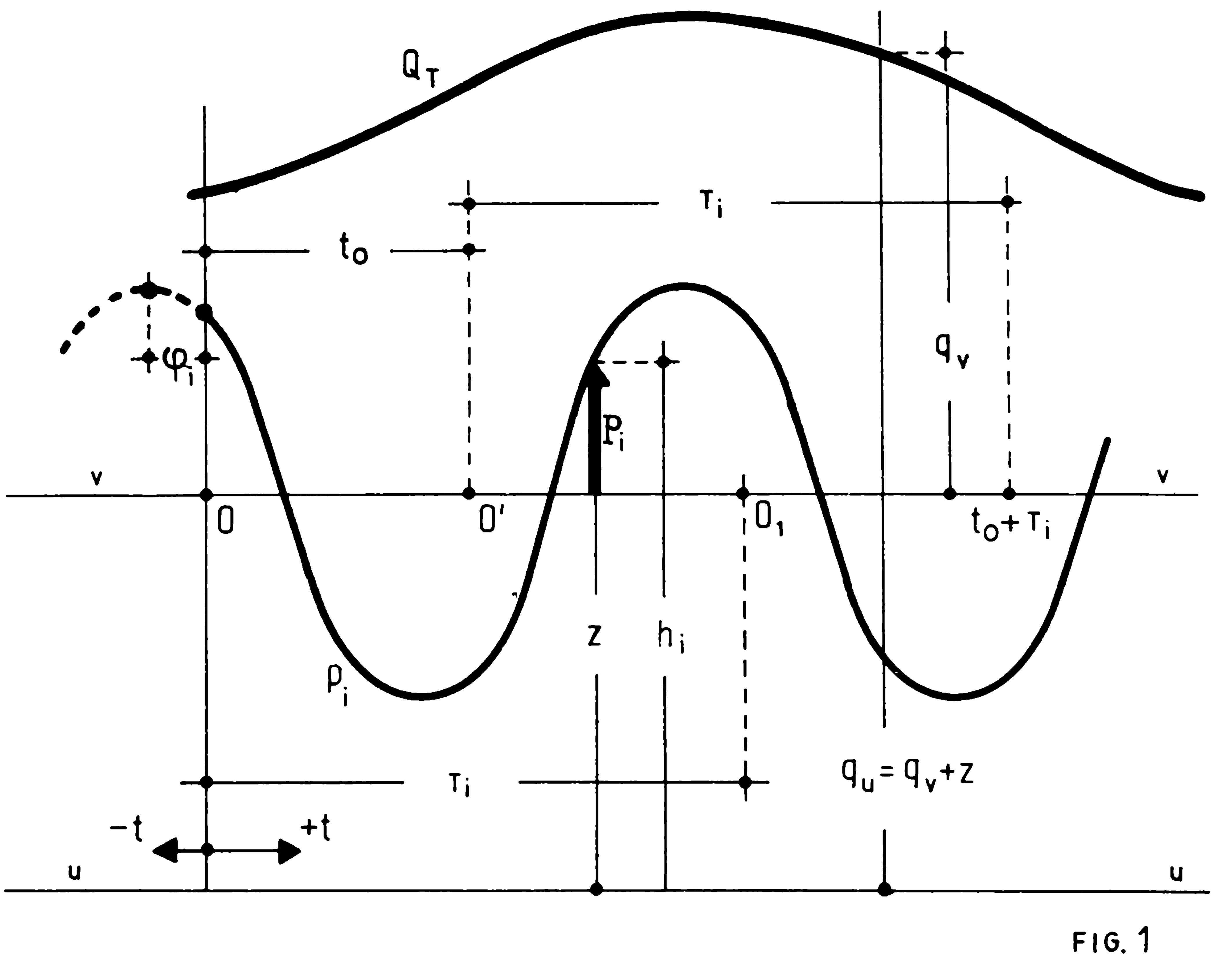


FIG. 1

This equation has two unknowns, R_i and φ_i , so that two independent equations of the same type must be available to evaluate them.
Equation (1) can be expanded as follows:

$$P_i = R_i \cos \varphi_i \cos \omega_i . t - R_i \operatorname{sen} \varphi_i \operatorname{sen} \omega_i . t \qquad (2)$$

and respectively calling

$$\begin{aligned} a_i &= \cos \omega_i . t \\ A_i &= R_i \cos \varphi_i \\ b_i &= \operatorname{sen} \omega_i . t \\ B_i &= R_i \operatorname{sen} \varphi_i \end{aligned} \qquad (3)$$

equation (2) becomes

$$P_i = a_i \; A_i - b_i \; B_i \qquad (4)$$

The a_i and b_i values are always known, because ω_i is known for each particular tidal component.
Two independent equations as that of Eq. (4), furnish the solution of a tidal component. A_i and B_i are finally solved, and from there it follows

$$\operatorname{Tg} \varphi_i = \frac{B_i}{A_i} \qquad (5)$$

$$R_i = \sqrt{A_i^2 + B_i^2} = \frac{A_i}{\cos \varphi_i} = \frac{B_i}{\operatorname{sen} \varphi_i} \qquad (6)$$

If the origin 0 is settled in the record on a place distant an interval of time τ from the origin of the first reading on the record, then Eq. (1) becomes

$$P_i = R_i \cos \left[\omega_i (t - \tau) + \varphi_i \right] \qquad (1A)$$

and, in all the cases, the initial phase angle φ_i is at 0 , the position that corresponds at the instant when $t - \tau = 0$.

2. Starting at 0 , the addition of the elongations P_i within a full period T_i , fulfil the condition

$$\sum_{0}^{T_i} P_i = 0 \qquad (7)$$

Same condition comes true starting from some other arbitrary origin, as 0' , distant from 0 by an interval of time t_o . Therefore

$$\sum_{t_o}^{T_i+t_o} P_i = 0 \qquad (7A)$$

Both equations, (7) and (7A), introduce, as a disgression, several consequences.

It can be written

$$\sum_{o}^{T_i} P_i = \sum_{t_o}^{T_i+t_o} P_i$$

and adding to each member the value $\sum_{T_i}^{t_o} P_i$ it is attained

$$\sum_{o}^{T_i} P_i + \sum_{T_i}^{t_o} P_i = \sum_{t_o}^{T_i+t_o} P_i + \sum_{T_i}^{t_o} P_i$$

$$\sum_{o}^{t_o} P_i = \sum_{T_i}^{T_i+t_o} P_i$$

whence

$$\boxed{\sum_{T_i}^{T_i+t_o} P_i - \sum_{o}^{t_o} P_i = 0} \qquad (8)$$

Furthermore, taking into account Eq. (7), the condition expressed by Eq. (8) is also maintained if it is written in the following form:

$$\sum_{nT_i}^{nT_i+t_o} P_i - \sum_{o}^{t_o} P_i = 0 \qquad (8A)$$

for every $n = 1 , 2 , 3 , 4 ....$, etc.

Equation (7), can also be expressed in this way:

$$\sum_{o}^{T_i} P_i = \sum_{o}^{\frac{1}{2}T_i} P_i + \sum_{\frac{1}{2}T_i}^{T_i} P_i = 0$$

which means

$$\sum_{0}^{\frac{1}{2}T_i} P_i = -\sum_{\frac{1}{2}T_i}^{T_i} P_i$$

Let us add an space of time m at the end of the upper limits of the sums in each member. Then, on keeping the equality, it follows that

$$\sum_{0}^{\frac{1}{2}T_i} P_i + \sum_{\frac{1}{2}T_i}^{\frac{1}{2}T_i+m} P_i = -\left(\sum_{\frac{1}{2}T_i}^{T_i} P_i + \sum_{T_i}^{T_i+m} P_i\right)$$

$$\sum_{0}^{\frac{1}{2}T_i+m} P_i = -\sum_{\frac{1}{2}T_i}^{T_i+m} P_i = -\sum_{\frac{1}{2}T_i}^{\frac{1}{2}T_i+\frac{1}{2}T_i+m} P_i$$

If it is now named

$$t_o = \frac{1}{2}T_i + m$$

then

$$\sum_{0}^{t_o} P_i = -\sum_{\frac{1}{2}T_i}^{\frac{1}{2}T_i+t_o} P_i$$

whence

$$\boxed{\sum_{0}^{t_o} P_i + \sum_{\frac{1}{2}T_i}^{\frac{1}{2}T_i+t_o} P_i = 0} \qquad (8B)$$

Taking into account Eq. (7), the latter can also be written as follows:

$$\sum_{0}^{t_o} P_i + \sum_{\frac{n}{2}T_i}^{\frac{n}{2}T_i+t_o} P_i = 0 \qquad (8C)$$

for every odd $n = 1, 3, 5, 7 \ldots\ldots$, etc.

Equations (8), (8A), (8B), (8C), afford the conditions to delineate two entirely different systems of absolute wave's filtering, and they are the basis of the schemes that will be explained in Chapters C and D, although, for the reasons given in Sec.31, it will be used the scheme given in Chapter C, and next a reiteration of itself.

As it is shown in Fig. 1, the origin 0 can be chosen at any phase φ_i of the wave p_i, and in all the cases the written conditions are maintained.

3. In general, every wave p_1 is one of the many tidal components of the tidal wave Q_T, which is actually obtained by means of an automatic instrumental record; but what is really unknown is the actual v-v axis, (Fig. 1), and the more it can be done to draw out deductions is to use q_u ordinates, starting from another arbitrary u-u axis.

Let us call z the distance between both axes; from here on z will be considered as a constant, although not necessarily. (See Chapter I).

In Fig. 1 it is shown that

$$q_u = q_v + Z$$

But, $q_v = P_1 + P_2 + P_3 + \ldots = \sum P_i$, Therefore, if equally spaced q_u ordinates are added during an interval of time t_o, it is found that

$$\sum_{o}^{t_o} q_u = \sum_{o}^{t_o} (P_1 + P_2 + P_3 + \ldots) + \sum_{o}^{t_o} z = Q \qquad (9)$$

and by calling

$$Z_{t_o} = \sum_{o}^{t_o} z \qquad (10)$$

it follows that

$$\sum_{o}^{t_o} q_u = \sum_{o}^{t} (P_1 + P_2 + P_3 + \ldots) + Z_{t_o} = Q \qquad (11)$$

It is understood without any other reason that the value Z_{t_o} depends only of the interval of time t_o, whatever the origin be. So, from some other origin starting at time T_o, it follows

$$\sum_{T_o}^{T_o+t_o} z = \sum_{o}^{t_o} z = Z_{t_o} \qquad (12)$$

4. As it is shown in Fig. 2, let p_i be, once again, one tidal component of the tidal wave Q_T.

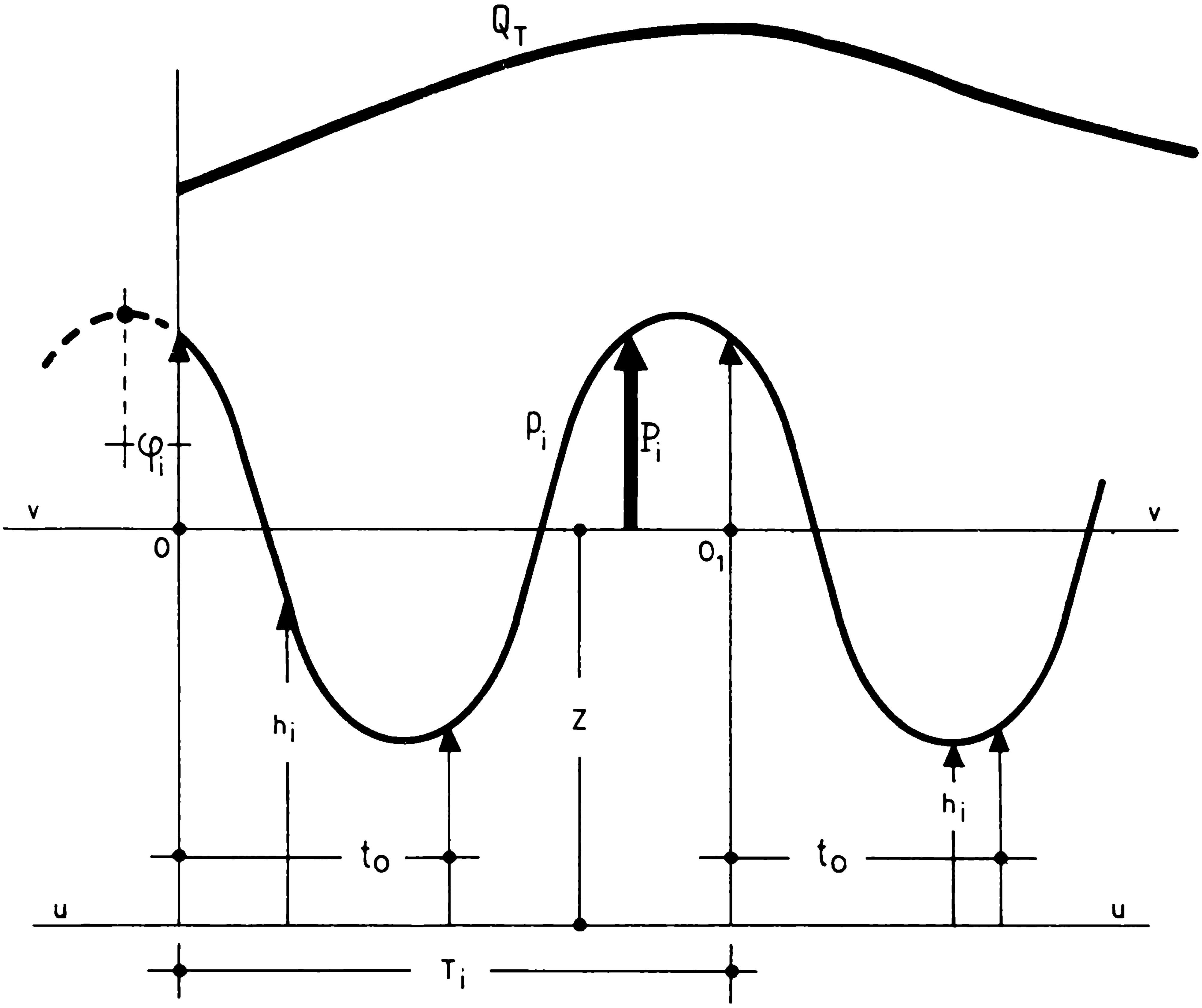


FIG. 2

If by starting from the origin 0, it is performed the addition of equally spaced h_i ordinates within the interval of time t_o, a value H_i will be obtained, so that

$$\sum_{0}^{t_o} h_i = H_i \qquad (13)$$

Performing exactly the same, but now starting at origin 0_1, which is separated from 0 by one or several full periods T_i, circunstances are again repeated and evidently

$$\sum_{T_i}^{T_i + t_o} h_i = H_i \qquad (14)$$

Since it is always

$$h_i = P_i + z$$

then by expanding the Eq. (13) and (14), it follows that

$$\sum_{0}^{t_0} P_i + \sum_{0}^{t} z = \sum_{T_i}^{T_i+t_0} P_i + \sum_{T_i}^{T_i+t_0} z \qquad (15)$$

and by virtue of Eq. (12),

$$\sum_{0}^{t_0} P_i = \sum_{T_i}^{T_i+t_0} P_i \qquad (16)$$

which merely amounts to a verification of Eq. (8)

5. In practice it is impossible to read directly the h_i ordinates, since on the record it is only known the tidal wave Q_T , which at the same time is the resultant of all the p_i components.

Then, the most it can be done is to read $q_u = q_v + z$ ordinates in accordance with what was said in Sec. 2, recalling that q_v is the resultant addition of all the elongations P_i.

As Fig. 2 shows, during the two spaces of time t_0 , starting respectively at origins 0 and 0_1 , equations type (11) can be written.

Then

$$\sum_{0}^{t_0} q_u = \sum_{0}^{t_0} (P_1 + P_2 + P_3 + \cdots) + z_{t_0} = Q_1 \qquad (17)$$

$$\sum_{T_i}^{T_i+t_0} q_u = \sum_{T_i}^{T_i+t_0} (P_1 + P_2 + P_3 + \cdots) + z_{t_0} = Q_2 \qquad (18)$$

To filter the tidal components p_i one after another, it will be necessary to state many equations. On each one, precision increases when the number of equally spaced ordinates within the interval of time t_0 is also increased, that is to say, when the space of time between successive ordinates becomes smaller and smaller.

In Fig. 3, the several terms of the former equations are identified.

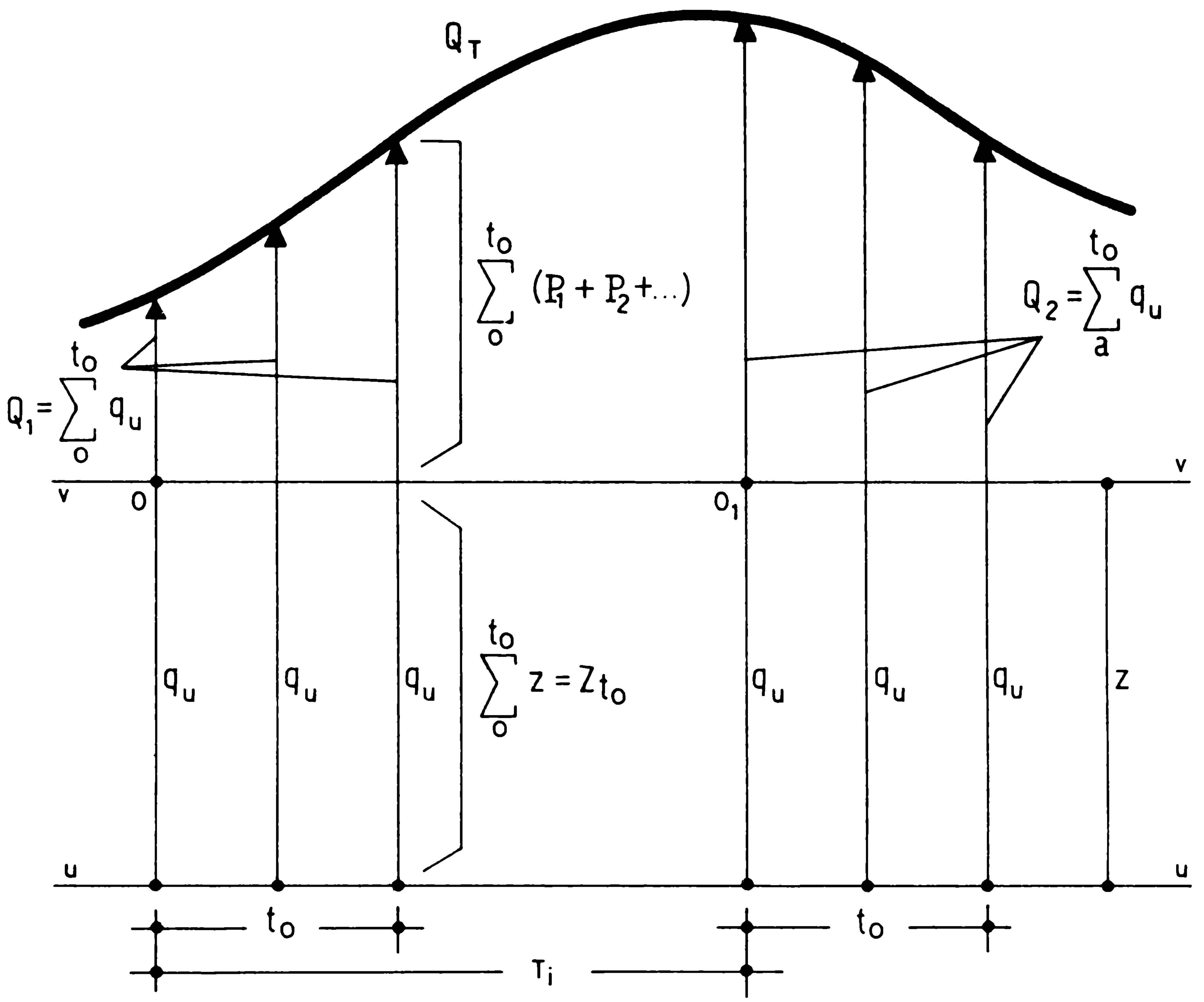


FIG. 3

B) ABSOLUTE FILTERING OF TIDAL COMPONENTS

6. Let P_1, P_2, P_3... be the several tidal components of the tidal wave Q_T, whose particular periods respectively are T_1, T_2, T_3... . It will be shown that it is always possible to filter, absolutely, one wave after another up to a limit in which the final equation has only the elements of just one wave, which then can be easily solved.

Attention is again called to the fact that in every case only one equation with two unknowns is attained. (See Sec. 1 and 19).

From now on, the condition expressed by Eq. (8) will be used. If $T_i \equiv T_1$ equations (18) and (19) become

$$\sum_{0}^{t_0} (P_1 + P_2 + P_3 + \ldots) + z_{t_0} = Q_1 \qquad (20)$$

$$\sum_{T_1}^{T_1+t_0} (P_1 + P_2 + P_3 + \ldots) + Z_{t_0} = Q_2 \qquad (21)$$

By subtracting the two latter equations, and taking into account what was already shown in Sec. 4, Eq. (16), it follows that

$$\sum_{T_1}^{T_1+t_0} (P_2 + P_3 + \ldots) - \sum_{0}^{t_0} (P_2 + P_3 + \ldots) = Q_2 - Q_1 \qquad (22)$$

which shows that the wave component p_1 , whose period is T_1 , has been absolutely filtered. Moreover, the term Z_{t_0} has been removed in such a way that Eq. (22) is now automatically referred to its real own v-v axis.

It is here said that the wave component p_1 has been absolutely filtered because the origins of both terms in the first member of Eq. (22) are exactly separated by one full period T_1.

This simple system permits to enter upon a general scheme of filtering to eliminate, one by one, as many tidal components as it would be necessary, stopping the process when only one tidal component remains.

7. It must be also pointed out that the filtering scheme is always valid whether instead of one full period T_1 , several of them are taken. This is true for any particular tidal component, and merely corresponds to the condition expressed by Eq. (8A).

C) GENERAL SCHEME OF FILTERING

8. Suposse that p_1, p_2, p_3, p_4, p_5, p_6, are six tidal components of the total tidal wave Q_T , (Fig. 4), whose respective periods T_1, T_2, T_3, T_4 T_5, T_6, have been represented, just for clearness, by different straight lines, having each of them a lenght which can be assumed proportional to its own period. Within the general theory, these periods are, of course, arbitrary ones, and the further problem is to solve every wave component by knowing the actual semiamplitude R_i and the initial phase angle φ_i at the arbitrary origin 0 , which in turn it is also the origin of the time, either t , (Eq.1) or $t' = [t - \tau]$, (Eq. 1A).

9. Throughout the adding process of ordinates it will be always taken the same interval t_0 . Thus, it should be observed that into the scheme of filtering shown in Fig. 4, all the positions, from [1] to [32] show an equal interval

of time, t_o

Within the several t_o intervals, in all the cases, the same number of repeated separation of ordinates ought to be considered.

10. At the position [1], a value Q_1 is obtained, which corresponds to the sum of chosen ordinates between the arbitrary u-u axis and the tidal wave Q_T within the interval of time t_o. In position [2], at the end of a full period T_1, a value Q_2 is attained.

As it has been explained in Sec. 6, the difference $Q_2 - Q_1$ filters the tidal component p_1 and also removes Z_{t_o}, remaining the equation referred to the real v-v axis.

11. If a system identical to [1], [2], is repeated as a whole at any position of the tidal wave record, then it filters, by subtraction, the component wave p_1.

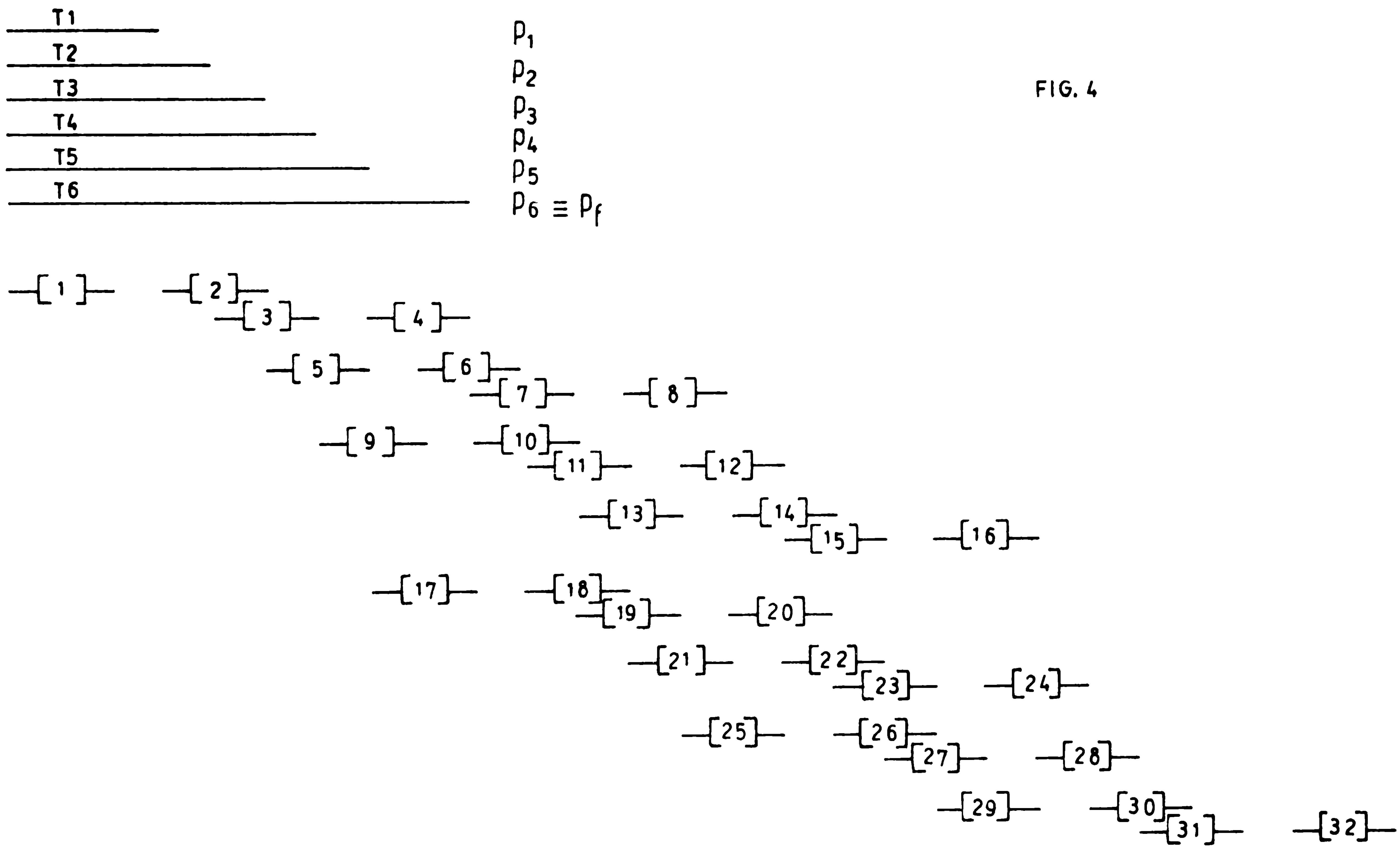
12. Let us consider a system [3], [4], whose relative positions are identical with those of the system [1], [2], that is now placed at the end of a full period T_2 of the tidal component p_2. By subtraction, this new system filters the tidal component p_1, (Sec. 11), but as both systems are located at the beginning and end of a full period T_2, if they are subtracted, a filtering of the tidal component p_2 also occurs. Now, two waves, p_1 and p_2, have been filtered.

13. Another system [5], [6], [7], [8], placed at the end of a full period T_3, whose relative positions are identical with those of the system [1], [2], [3], [4], by using the same method as before, also filters the waves p_1 and p_2. Since both systems are located at the beginning and end of a full period T_3, the wave p_3 will also be absolutely filtered if both systems are subtracted. This means that now there are three tidal constituents filtered, say p_1, p_2, p_3.

14. By an argument entirely similar, a system [9], [10],.......[15],[16], placed at the end of a full period T_4, whose relative positions are identical with those of the system [1], [2]..........[7], [8], by appying the same method, also filters the wave p_1, p_2, p_3.

But both systems are placed at the beginning and end of a full period T_4 so that, if they are subtracted, it results that the tidal component p_4 is also filtered. Then, four tidal constituents (p_1, p_2, p_3, p_4) have been eliminated.

FIG. 4


T1
T2
T3
T4
T5
T6
p_1
p_2
p_3
p_4
p_5
$p_6 \equiv p_f$
1
2
3
4
5
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15. Let us finally consider a system [17], [18] ...... [31], [32] which is placed at the end of a full period T_5, whose relative positions are identical with those of the system [1], [2] .... [15], [16]. Following the same arguments and methods given before, both systems absolutely filter the tidal waves components p_1, p_2, p_3, p_4. As both systems are located at the beginning and end of a full period T_5, if they are subtracted, the p_5 tidal component is also filtered, and now five waves constituents (p_1, p_2, p_3, p_4, p_5) have been filtered.

16. Since the scheme on Fig. 4 shows only six tidal components, it follows that, in the final equation, only one tidal component, p_6, will remain, that can be now solved as a tidal component fully independent from all the others. We will call p_f this final component.

17. Following the same method, and by quite similar arguments, it is possible to filter as many tidal components as necessary, no matter how many of them. However, it must be pointed out that the number of Q to be read from the tidal record, increases geometrically with the number of tidal components to be filtered.

18. Once the wave $p_f \equiv (p_6)$ is known, the other waves should be solved one by one, by going back to the equations set. Neverthless, a better method will be explained in next Chapter.

D) MATHEMATICAL DEVELOPMENT OF THE SCHEME OF FILTERING.

19. The values $Q = \sum q_u$ to be read within the interval of time t_0 are shown in Fig. 3; and the scheme of filtering follows the order shown in Fig. 4, which has been explained in Chapter C.

For brevity of notation, the periods T_i (or $n\,T_i$) will be indicated only by its numbers. For example

$$\sum_{3.2.1}^{3.2.1.t_0} (P_i) \quad \text{means} \quad \sum_{T_3+T_2+T_1}^{T_3+T_2+T_1+t_0} (P_i)$$

wherein the letter T and the sign $(+)$ have been removed.

The initial phase angles φ_i of the several tidal components are given at an arbitrary origin 0, (which can be located either inside or outside of the tidal record), whose position corresponds to the moment when in equation (1) or (1A), it be either $t = 0$ or $t' = (t - \tau) = 0$, as it is evident.

In position [1], (Fig. 4), first equation is

$$\sum_{0}^{t_0} (P_1 + P_2 + P_3 + \ldots) + Z_{t_0} = Q_1 \qquad (24) = (20)$$

In position [2], at the end of a full period T_1, it is

$$\sum_{1}^{1.t_0} (P_1 + P_2 + P_3 + \ldots) + Z_{t_0} = Q_2 \qquad (25) = (21)$$

As it has been explained in Chapter B, and Sec. 4, Eq. (16), it is

$$\sum_{0}^{t_0} P_1 = \sum_{1}^{1.t_0} P_1$$

Then, by subtracting Eq. (24) from Eq. (25), it follows

$$\left[\sum_{1}^{1.t_0} - \sum_{0}^{t_0} \right] (P_2 + P_3 + P_4 + \ldots) = Q_2 - Q_1 \qquad (26) = (22)$$

and considerations in Sec. 6 are also valid here.

The positions [3], [4], are placed as a whole at the end of a full period T_2, and being identical with those of the system [1], [2], their equations can immediately be written. Hence

$$\sum_{2}^{2.t_0} (P_1 + P_2 + P_3 + \ldots) + Z_{t_0} = Q_3 \qquad (27)$$

$$\sum_{2.1}^{2.1.t_0} (P_1 + P_2 + P_3 + \ldots) + Z_{t_0} = Q_4 \qquad (28)$$

Being
$$\sum_{2}^{2.t_o} P_1 = \sum_{2.1}^{2.1.t} P_1$$

by subtracting Eq. (27) from Eq. (28), it results

$$\left[\sum_{2.1}^{2.1.t_o} - \sum_{2}^{2.t_o}\right](P_2 + P_3 + P_4 + \ldots) = Q_4 - Q_3 \qquad (29)$$

Systems expressed by Eqs. (26) and (29), are respectively at the beginning and end of a full period T_2, so that, after what was said in Sec. 12, if Eq. (26) is subtracted from Eq. (29), the tidal component p_2 is also absolutely filtered. Then

$$\left[+\sum_{2.1}^{2.1.t_o} - \sum_{2}^{2.t_o} - \sum_{1}^{1.t_o} + \sum_{0}^{t_o}\right](P_3 + P_4 + \ldots) = Q_4 - Q_3 - Q_2 + Q_1 \qquad (30)$$

which shows that the tidal components p_1 and p_2 have been fully filtered.

It should be observed in Fig.4, that the system [5],[6],[7],[8], that is located, as a whole, at the end of a full period T_3, is identical with the system [1],[2],[3],[4].

Then, by using the same method as before, it also filters the waves p_1 and p_2, so that it can be written

$$\left[+\sum_{3.2.1}^{3.2.1.t_o} - \sum_{3.2}^{3.2.t_o} - \sum_{3.1}^{3.1.t_o} + \sum_{3}^{3.t_o}\right](P_3 + P_4 + \ldots) = Q_8 - Q_7 - Q_6 + Q_5 \qquad (31)$$

Equations (30) and (31) correspond to two identical systems, each one respectively placed as a whole at the beginning and end of a full period T_3. As it was shown in Sec. 13, if the first equation is subtracted from the other, the wave p_3 is now also filtered, and so it is found that

$$\left[\begin{matrix} +\sum_{3.2.1}^{3.2.1.t_o} - \sum_{3.2}^{3.2.t_o} - \sum_{3.1}^{3.1.t_o} + \sum_{3}^{3.t_o} \\ -\sum_{2.1}^{2.1.t_o} + \sum_{2}^{2.t_o} + \sum_{1}^{1.t_o} - \sum_{0}^{t_o} \end{matrix}\right](P_4 + P_5 + \ldots) = \begin{matrix} Q_8 - Q_7 - Q_6 + Q_5 \\ -Q_4 + Q_3 + Q_2 - Q_1 \end{matrix} \qquad (32)$$

In a quite similar manner, it should be also observed in Fig. 4, that the system [9],[10]........[15],[16], that is located, as a whole, at the end of a full period T_4, is identical with the system [1],[2]......[7], [8], also considered as a whole. Then, with the same method as used above, it will also filter the waves p_1, p_2, p_3.

On replacing the sum limits within the new region of summation, a similar equation as that shown by Eq. (32), can be written, thus becoming:

$$\left[\begin{array}{l} +\sum_{4.3.2.1}^{4.3.2.1.t_0} - \sum_{4.3.2}^{4.3.2.t_0} - \sum_{4.3.1}^{4.3.1.t_0} + \sum_{4.3}^{4.3.t_0} \\ -\sum_{4.2.1}^{4.2.1.t_0} + \sum_{4.2}^{4.2.t_0} + \sum_{4.1}^{4.1.t_0} - \sum_{4}^{4.t_0} \end{array}\right] (P_4 + P_5 + \ldots) = \begin{array}{l} Q_{16} - Q_{15} - Q_{14} + Q_{13} \\ - Q_{12} + Q_{11} + Q_{10} - Q_9 \end{array} \quad (33)$$

The two latter equations are the expressions of two identical systems, both of them filtering the tidal components p_1, p_2 and p_3 But, since each system is respectively located at the beginning and end of a full period T_4 by means of a simple subtraction the wave component p_4 is also filtered. Hence

$$\left[\begin{array}{l} \sum_{4.3.2.1}^{4.3.2.1.t_0} - \sum_{4.3.2}^{4.3.2.t_0} - \sum_{4.3.1}^{4.3.1.t_0} + \sum_{4.3}^{4.3.t_0} \\ -\sum_{4.2.1}^{4.2.1.t_0} + \sum_{4.2}^{4.2.t_0} + \sum_{4.1}^{4.1.t_0} - \sum_{4}^{4.t_0} \\ -\sum_{3.2.1}^{3.2.1.t_0} + \sum_{3.2}^{3.2.t_0} + \sum_{3.1}^{3.1.t_0} - \sum_{3}^{3.t_0} \\ +\sum_{2.1}^{2.1.t_0} - \sum_{2}^{2.t_0} - \sum_{1}^{1.t_0} + \sum_{0}^{t_0} \end{array}\right] (P_5 + P_f) = \begin{array}{l} Q_{16} - Q_{15} - Q_{14} + Q_{13} \\ - Q_{12} + Q_{11} + Q_{10} - Q_9 \\ - Q_8 + Q_7 + Q_6 - Q_5 \\ + Q_4 - Q_3 - Q_2 + Q_1 \end{array} \quad (34)$$

In Sec. 15 it had been said that a system [17],[18].......[31],[32], that is localted, as a whole, at the end of a full period T_5, and it is also a repetion of the system [1],[2]............[15],[16], will also filters the waves p_1 , p_2 , p_3 and p_4 . Therefore, the corresponding equation within the new region of summation will be similar to the latter, and so it results:

$$\left[\sum_{5.4.3.2.1}^{5.4.3.2.1.t_o} - \sum_{5.4.3.2}^{5.4.3.2.t_o} - \sum_{5.4.3.1}^{5.4.3.1.t_o} + \sum_{5.4.3}^{5.4.3.t_o} - \sum_{5.4.2.1}^{5.4.2.1.t_o} + \sum_{5.4.2}^{5.4.2.t_o} + \sum_{5.4.1}^{5.4.1.t_o} - \sum_{5.4}^{5.4.t_o} - \sum_{5.3.2.1}^{5.3.2.1.t_o} + \sum_{5.3.2}^{5.3.2.t_o} + \sum_{5.3.1}^{5.3.1.t_o} - \sum_{5.3}^{5.3.t_o} + \sum_{5.2.1}^{5.2.1.t_o} - \sum_{5.2}^{5.2.t_o} - \sum_{5.1}^{5.1.t_o} + \sum_{5}^{5.t_o} \right] (P_5 + P_f) =$$

$$= Q_{32} - Q_{31} - Q_{30} + Q_{29} - Q_{28} + Q_{27} + Q_{26} - Q_{25} - Q_{24} + Q_{23} + Q_{22} - Q_{21} + Q_{20} - Q_{19} - Q_{18} + Q_{17} \qquad (35)$$

Once more and finally, Eqs. (34) and (35) refer to two identical systems, each one filtering the tidal components p_1 , p_2 , p_3 and p_4 . It has already been observed that both systems are respectively located, as a whole, at the beginning and end of a full period T_5 . According to Sec. 15, by subtracting Eq.(34) from Eq. (35), the fifth component wave p_5 is also filtered and so it is finally obtained an equation with only one wave $p_f \equiv p_6$ as unknown, wholly expressed in terms of all the addition of ordinates, Q , and of sums depending on its own pulsation.

The final result is:

$$
\left[\begin{array}{l}
\sum_{5.4.3.2.1}^{5.4.3.2.1.t_o}[32] - \sum_{5.4.3.2}^{5.4.3.2.t_o}[31] - \sum_{5.4.3.1}^{5.4.3.1.t_o}[30] + \sum_{5.4.3}^{5.4.3.t_o}[29] \\
- \sum_{5.4.2.1}^{5.4.2.1.t_o}[28] + \sum_{5.4.2}^{5.4.2.t_o}[27] + \sum_{5.4.1}^{5.4.1.t_o}[26] - \sum_{5.4}^{5.4.t_o}[25] \\
- \sum_{5.3.2.1}^{5.3.2.1.t_o}[24] + \sum_{5.3.2}^{5.3.2.t_o}[23] + \sum_{5.3.1}^{5.3.1.t_o}[22] - \sum_{5.3}^{5.3.t_o}[21] \\
+ \sum_{5.2.1}^{5.2.1.t_o}[20] - \sum_{5.2}^{5.2.t_o}[19] - \sum_{5.1}^{5.1.t_o}[18] + \sum_{5}^{5.t_o}[17] \\
- \sum_{4.3.2.1}^{4.3.2.1.t_o}[16] + \sum_{4.3.2}^{4.3.2.t_o}[15] + \sum_{4.3.1}^{4.3.1.t_o}[14] - \sum_{4.3}^{4.3.t_o}[13] \\
+ \sum_{4.2.1}^{4.2.1.t_o}[12] - \sum_{4.2}^{4.2.t_o}[11] - \sum_{4.1}^{4.1.t_o}[10] + \sum_{4}^{4.t_o}[9] \\
+ \sum_{3.2.1}^{3.2.1.t_o}[8] - \sum_{3.2}^{3.2.t_o}[7] - \sum_{3.1}^{3.1.t_o}[6] + \sum_{3}^{3.t_o}[5] \\
- \sum_{2.1}^{2.1.t_o}[4] + \sum_{2}^{2.t_o}[3] + \sum_{1}^{1.t_o}[2] - \sum_{0}^{t_o}[1]
\end{array}\right] P_f =
$$

$$
= \begin{array}{l}
Q_{32} - Q_{31} - Q_{30} + Q_{29} - Q_{28} + Q_{27} + Q_{26} - Q_{25} \\
- Q_{24} + Q_{23} + Q_{22} - Q_{21} + Q_{20} - Q_{19} - Q_{18} + Q_{17} \\
- Q_{16} + Q_{15} + Q_{14} - Q_{13} + Q_{12} - Q_{11} - Q_{10} + Q_{9} \\
+ Q_{8} - Q_{7} - Q_{6} + Q_{5} - Q_{4} + Q_{3} + Q_{2} - Q_{1}
\end{array} \qquad (36)
$$

This equation brings out one solution of

$$P_f = a_f \ A_f \ - \ b_f \ B_f \qquad (4)$$

after it was cleared up in Sec. 1. Since there are two unknowns, (A_f and B_f), still remains the problem of evaluating them. Another independent equation is needed to solve up the wave p_f.

20. In the Eq. (36), it has been written within each sum the position that it occupies on the scheme of Fig. 4, and in each case, the sum limits are as shown. Henceforth, the notation is much simplified by indicating the sums only by their positions.

21. The solution of the tidal component $p_f \equiv p_6$, depending of the ad - dition of ordinates Q_1, Q_2 .... Q_{31}, Q_{32}, and of the sums shown in Eq. (36), is one and only one. It is not only absolutely independent of all the previously filtered waves, but it is also independent with the order chosen to filter them. This means that the order of filtering of the tidal components p_1, p_2, p_3, p_4, p_5, can be permutated in Fig. 4, while the final result, brought out to the tidal component p_f by Eq. (36), remains the same in all the cases. This can be easily shown in mathematics and, moreover, it cannot be otherwise.

This circumstance furnishes the solution of every component wave with the same weight, and always as a function of the same values Q , (Q_1, Q_2 .... Q_{31}, Q_{32}) , which in every case must be taken in an appro- piated order and sign. Chapter E expresses the equations of every tidal com - ponent, each one in all the cases depending of the wave $p_f \equiv p_6$.

22. There exist some other ways to solve every wave; but the equations depend on more than one tidal component that must be previously resolved.

Next chapter shows the equations of all the waves, the method used being as the one explained above.

E) EQUATIONS OF THE TIDAL COMPONENTS.

23. WAVE p_f $\left[P_f = R_f \cos(\omega_f t - \varphi) = a_f A_f - b_f B_f\right]$

Its solution is that of Eq. (38), which, with the simplified notation already mentioned in Sec. 20, can now be written:

$$\begin{bmatrix} +\sum[32]-\sum[31]-\sum[30]+\sum[29]-\sum[28]+\sum[27]+\sum[26]-\sum[25] \\ -\sum[24]+\sum[23]+\sum[22]-\sum[21]+\sum[20]-\sum[19]-\sum[18]+\sum[17] \\ -\sum[16]+\sum[15]+\sum[14]-\sum[13]+\sum[12]-\sum[11]-\sum[10]+\sum[9] \\ +\sum[8]-\sum[7]-\sum[6]+\sum[5]-\sum[4]+\sum[3]+\sum[2]-\sum[1] \end{bmatrix} P_f =$$

$$= \begin{matrix} +Q_{32} - Q_{31} - Q_{30} + Q_{29} - Q_{28} + Q_{27} + Q_{26} - Q_{25} \\ -Q_{24} + Q_{23} + Q_{22} - Q_{21} + Q_{20} - Q_{19} - Q_{18} + Q_{17} \\ -Q_{16} + Q_{15} + Q_{14} - Q_{13} + Q_{12} - Q_{11} - Q_{10} + Q_{9} \\ +Q_{8} - Q_{7} - Q_{6} + Q_{5} - Q_{4} + Q_{3} + Q_{2} - Q_{1} \end{matrix} \qquad (37)$$

24. WAVE p_5 $\left[P_5 = R_5 \cos(\omega_5 t + \varphi_5) = a_5 A_5 - b_5 B_5\right]$

Since the wave p_f is previously known, by means of every one of the Eqs. (34) and (35) the wave p_5, (another tidal component), can be resolved.

All of the sums which appear in the left member of both equations, (34) and (35), are of course exactly the same, in reason that if both equations are subtracted the wave p_5 is filtered. Being that so, by adding both equations the wave p_5 is solved as a function of all the Q and of the wave p_f.

After ordering it is found that

$$2\begin{bmatrix} +\sum[16]-\sum[15]-\sum[14]+\sum[13]-\sum[12]+\sum[11]+\sum[10]-\sum[9] \\ -\sum[8]+\sum[7]+\sum[6]-\sum[5]+\sum[4]-\sum[3]-\sum[2]+\sum[1] \end{bmatrix} P_5 = \tag{38}$$

$$\begin{aligned} = &+Q_{32} - Q_{31} - Q_{30} + Q_{29} - Q_{28} + Q_{27} + Q_{26} - Q_{25} \\ &- Q_{24} + Q_{23} + Q_{22} - Q_{21} + Q_{20} - Q_{19} - Q_{18} + Q_{17} \\ &+ Q_{16} - Q_{15} - Q_{14} + Q_{13} - Q_{12} + Q_{11} + Q_{10} - Q_9 \\ &- Q_8 + Q_7 + Q_6 - Q_5 + Q_4 - Q_3 - Q_2 + Q_1 - \end{aligned}$$

$$-\begin{bmatrix} +\sum[32]-\sum[31]-\sum[30]+\sum[29]-\sum[28]+\sum[27]+\sum[26]-\sum[25] \\ -\sum[24]+\sum[23]+\sum[22]-\sum[21]+\sum[20]-\sum[19]-\sum[18]+\sum[17] \\ +\sum[16]-\sum[15]-\sum[14]+\sum[13]-\sum[12]+\sum[11]+\sum[10]-\sum[9] \\ -\sum[8]+\sum[7]+\sum[6]-\sum[5]+\sum[4]-\sum[3]-\sum[2]+\sum[1] \end{bmatrix} P_f$$

25. WAVE p_4 $\quad \left[P_4 = R_4 \cos(\omega_4 t + \varphi_4) = a_4 A_4 - b_4 B_4\right]$

If within the scheme of Fig. 4 it is permutated the order of the tidal components p_4 and p_5 , it now means that the wave p_4 actually occupies the position that the wave p_5 had before. In accordance with the reference already set up in Sec. 21, it immediately follows:

$$(39)$$

$$2\begin{bmatrix} +\sum[24]-\sum[23]-\sum[22]+\sum[21]-\sum[20]+\sum[19]+\sum[18]-\sum[17] \\ \\ -\sum[8]+\sum[7]+\sum[6]-\sum[5]+\sum[4]-\sum[3]-\sum[2]+\sum[1] \end{bmatrix} P_4 =$$

$$\begin{aligned} = &+Q_{32} - Q_{31} - Q_{30} + Q_{29} - Q_{28} + Q_{27} + Q_{26} - Q_{25} \\ &- Q_{16} + Q_{15} + Q_{14} - Q_{13} + Q_{12} - Q_{11} - Q_{10} + Q_9 \\ &+ Q_{24} - Q_{23} - Q_{22} + Q_{21} - Q_{20} + Q_{19} + Q_{18} - Q_{17} \\ &- Q_8 + Q_7 + Q_6 - Q_5 + Q_4 - Q_3 - Q_2 + Q_1 - \end{aligned}$$

$$-\begin{bmatrix} +\sum[32]-\sum[31]-\sum[30]+\sum[29]-\sum[28]+\sum[27]+\sum[26]-\sum[25] \\ \\ -\sum[16]+\sum[15]+\sum[14]-\sum[13]+\sum[12]-\sum[11]-\sum[10]+\sum[9] \\ \\ +\sum[24]-\sum[23]-\sum[22]+\sum[21]-\sum[20]+\sum[19]+\sum[18]-\sum[17] \\ \\ -\sum[8]+\sum[7]+\sum[6]-\sum[5]+\sum[4]-\sum[3]-\sum[2]+\sum[1] \end{bmatrix} P_f$$

26. WAVE p_3 $\quad\left[P_3 = R_3 \cos(\omega_3 t + \varphi_3) = a_3 A_3 - b_3 B_3\right]$

By an argument entirely similar as the former one, if the tidal component p_3 is placed at the position that the tidal component p_5 had before in the scheme of Fig. 4, it is seen that

(40)

$$2\begin{bmatrix} +\sum[28]-\sum[27]-\sum[26]+\sum[25]-\sum[20]+\sum[19]+\sum[18]-\sum[17] \\ -\sum[12]+\sum[11]+\sum[10]-\sum[9]+\sum[4]-\sum[3]-\sum[2]+\sum[1] \end{bmatrix} P_3 =$$

$$\begin{aligned} = &+Q_{32} - Q_{31} - Q_{30} + Q_{29} - Q_{24} + Q_{23} + Q_{22} - Q_{21} \\ &- Q_{16} + Q_{15} + Q_{14} - Q_{13} + Q_8 - Q_7 - Q_6 + Q_5 \\ &+ Q_{28} - Q_{27} - Q_{26} + Q_{25} - Q_{20} + Q_{19} + Q_{18} - Q_{17} \\ &- Q_{12} + Q_{11} + Q_{10} - Q_9 + Q_4 - Q_3 - Q_2 + Q_1 - \end{aligned}$$

$$-\begin{bmatrix} +\sum[32]-\sum[31]-\sum[30]+\sum[29]-\sum[24]+\sum[23]+\sum[22]-\sum[21] \\ -\sum[16]+\sum[15]+\sum[14]-\sum[13]+\sum[8]-\sum[7]-\sum[6]+\sum[5] \\ +\sum[28]-\sum[27]-\sum[26]+\sum[25]-\sum[20]+\sum[19]+\sum[18]-\sum[17] \\ -\sum[12]+\sum[11]+\sum[10]-\sum[9]+\sum[4]-\sum[3]-\sum[2]+\sum[1] \end{bmatrix} P_f$$

27. WAVE p_2 $\quad \left[P_2 = R_2 \cos\,(\omega_2 t + \varphi_2) = a_2 A_2 - b_2 B_2\right]$

By reasoning in the same way, if within the scheme of Fig. 4 the tidal component p_2 is now occupying the position that the wave p_5 had before, it is easily seen that its equation becomes

(41)

$$2\begin{bmatrix} +\sum[30]-\sum[29]-\sum[26]+\sum[25]-\sum[22]+\sum[21]+\sum[18]-\sum[17] \\ \\ -\sum[14]+\sum[13]+\sum[10]-\sum[9]+\sum[6]-\sum[5]-\sum[2]+\sum[1] \end{bmatrix} P_2 =$$

$$\begin{aligned} = &+ Q_{32} - Q_{31} - Q_{28} + Q_{27} - Q_{24} + Q_{23} + Q_{20} - Q_{19} \\ &- Q_{16} + Q_{15} + Q_{12} - Q_{11} + Q_{8} - Q_{7} - Q_{4} + Q_{3} \\ &+ Q_{30} - Q_{29} - Q_{26} + Q_{25} - Q_{22} + Q_{21} + Q_{18} - Q_{17} \\ &- Q_{14} + Q_{13} + Q_{10} - Q_{9} + Q_{6} - Q_{5} - Q_{2} + Q_{1} - \end{aligned}$$

$$-\begin{bmatrix} +\sum[32]-\sum[31]-\sum[28]+\sum[27]-\sum[24]+\sum[23]+\sum[20]-\sum[19] \\ \\ -\sum[16]+\sum[15]+\sum[12]-\sum[11]+\sum[8]-\sum[7]-\sum[4]+\sum[3] \\ \\ +\sum[30]-\sum[29]-\sum[26]+\sum[25]-\sum[22]+\sum[21]+\sum[18]-\sum[17] \\ \\ -\sum[14]+\sum[13]+\sum[10]-\sum[9]+\sum[6]-\sum[5]-\sum[2]+\sum[1] \end{bmatrix} P_f$$

28. <u>WAVE p_1</u> $\left[P_1 = R_1 \cos\,(\omega_1 t + \varphi_1) = a_1\, A_1 - b_1\, B_1 \right]$

Repeating once again the same process, that is to say, if the tidal component p_1 is located at the place that the wave p_5 had before, always within the scheme of Fig.4, the equation of the wave p_1 evidently becomes

(42)

$$
2\begin{bmatrix} +\sum[31] -\sum[29] -\sum[27] +\sum[25] -\sum[23] +\sum[21] +\sum[19] -\sum[17] \\ \\ -\sum[15] +\sum[13] +\sum[11] -\sum[9] +\sum[7] -\sum[5] -\sum[3] +\sum[1] \end{bmatrix} P_1 =
$$

$$
\begin{aligned}
= &+ Q_{32} - Q_{30} - Q_{28} + Q_{26} - Q_{24} + Q_{22} + Q_{20} - Q_{18} \\
&- Q_{16} + Q_{14} + Q_{12} - Q_{10} + Q_{8} - Q_{6} - Q_{4} + Q_{2} \\
&+ Q_{31} - Q_{29} - Q_{27} + Q_{25} - Q_{23} + Q_{21} + Q_{19} - Q_{17} \\
&- Q_{15} + Q_{13} + Q_{11} - Q_{9} + Q_{7} - Q_{5} - Q_{3} + Q_{1} -
\end{aligned}
$$

$$
-\begin{bmatrix} +\sum[32] -\sum[30] -\sum[28] +\sum[26] -\sum[24] +\sum[22] +\sum[20] -\sum[18] \\ \\ -\sum[16] +\sum[14] +\sum[12] -\sum[10] +\sum[8] -\sum[6] -\sum[4] +\sum[2] \\ \\ +\sum[31] -\sum[29] -\sum[27] +\sum[25] -\sum[23] +\sum[21] +\sum[19] -\sum[17] \\ \\ -\sum[15] +\sum[13] +\sum[11] -\sum[9] +\sum[7] -\sum[5] -\sum[3] +\sum[1] \end{bmatrix} P_f
$$

29. From now on it is very convenient to make clear that all the equations written for the several tidal components are in all the cases referred to the real own v-v axis, since all of them, besides that the value Z_{t_0} has been removed, are also absolutely independent of the instrumental drift, whatever its value be just in case this can be considered as linear. This condition is discussed in extent in Sec. 47.

F) REITERATION OF THE GENERAL SCHEME OF FILTERING.

30. As it has been said at the end of Sec. 19, the scheme of filtering of the several tidal components brings out, in every case, one expression which involves two unknowns, A_i and B_i .

Another independent equation must be furnished to evaluate them.

31. In order to obtain another fully independent set of equations, it could be used the condition settled by Eqs. (8A) or (8B). Though this is possible, it is easily demonstrated in mathematics, that the new set of equations is not independent of the instrumental drift, even this being a linear one. Unless it could be asserted that the instrumental drift is null all the time, this new system of filtering must be put aside.

32. Therefore, the best it can be done to attain another set of equations, is to repeat the same system of filtering as the one previously explained, starting this time on the tidal record from some other beginning but keeping the same origin of the time as before, in order to maintain invariables, in both systems of filtering, the initial phase angles, φ_i .

33. Let p_1 , p_2 , p_3 , p_4 , p_5 , p_f , (Fig. 5) be the same six tidal components as in Fig. 4, having the same origin and represented in the same way that was already said in Sec. 8.

The scheme of filtering is exactly a repetition of the one explained in Chapter C, mathematically developed in Chapter D and having the explicit equations written in Chapter E.

On comparing both schemes, (Figs. 4 and 5), it is found that they have only changed their relative positions, being separated, as a whole, by an interval of time T_0 , that for best results must be wisely chosen. (See example at the end).

Within the reiterated scheme, the new summations Q will be identified as $Q^{\bullet}$, and in turn, all the sums on the left member of the equations, which are the same as those of Eq. (36), but advanced now in T_0 , will be denoted in their own order as follows:

$$\sum_{T_0.5.4.3.2.1}^{T_0.5.4.3.2.1.t_0} = \sum [32^{\bullet}]$$

$$\sum_{T_0.5.4.3.2}^{T_0.5.4.3.2.t_0} = \sum [31^{\bullet}]$$

$$\sum_{T_0.5.4.3.1}^{T_0.5.4.3.1.t_0} = \sum [30^{\bullet}]$$

$$\sum_{T_0.5.4.3}^{T_0.5.4.3.t_0} = \sum [29^{\bullet}]$$

$$\sum_{T_0.5.4.2.1}^{T_0.5.4.2.1.t_0} = \sum [28^{\bullet}]$$

$$\sum_{T_0.5.4.2}^{T_0.5.4.2.t_0} = \sum [27^{\bullet}]$$

$$\sum_{T_0.5.4.1}^{T_0.5.4.1.t_0} = \sum [26^{\bullet}]$$

$$\sum_{T_0.5.4}^{T_0.5.4.t_0} = \sum [25^{\bullet}]$$

$$\sum_{T_0.5.3.2.1}^{T_0.5.3.2.1.t_0} = \sum [24^{\bullet}]$$

$$\sum_{T_0.5.3.2}^{T_0.5.3.2.t_0} = \sum [23^{\bullet}]$$

$$\sum_{T_0.5.3.1}^{T_0.5.3.1.t_0} = \sum [22^{\bullet}]$$

$$\sum_{T_0.5.3}^{T_0.5.3.t_0} = \sum [21^{\bullet}]$$

$$\sum_{T_0.5.2.1}^{T_0.5.2.1.t_0} = \sum [20^{\bullet}]$$

$$\sum_{T_0.5.2}^{T_0.5.2.t_0} = \sum [19^{\bullet}]$$

$$\sum_{T_0.5.1}^{T_0.5.1.t_0} = \sum [18^{\bullet}]$$

$$\sum_{T_0.5}^{T_0.5.t_0} = \sum [17^{\bullet}]$$

$$\sum_{T_0.4.3.2.1}^{T_0.4.3.2.1.t_0} = \sum [16^{\bullet}]$$

$$\sum_{T_0.4.3.2}^{T_0.4.3.2.t_0} = \sum [15^{\bullet}]$$

$$\sum_{T_0.4.3.1}^{T_0.4.3.1.t_0} = \sum [14^{\bullet}]$$

$$\sum_{T_0.4.3}^{T_0.4.3.t_0} = \sum [13^{\bullet}]$$

$$\sum_{To.4.2.1}^{To.4.2.1.to} = \sum [12^{\bullet}]$$

$$\sum_{To.4.2}^{To.4.2.to} = \sum [11^{\bullet}]$$

$$\sum_{To.4.1}^{To.4.1.to} = \sum [10^{\bullet}]$$

$$\sum_{To.4}^{To.4.to} = \sum [9^{\bullet}]$$

$$\sum_{To.3.2.1}^{To.3.2.1.to} = \sum [8^{\bullet}]$$

$$\sum_{To.3.2}^{To.3.2.to} = \sum [7^{\bullet}]$$

$$\sum_{To.3.1}^{To.3.1.to} = \sum [6^{\bullet}]$$

$$\sum_{To.3}^{To.3.to} = \sum [5^{\bullet}]$$

$$\sum_{To.2.1}^{To.2.1.to} = \sum [4^{\bullet}]$$

$$\sum_{To.2}^{To.2.to} = \sum [3^{\bullet}]$$

$$\sum_{To.1}^{To.1.to} = \sum [2^{\bullet}]$$

$$\sum_{To}^{To.to} = \sum [1^{\bullet}] \qquad (43)$$

To identify which of both schemes is used, in the scheme reiterated the elongations will be denoted by

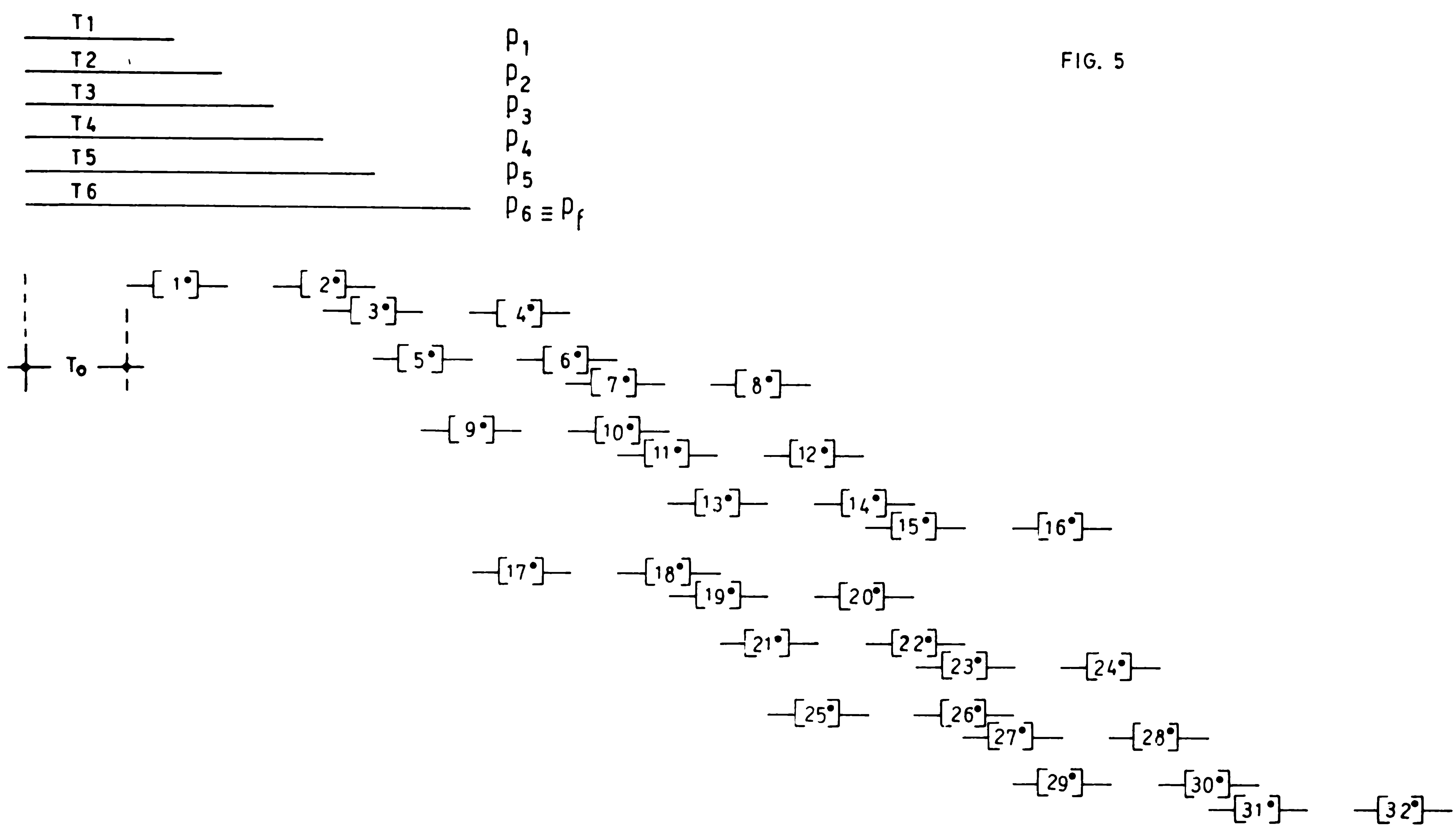
$$P_i^{\bullet} = R_i \cos(\omega_i t + \varphi) = a_i^{\bullet} A_i - b_i^{\bullet} B_i$$

in accordance with Eq. (1), or

$$P_i^{\bullet} = R_i \left[\cos \omega_i (t - \tau) + \varphi_i\right] = a_i^{\bullet} A_i - b_i^{\bullet} B_i$$

which is in accordance with Eq. (1A). (See also Sec. 1·9)

FIG. 5


T1
T2
T3
T4
T5
T6
P_1
P_2
P_3
P_4
P_5
$P_6 \equiv P_f$
T_o
1
2
3
4
5
6
7
8
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G) EQUATIONS OF THE TIDAL COMPONENT WAVES IN THE REITERATION OF THE GENERAL SCHEME OF FILTERING.

The expressions for every component wave are identical with those in the Chapter E, and they are written below without any further commentary.

34. WAVE p_f $\qquad P_f^{\bullet} = R_f \cos(\omega_f t + \varphi_f) = a_f^{\bullet} A_f - b_f^{\bullet} B_f$

(44)

$$\begin{bmatrix} +\sum[32^{\bullet}] - \sum[31^{\bullet}] - \sum[30^{\bullet}] + \sum[29^{\bullet}] - \sum[28^{\bullet}] + \sum[27^{\bullet}] + \sum[26^{\bullet}] - \sum[25^{\bullet}] \\ -\sum[24^{\bullet}] + \sum[23^{\bullet}] + \sum[22^{\bullet}] - \sum[21^{\bullet}] + \sum[20^{\bullet}] - \sum[19^{\bullet}] - \sum[18^{\bullet}] + \sum[17^{\bullet}] \\ -\sum[16^{\bullet}] + \sum[15^{\bullet}] + \sum[14^{\bullet}] - \sum[13^{\bullet}] + \sum[12^{\bullet}] - \sum[11^{\bullet}] - \sum[10^{\bullet}] + \sum[9^{\bullet}] \\ +\sum[8^{\bullet}] - \sum[7^{\bullet}] - \sum[6^{\bullet}] + \sum[5^{\bullet}] - \sum[4^{\bullet}] + \sum[3^{\bullet}] + \sum[2^{\bullet}] - \sum[1^{\bullet}] \end{bmatrix} P_f$$

$$\begin{aligned} = &+ Q^{\bullet}_{32} - Q^{\bullet}_{31} - Q^{\bullet}_{30} + Q^{\bullet}_{29} - Q^{\bullet}_{28} + Q^{\bullet}_{27} + Q^{\bullet}_{26} - Q^{\bullet}_{25} \\ &- Q^{\bullet}_{24} + Q^{\bullet}_{23} + Q^{\bullet}_{22} - Q^{\bullet}_{21} + Q^{\bullet}_{20} - Q^{\bullet}_{19} - Q^{\bullet}_{18} + Q^{\bullet}_{17} \\ &- Q^{\bullet}_{16} + Q^{\bullet}_{15} + Q^{\bullet}_{14} - Q^{\bullet}_{13} + Q^{\bullet}_{12} - Q^{\bullet}_{11} - Q^{\bullet}_{10} + Q^{\bullet}_{9} \\ &+ Q^{\bullet}_{8} - Q^{\bullet}_{7} - Q^{\bullet}_{6} + Q^{\bullet}_{5} - Q^{\bullet}_{4} + Q^{\bullet}_{3} + Q^{\bullet}_{2} - Q^{\bullet}_{1} \end{aligned}$$

35. <u>WAVE p_5</u> $\left[P_5^\bullet = R_5 \cos(\omega_5 t + \varphi_5) = a_5^\bullet A_5 - b_5^\bullet B_5\right]$

(45)

$$2\begin{bmatrix} +\sum[16^\bullet]-\sum[15^\bullet]-\sum[14^\bullet]+\sum[13^\bullet]-\sum[12^\bullet]+\sum[11^\bullet]+\sum[10^\bullet]-\sum[9^\bullet] \\ -\sum[8^\bullet]+\sum[7^\bullet]+\sum[6^\bullet]-\sum[5^\bullet]+\sum[4^\bullet]-\sum[3^\bullet]-\sum[2^\bullet]+\sum[1^\bullet] \end{bmatrix} P_5 =$$

$$\begin{aligned} = \; & + Q_{32}^\bullet - Q_{31}^\bullet - Q_{30}^\bullet + Q_{29}^\bullet - Q_{28}^\bullet + Q_{27}^\bullet + Q_{26}^\bullet - Q_{25}^\bullet \\ & - Q_{24}^\bullet + Q_{23}^\bullet + Q_{22}^\bullet - Q_{21}^\bullet + Q_{20}^\bullet - Q_{19}^\bullet - Q_{18}^\bullet + Q_{17}^\bullet \\ & + Q_{16}^\bullet - Q_{15}^\bullet - Q_{14}^\bullet + Q_{13}^\bullet - Q_{12}^\bullet + Q_{11}^\bullet + Q_{10}^\bullet - Q_{9}^\bullet \\ & - Q_{8}^\bullet + Q_{7}^\bullet + Q_{6}^\bullet - Q_{5}^\bullet + Q_{4}^\bullet - Q_{3}^\bullet - Q_{2}^\bullet + Q_{1}^\bullet - \end{aligned}$$

$$-\begin{bmatrix} +\sum[32^\bullet]-\sum[31^\bullet]-\sum[30^\bullet]+\sum[29^\bullet]-\sum[28^\bullet]+\sum[27^\bullet]+\sum[26^\bullet]-\sum[25^\bullet] \\ -\sum[24^\bullet]+\sum[23^\bullet]+\sum[22^\bullet]-\sum[21^\bullet]+\sum[20^\bullet]-\sum[19^\bullet]-\sum[18^\bullet]+\sum[17^\bullet] \\ +\sum[16^\bullet]-\sum[15^\bullet]-\sum[14^\bullet]+\sum[13^\bullet]-\sum[12^\bullet]+\sum[11^\bullet]+\sum[10^\bullet]-\sum[9^\bullet] \\ -\sum[8^\bullet]+\sum[7^\bullet]+\sum[6^\bullet]-\sum[5^\bullet]+\sum[4^\bullet]-\sum[3^\bullet]-\sum[2^\bullet]+\sum[1^\bullet] \end{bmatrix} P_f$$

36. <u>WAVE p_4</u> $\left[P_4^\bullet = R_4 \cos(\omega_4 t + \varphi_4) = a_4^\bullet A_4 - b_4^\bullet B_4\right]$

(46)

$$2\begin{bmatrix} +\sum[24^\bullet] -\sum[23^\bullet] -\sum[22^\bullet] +\sum[21^\bullet] -\sum[20^\bullet] +\sum[19^\bullet] +\sum[18^\bullet] -\sum[17^\bullet] \\ -\sum[8^\bullet] +\sum[7^\bullet] +\sum[6^\bullet] -\sum[5^\bullet] +\sum[4^\bullet] -\sum[3^\bullet] -\sum[2^\bullet] +\sum[1^\bullet] \end{bmatrix} P_4 =$$

$$\begin{aligned} = &+ Q_{32}^\bullet - Q_{31}^\bullet - Q_{30}^\bullet + Q_{29}^\bullet - Q_{28}^\bullet + Q_{27}^\bullet + Q_{26}^\bullet - Q_{25}^\bullet \\ &- Q_{16}^\bullet + Q_{15}^\bullet + Q_{14}^\bullet - Q_{13}^\bullet + Q_{12}^\bullet - Q_{11}^\bullet - Q_{10}^\bullet + Q_{9}^\bullet \\ &+ Q_{24}^\bullet - Q_{23}^\bullet - Q_{22}^\bullet + Q_{21}^\bullet - Q_{20}^\bullet + Q_{19}^\bullet + Q_{18}^\bullet - Q_{17}^\bullet \\ &- Q_{8}^\bullet + Q_{7}^\bullet + Q_{6}^\bullet - Q_{5}^\bullet + Q_{4}^\bullet - Q_{3}^\bullet - Q_{2}^\bullet + Q_{1}^\bullet - \end{aligned}$$

$$-\begin{bmatrix} +\sum[32^\bullet] -\sum[31^\bullet] -\sum[30^\bullet] +\sum[29^\bullet] -\sum[28^\bullet] +\sum[27^\bullet] +\sum[26^\bullet] -\sum[25^\bullet] \\ -\sum[16^\bullet] +\sum[15^\bullet] +\sum[14^\bullet] -\sum[13^\bullet] +\sum[12^\bullet] -\sum[11^\bullet] -\sum[10^\bullet] +\sum[9^\bullet] \\ +\sum[24^\bullet] -\sum[23^\bullet] -\sum[22^\bullet] +\sum[21^\bullet] -\sum[20^\bullet] +\sum[19^\bullet] +\sum[18^\bullet] -\sum[17^\bullet] \\ -\sum[8^\bullet] +\sum[7^\bullet] +\sum[6^\bullet] -\sum[5^\bullet] +\sum[4^\bullet] -\sum[3^\bullet] -\sum[2^\bullet] +\sum[1^\bullet] \end{bmatrix} P_f$$

37. WAVE p_3 $\qquad \left[P_3^\bullet = R_3 \cos(\omega_3 t + \varphi_3) = a_3^\bullet A_3 - b_3^\bullet B_3\right]$

(47)

$$2\begin{bmatrix} +\sum[28^\bullet] -\sum[27^\bullet] -\sum[26^\bullet] +\sum[25^\bullet] -\sum[20^\bullet] +\sum[19^\bullet] +\sum[18^\bullet] -\sum[17^\bullet] \\ -\sum[12^\bullet] +\sum[11^\bullet] +\sum[10^\bullet] -\sum[9^\bullet] +\sum[4^\bullet] -\sum[3^\bullet] -\sum[2^\bullet] +\sum[1^\bullet] \end{bmatrix} P_3 =$$

$$\begin{aligned} = &+ Q_{32}^\bullet - Q_{31}^\bullet - Q_{30}^\bullet + Q_{29}^\bullet - Q_{24}^\bullet + Q_{23}^\bullet + Q_{22}^\bullet - Q_{21}^\bullet \\ &- Q_{16}^\bullet + Q_{15}^\bullet + Q_{14}^\bullet - Q_{13}^\bullet + Q_{8}^\bullet - Q_{7}^\bullet - Q_{6}^\bullet + Q_{5}^\bullet \\ &+ Q_{28}^\bullet - Q_{27}^\bullet - Q_{26}^\bullet + Q_{25}^\bullet - Q_{20}^\bullet + Q_{19}^\bullet + Q_{18}^\bullet - Q_{17}^\bullet \\ &- Q_{12}^\bullet + Q_{11}^\bullet + Q_{10}^\bullet - Q_{9}^\bullet + Q_{4}^\bullet - Q_{3}^\bullet - Q_{2}^\bullet + Q_{1}^\bullet - \end{aligned}$$

$$-\begin{bmatrix} +\sum[32^\bullet] -\sum[31^\bullet] -\sum[30^\bullet] +\sum[29^\bullet] -\sum[24^\bullet] +\sum[23^\bullet] +\sum[22^\bullet] -\sum[21^\bullet] \\ -\sum[16^\bullet] +\sum[15^\bullet] +\sum[14^\bullet] -\sum[13^\bullet] +\sum[8^\bullet] -\sum[7^\bullet] -\sum[6^\bullet] +\sum[5^\bullet] \\ +\sum[28^\bullet] -\sum[27^\bullet] -\sum[26^\bullet] +\sum[25^\bullet] -\sum[20^\bullet] +\sum[19^\bullet] +\sum[18^\bullet] -\sum[17^\bullet] \\ -\sum[12^\bullet] +\sum[11^\bullet] +\sum[10^\bullet] -\sum[9^\bullet] +\sum[4^\bullet] -\sum[3^\bullet] -\sum[2^\bullet] +\sum[1^\bullet] \end{bmatrix} P_f$$

38. WAVE p_2

$$\left[P_2^{\bullet} = R_2 \cos\left(\omega_2 t + \varphi_2\right) = a_2^{\bullet} A_2 - b_2^{\bullet} B_2\right]$$

(48)

$$2\left[\begin{array}{l} +\sum[30^{\bullet}] -\sum[29^{\bullet}] -\sum[26^{\bullet}] +\sum[25^{\bullet}] -\sum[22^{\bullet}] +\sum[21^{\bullet}] +\sum[18^{\bullet}] -\sum[17^{\bullet}] \\ \\ -\sum[14^{\bullet}] +\sum[13^{\bullet}] +\sum[10^{\bullet}] -\sum[9^{\bullet}] +\sum[6^{\bullet}] -\sum[5^{\bullet}] -\sum[2^{\bullet}] +\sum[1^{\bullet}] \end{array}\right] P_2 =$$

$$\begin{array}{l} = + Q_{32}^{\bullet} - Q_{31}^{\bullet} - Q_{28}^{\bullet} + Q_{27}^{\bullet} - Q_{24}^{\bullet} + Q_{23}^{\bullet} + Q_{20}^{\bullet} - Q_{19}^{\bullet} - \\ \quad - Q_{16}^{\bullet} + Q_{15}^{\bullet} + Q_{12}^{\bullet} - Q_{11}^{\bullet} + Q_{8}^{\bullet} - Q_{7}^{\bullet} - Q_{4}^{\bullet} + Q_{3}^{\bullet} \\ \quad + Q_{30}^{\bullet} - Q_{29}^{\bullet} - Q_{26}^{\bullet} + Q_{25}^{\bullet} - Q_{22}^{\bullet} + Q_{21}^{\bullet} + Q_{18}^{\bullet} - Q_{17}^{\bullet} \\ \quad - Q_{14}^{\bullet} + Q_{13}^{\bullet} + Q_{10}^{\bullet} - Q_{9}^{\bullet} + Q_{6}^{\bullet} - Q_{5}^{\bullet} - Q_{2}^{\bullet} + Q_{1}^{\bullet} \end{array}$$

$$-\left[\begin{array}{l} +\sum[32^{\bullet}] -\sum[31^{\bullet}] -\sum[28^{\bullet}] +\sum[27^{\bullet}] -\sum[24^{\bullet}] +\sum[23^{\bullet}] +\sum[20^{\bullet}] -\sum[19^{\bullet}] \\ \\ -\sum[16^{\bullet}] +\sum[15^{\bullet}] +\sum[12^{\bullet}] -\sum[11^{\bullet}] +\sum[8^{\bullet}] -\sum[7^{\bullet}] -\sum[4^{\bullet}] +\sum[3^{\bullet}] \\ \\ +\sum[30^{\bullet}] -\sum[29^{\bullet}] -\sum[26^{\bullet}] +\sum[25^{\bullet}] -\sum[22^{\bullet}] +\sum[21^{\bullet}] +\sum[18^{\bullet}] -\sum[17^{\bullet}] \\ \\ -\sum[14^{\bullet}] +\sum[13^{\bullet}] +\sum[10^{\bullet}] -\sum[9^{\bullet}] +\sum[6^{\bullet}] -\sum[5^{\bullet}] -\sum[2^{\bullet}] +\sum[1^{\bullet}] \end{array}\right] P_f$$

39. WAVE P_1

$$\left[P_1^\bullet = R_1 \cos\left(\omega_1 t + \varphi_1\right) = a_1^\bullet A_1 - b_1^\bullet B_1\right] \tag{49}$$

$$2\left[\begin{array}{l} +\sum[31^\bullet] -\sum[29^\bullet] -\sum[27^\bullet] +\sum[25^\bullet] -\sum[23^\bullet] +\sum[21^\bullet] +\sum[19^\bullet] -\sum[17^\bullet] \\ \\ -\sum[15^\bullet] +\sum[13^\bullet] +\sum[11^\bullet] -\sum[9^\bullet] +\sum[7^\bullet] -\sum[5^\bullet] -\sum[3^\bullet] +\sum[1^\bullet] \end{array}\right] P_1 =$$

$$\begin{aligned} = &+Q_{32}^\bullet - Q_{30}^\bullet - Q_{28}^\bullet + Q_{26}^\bullet - Q_{24}^\bullet + Q_{22}^\bullet + Q_{20}^\bullet - Q_{18}^\bullet \\ &- Q_{16}^\bullet + Q_{14}^\bullet + Q_{12}^\bullet - Q_{10}^\bullet + Q_{8}^\bullet - Q_{6}^\bullet - Q_{4}^\bullet + Q_{2}^\bullet \\ &+ Q_{31}^\bullet - Q_{29}^\bullet - Q_{27}^\bullet + Q_{25}^\bullet - Q_{23}^\bullet + Q_{21}^\bullet + Q_{19}^\bullet - Q_{17}^\bullet \\ &- Q_{15}^\bullet + Q_{13}^\bullet + Q_{11}^\bullet - Q_{9}^\bullet + Q_{7}^\bullet - Q_{5}^\bullet - Q_{3}^\bullet + Q_{1}^\bullet - \end{aligned}$$

$$-\left[\begin{array}{l} +\sum[32^\bullet] -\sum[30^\bullet] -\sum[28^\bullet] +\sum[26^\bullet] -\sum[24^\bullet] +\sum[22^\bullet] +\sum[20^\bullet] -\sum[18^\bullet] \\ \\ -\sum[16^\bullet] +\sum[14^\bullet] +\sum[12^\bullet] -\sum[10^\bullet] +\sum[8^\bullet] -\sum[6^\bullet] -\sum[4^\bullet] +\sum[2^\bullet] \\ \\ +\sum[31^\bullet] -\sum[29^\bullet] -\sum[27^\bullet] +\sum[25^\bullet] -\sum[23^\bullet] +\sum[21^\bullet] +\sum[19^\bullet] -\sum[17^\bullet] \\ \\ -\sum[15^\bullet] +\sum[13^\bullet] +\sum[11^\bullet] -\sum[9^\bullet] +\sum[7^\bullet] -\sum[5^\bullet] -\sum[3^\bullet] +\sum[1^\bullet] \end{array}\right] P_f$$

H) EXPLICIT SOLUTIONS OF THE TIDAL COMPONENT WAVES.

40. Equations (37) and (44), fully solve the wave $p_f = (p_6)$
" (38) " (45), " " " " p_5
" (39) " (46), " " " " p_4
" (40) " (47), " " " " p_3
" (41) " (48), " " " " p_2
" (42) " (49), " " " " p_1

41. To set an example, Eqs. (37) and (44) can be written in short as follows:

$$\sum \left[\sum [\ ] \right] p_f = \sum Q \tag{37A}$$

$$\sum \left[\sum [\ \cdot] \right] p_f^{\cdot} = \sum Q^{\cdot} \tag{44A}$$

where the sums must be taken under the real signs they have into the detailed Eqs. (37) and (44). Expanding both of them by introducing the more explicit form given by Eq. (4), it is found that

$$\sum \left[\sum a_f \right] A_f - \sum \left[\sum b_f \right] B_f = \sum Q \tag{37B}$$

$$\sum \left[\sum a_f^{\cdot} \right] A_f - \sum \left[\sum b_f^{\cdot} \right] B_f = \sum Q^{\cdot} \tag{44B}$$

All the terms and coefficients written in these equations are known, except A_f and B_f which now can be solved and from them to bring out the values of φ_f and R_f by means of Eqs. (5) and (6).

42. Once A_f and B_f have been determined, in a quite similar way are solved the other components waves by using the pairs of equations mentioned in Sec. 40.

43. If in order to analyse the SAME tidal components from time to time the SAME filtering schemes are used, say as those schemes shown in Figs. 4 and 5, and moreover, if the summation's interval of time t_0 is ALWAYS kept the SAME, then the values of the sums $\sum[a_i], \sum[b_i], \sum[a_i^{\bullet}], \sum[b_i^{\bullet}]$ in their several positions given in Eqs. (36) and (43), will ALWAYS remain the SAME; then, they might be computed once and for ever.

Therefore, on a new tidal record will be necessary to compute only the sum of ordinates, Q and $Q^{\bullet}$, which are achived very fast, if the zone to be measured is framed by a template whose free gap be as wide as exactly t_0 and within it are marked the position of the ordinates to add.

It has been observed that in all the cases there must be solved only two simple linear equations, with two unknows each one. This is very easily performed with a small desk computer, thus avoiding the study of complicated programs and the preparation of punched cards to feed the modern electronic computers.

Moreover, the computation control is better made while reckoning is progressing.

I) INSTRUMENTAL DRIFT.

44. Throughout the theory developed to filter one by one the various tidal components of the tidal wave, it has been considered as reference an arbitrary u-u axis parallel to the true v-v axis, (Figs. 1,2,3) , thus meaning in turn that z is constant.

However, the equipments are, by their own making characteristics and stability of the materials used in, sensitive to aging, in such a way that a drift of the zero instrumental occurs.

An study of the variation of the drift through the time brings out a close idea about the excellence and reliability of the equipment itself.

Drift can be linear, parabolic, erratic and sometimes periodic. In a general sense, a good instrument has a linear drift whose value must be very small with almost no variations. Erratic or periodic drift are very difficult to eliminate within the equations set and they characterize unreliable instruments.

Henceforth, linear drift will be discussed; it will be demonstrated that within the absolute general filtering scheme in this work a linear drift is also absolutely filtered, and the results are free of linear drift, whatever it be.

45. Suppose that Q_T be the recorded tidal wave, (Fig. 6), and u-u the arbitrary axis of reference from which values Q_A and Q_B are obtained within the interval of time t_o, in accordance with the theory formerly developed. Let also that $T_{(i)}$ be the space of time between their origins.

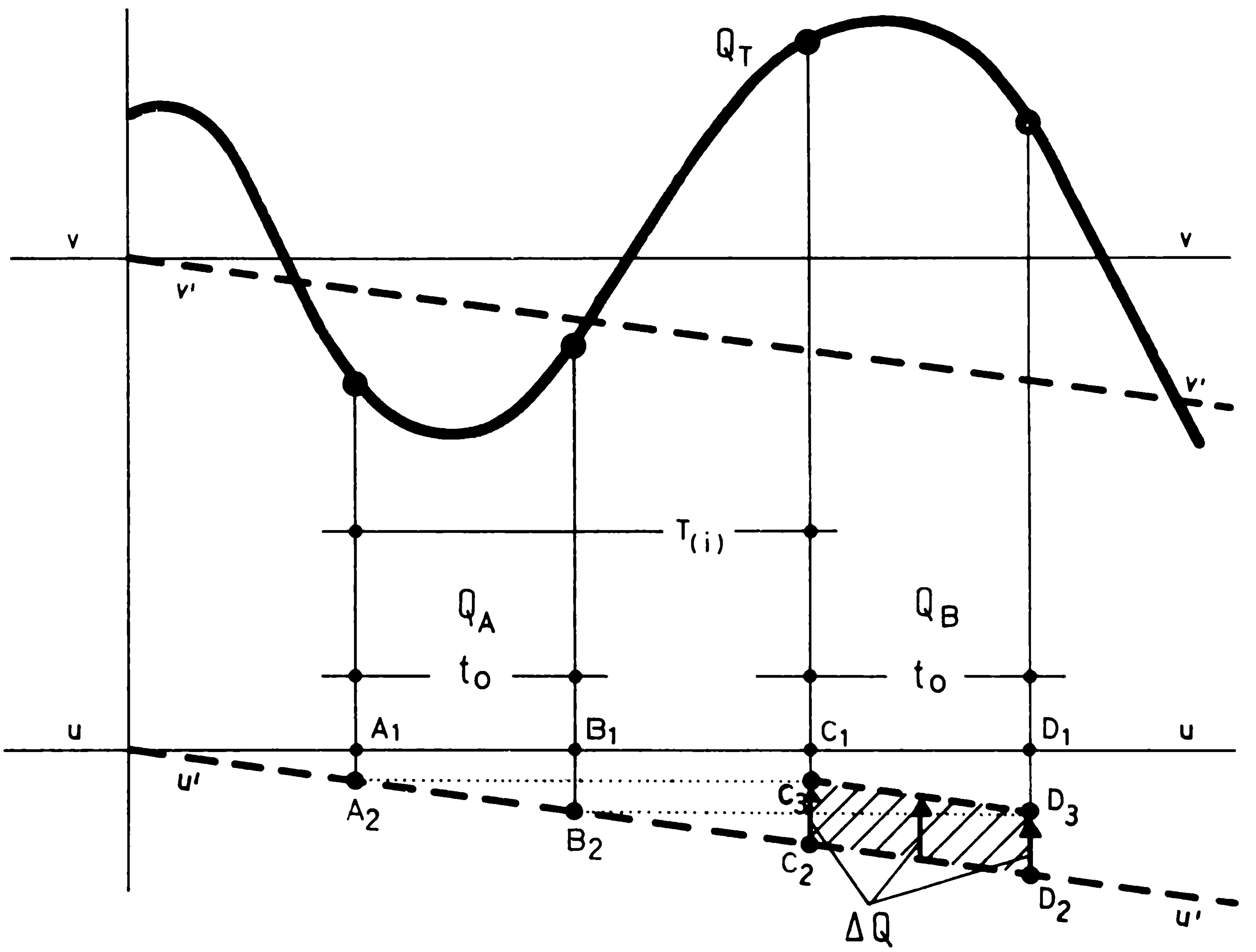

FIG. 6

If the instrument has a linear drift, then the true reference axis should be, say, v'-v' , instead of the v-v axis. Then, the sum of ordinates Q_A and Q_B had to be computed from a new u'-u' axis, parallel to the v'-v' axis. But as the u'-u' axis is fully unknown, the sums Q_A and Q_B have errors respectively given by the ordinates within the trapezoidal surfaces $A_1\ A_2\ B_1\ B_2$ and $C_1 C_2 D_1 D_2$

Individually, each error is different and the difference between both of them is the sum of ordinates within the quadrangular surface $C_2\ C_3\ D_2\ D_3$, which from here on shall be denoted by ΔQ.

46. If an identical scheme as that shown in Fig. 6 is repeated at any place of the record, other values Q'_A and Q'_B should be attained; but the difference of errors, by using the arbitrary u–u axis instead of the true u'–u' axis, will still remain the same as ΔQ.

47. AUTOMATIC ELIMINATION OF A LINEAR INSTRUMENTAL DRIFT, WITHIN THE GENERAL SCHEME OF FILTERING.

Let us consider the Eqs. (37) and (44) which yield to the solution of the tidal component p_f . From here on, what is to be said for the component wave p_f is also valid for every other tidal component.

It should be observed that in the equations mentioned above, there exist 16 pairs of identical positions located at different places of the tidal record. (Figs. 4 and 5). They respectively are:

[1] , [2]	[1•] , [2•]
[3] , [4]	[3•] , [4•]
---------	---------
---------	---------
[29] , [30]	[29•] , [30•]
[31] , [32]	[31•] , [32•]

According to the expressions (37) and (44) , these 16 pairs of positions enter in the equations with the following signs:

[32] – [31]	[32•] – [31•]
– ([30] – [29])	– ([30•] – [29•])
– ([28] – [27])	– ([28•] – [27•])
[26] – [25]	[26•] – [25•]
– ([24] – [23])	– ([24•] – [23•])

$$
\begin{array}{ll}
[22] - [21] & [22^{\bullet}] - [21^{\bullet}] \\
[20] - [19] & [20^{\bullet}] - [19^{\bullet}] \\
-([18] - [17]) & -([18^{\bullet}] - [17^{\bullet}]) \\
-([16] - [15]) & -([16^{\bullet}] - [15^{\bullet}]) \\
[14] - [13] & [14^{\bullet}] - [13^{\bullet}] \\
[12] - [11] & [12^{\bullet}] - [11^{\bullet}] \\
-([10] - [9]) & -([10^{\bullet}] - [9^{\bullet}]) \\
[8] - [7] & [8^{\bullet}] - [7^{\bullet}] \\
-([6] - [5]) & -([6^{\bullet}] - [5^{\bullet}]) \\
-([4] - [3]) & -([4^{\bullet}] - [3^{\bullet}]) \\
[2] - [1] & [2^{\bullet}] - [1^{\bullet}]
\end{array}
$$

In all the cases each pair supplies, by subtraction, a difference of errors which amounts ΔQ . Since there are 8 positive and 8 negative pairs, the final addition cancels each other all the ΔQ , and the equations remain referred to the real own v-v axis.

This is also true, if instead of six tidal constituents a different number of them is considered.

J) TIDAL COMPONENTS.

48. It is easily understood that the schemes of absolute filtering already explained, can be expanded to any number of independent tidal components, no matter how many of them. (Sec.6).

However, in a general way, only eight waves, known as principal tidal constituents, are analysed from the tidal record. Particularly, four semi-diurnal and four diurnal tidal components are chosen. They are

TABLE 1

CLASS	Symbol	$\mathcal{R}_i$ (1) Mean semi-amplitude	T_i PERIOD (Mean time)	NAME
SEMI-DIURNAL	K_2	0.1152	11.96724 hs	LUNI-SOLAR
	S_2	0.4227	12.00000 "	PRINCIPAL SOLAR
	M_2	0.9085	12.42060 "	PRINCIPAL LUNAR
	N_2	0.1759	12.65835 "	LUNAR ELLIPTICITY
DIURNAL	K_1	0.5305	23.93447 "	LUNI - SOLAR
	P_1	0.1755	24.06589 "	SOLAR DECLINATION
	O_1	0.3771	25.81935 "	LUNAR DECLINATION
	Q_1	0.0730	26.86836 "	LUNAR DIURNAL

(1) The mean value $\mathcal{R}_i$ is related to the actual value R_i by means of the node factor, f, being $R_i = f \cdot \mathcal{R}_i$

Since the semi-diurnal tidal components S_2 and K_2, and the diurnal tidal components K_1 and P_1, have almost the same period, to attain a good separation of each one in every pair it is necessary to depend on long period records, say at least one of a quarter of a year, unless another method be used. (See example at the end).

In practice, when a few days record is analysed, each constituent in the former pairs of tidal components separates each other so little, that without losing generality it is usual to involve each pair of tidal components as if it were only one wave acting with the average period of the couple considered, say

$$(K_2 + S_2) = 11.98362 \text{ hours}$$

$$(K_1 + P_1) = 24.00018 \text{ "}$$

These periods are even used on records lasting one month.

Having joined those tidal components, then the Table I is reduced to only six tidal components.

Since the theory has been developed for six component waves, the equations written are ready to be used by applying the absolute filtering scheme.

L) SOME IMPORTANT REMARKS.

49. According to the filtering system, only constituent tidal waves with constant amplitude and period, can be eliminated. Nevertheless, on a short period record, (one or two weeks long), such a restriction can be overlooked within some limits, and still obtain a good separation in tidal waves. (See the example at the end of this work).

50. If we have less than six waves on a filtering scheme, the equations from (37) to (42) and from (44) to (49) are still usables.

Suppose that in the schemes shown in Figs. 4 and 5, only three waves are present, say p_1 , p_2 and p_f . This means that the waves p_3 , p_4 and p_5 are null, which in turn means that positions from [5] to [32] and from [5•] to [32•] are zero. Then, the three solutions for each wave, are simply, from Fig. 4:

WAVE p_f (50)

$$\left(\sum[4]-\sum[3]-\sum[2]+\sum[1]\right) P_f = Q_4 - Q_3 - Q_2 + Q_1$$

WAVE p_2 (51)

$$\left(\sum[4]-\sum[3]+\sum[2]-\sum[1]\right) P_2 = Q_4 - Q_3 + Q_2 - Q_1 - \left(\sum[4]-\sum[3]+\sum[2]-\sum[1]\right) P_f$$

WAVE p_1 (52)

$$\left(\sum[4]+\sum[3]-\sum[2]-\sum[1]\right)P_1 = Q_4+Q_3-Q_2-Q_1-\left(\sum[4]+\sum[3]-\sum[2]-\sum[1]\right)P_f$$

Another set of similar equations must be written from a reiterated scheme, Fig. 5, in order to reach a full solution of each tidal component.

51. Values of Q and $Q^{\bullet}$ can be obtained from the tidal record, in every position shown in Figs. 4 and 5, either by reading only one ordinate or by the sum of equally spaced ordinates within the interval t_0.

Hence

$$Q = \sum_{1}^{n} q_u$$

$$a_i = \sum_{1}^{n} \cos \omega_i \left[t+(n-1)\Delta t\right]$$

$$b_i = \sum_{1}^{n} \operatorname{sen} \omega_i \left[t+(n-1)\Delta t\right]$$

where Δt is the time space chosen within the interval t_0 ; generally Δt is one or half an hour. It is understood without any further comment the more ordinates we add, the better the results.

As it depends on the quality of the tidal record, it is not possible to give a general rule.

52. In Sec. 21 it had been said that the filtering and final solution of a scheme with several tidal waves, was independent of the order they are considered.

However, in practice, some precautions must be taken. In fact: suppose two waves, p_y and p_x , whose periods are respectively T_y and T_x Assume that $T_y \equiv 2T_x$; then, if on a filtering scheme such as that of Fig. 4 or Fig. 5, the wave p_y is filtered before or after the wave p_x , this one is also absolutely filtered, since $T_y \equiv 2T_x$.

Neither an even number of T_x periods can be considered. In that case, when p_x is filtered, p_y is also automatically filtered.

When this special case is present, a full solution of the waves p_y and p_x is attained if p_y is solved as a final wave, say p_f , and p_x is taken at the moment of filtering with an odd number of periods, say 1, 3, 5, ... etc.

Moreover, if the waves p_x and p_y were the only two waves of a filtering scheme, then the wave p_y can be fully solved while p_x remains unsolved. A complete solution is reached by introducing a fictitious wave W_F whose amplitude should be zero, and its period chosen quite apart from p_x and p_y. The waves to filter should now be in this order: W_f , p_x and p_y .

The case considered corresponds to the waves K_2 and K_1 , and also practically to the waves $(S_2 + K_2)$ and $(P_1 + K_1)$.

M) APPLICATION OF ABSOLUTE FILTERING METHOD.

53. We have prepared a tidal record based on theoretical values, computed hour by hour, covering seven days; from May 1/1970 at 3 H. U.T. to May 8/1970 at 3 H.U.T.

The analysis method, starting from the tidal record, will be performed in two steps.

1º) Separating the lunar and solar diurnal and semidiurnal waves,

2º) Obtaining the principal tidal components from every of the former waves, taking into account that the analysis is performed from a short period record, namely seven days.

	1rs. Step	2nd. Step
Tidal Record	Lunar Diurnal Wave	O_1 , $[K_1]$, Q_1
	Lunar Semi - diurnal Wave	M_2 , N_2 , $[K_2]$
	Solar Diurnal Wave	P_1 , $[K_1]$
	Solar Semi - diurnal Wave	S_2 , $[K_2]$

54. Mean speed per solar hour, (ω_L), and period (T_L) of the Moon.

May 1/1970 at 3 H.U.T.	$\alpha = 22^h\ 53^m\ 23^s.940$
May 8/1970 at 3 H.U.T.	$\alpha = 5\ \ 04\ \ 34.728$
	$\Delta\alpha = 6\ \ 11\ \ 10.788$
	$= 371.1798$ min.

$\Delta\alpha$ / day $= 53^m.025686$

$\Delta\alpha^\circ$/ day $= 13^\circ.256421$

$\Delta\alpha^\circ$/ hour $= 0^\circ.552351$

Being $\omega = 15^\circ.041069$ the mean speed per hour of the sun, it must be $(15^\circ.041069 - 0^\circ.552351)\ T_L = 360^\circ$ and from there

$T_L = 24^h.8469$

$\omega_L = 14^\circ.48873$

Diurnal

it immediately follows that

$T_L = 12^h.42345$

$\omega_L = 28^\circ.97746$

Semidiurnal

55. Mean speed per solar hour, (ω_s) and Period (T_s) of the Sun.

May 1/1970 at 3 H.U.T.	$\alpha = 2^h\ 21^m\ 39^s.314$
May 8/1970 at 3 H.U.T.	$\alpha = 2\ \ 58\ \ 25.656$
	$\Delta\alpha = 26^m.939033$ min.

$\Delta\alpha /$ day $= 3^m.848433$

$\Delta\alpha^\circ /$ day $= 0^\circ.962108$

$\Delta\alpha^\circ /$ hour $= 0^\circ.040088$

Since $\omega = 15^\circ.041069$ is the mean hourly speed of the sun, then it must be $(15^\circ.041069 - 0^\circ.040088)\ T_s = 360^\circ$ and from there

$$T_s = 23^h.9984$$

$$\omega_s = 15^\circ.00100$$

Diurnal

and consequently

$$T_s = 11^h.9992$$

$$\omega_s = 30^\circ.00200$$

Semidiurnal

56. In Fig. 7, are represented the suitable schemes A and A' (covering practically seven days) to separate the following three tidal waves:

p_1 , solar diurnal, with a period $T_1 = 23^h.99840$

p_2 , lunar semidiurnal, with a period $T_2 = 12.42345$

p_3 , lunar diurnal, with a period $T_3 = 24.84690$

The tidal constituent p_0 (solar semidiurnal) cannot be separated from the former schemes, owing to the fact that at the moment that the tidal waves p_1 (solar diurnal) be filtered, the p_0 is also automatically filtered (See 52).

If we also want to separate the tidal component p_0 , then a fictitious wave can be set up in Fig. 7, on top of the wave p_0 (See 52). But then, we should need eight positions, [1],[2]....... [7],[8] in every scheme to solve the four waves. That means, either to use a record twice as long as the former one, (14 days), or to diminish the accuracy of the results if a seven days record is solely available.

57. In Fig. 7, it can be seen, that all the positions from [1] to [4'] are symmetrical with respect to a center settled on May 4/1970 at 15 H.U.T. From here on this date will be kept as origin af the time, t , which in turn means that all the initial phase angles are given there.

58. The highness of the tide, either produced by the moon or the sun, is expressed by

$$H_T = \frac{3}{4}\,\frac{m}{M}\,\frac{a^4}{\rho_m^3}\left(\frac{\rho_m}{\rho_a}\right)^3 \begin{bmatrix} 3\left(\frac{1}{3}-\mathrm{sen}^2\phi\right)\left(\frac{1}{3}-\mathrm{sen}^2\delta\right)+ & \text{permanent tide} \\ +\cos^2\delta\cos^2\phi\cos 2t_h+ & \text{semidiurnal tide} \\ +\mathrm{sen}\,2\delta\,\mathrm{sen}\,2\phi\cos t_h & \text{diurnal tide} \end{bmatrix} \quad (53)$$

In our latitude, $\phi = -34^\circ\, 54'.5$, the permanent tide term is practically nill, and will be neglected in all the cases. The former formula can be simplified by writing

$$H_T = R\left(\cos^2\delta\,\cos^2\phi\,\cos 2t_h + \mathrm{sen}\,2\delta\,\mathrm{sen}\,2\phi\,\cos t_h\right) \quad (54)$$

wherein R substituted the coefficient in (53), and will assume the values R_s or R_L if it is either the sun or the moon the body taken into account.

Also we must recall that

$$r = \frac{R_S}{R_L} = 0.46 \qquad (55)$$

59. At the recording place, both R and ϕ are constant; and if we rename

$$R_1 = R \cos^2 \phi$$
$$R_2 = R \text{ sen } 2\phi \qquad (56)$$

then

$$H_T = (R_1 \cos 2 t_h) \cos^2 \delta + (R_2 \cos t_h) \text{ sen } 2\delta \qquad (57)$$

The waves $R_1 \cos 2t_h$ and $R_2 \cos t_h$, depend only on the hour angle t_h We can refer t_h at any origin by knowing just the time t and the initial phase angle φ , and by introducing the hourly speed ω , either of the sun or the moon.

That means

$$R_1 \cos 2t_h = R_1 \cos(\omega_1 t + \varphi_1) = a_1 A_1 - b_1 B_1$$
$$R_2 \cos t_h = R_2 \cos(\omega_2 t + \varphi_2) = a_2 A_2 - b_2 B_2 \qquad (58)$$

$$\omega_1 = 2\,\omega_2$$

Going back to (57), it follows

$$H_T = (a_1 A_1 - b_1 B_1) \cos^2 \delta + \quad \text{.........semidiurnal tide}$$
$$+ (a_2 A_2 - b_2 B_2) \text{ sen } 2\delta \quad \text{.........diurnal tide} \qquad (59)$$

The filtering system already explained in the text, corresponds all along to waves with constant amplitude and hourly speed. This should be the case in (59) , if δ = constant.

But the tidal wave coming from a record has involved the semidiurnal and diurnal waves of the moon and the sun, whose declinations are shifting little by little while the time t is increasing. Nevertheless, being A and B constants, our problem will be fully solved if, for every position of the filtering schemes A and A' in Fig. 7, we know the δ_s' both of the moon and the sun. Those values are given in Tables 2 and 5.

In fact: if we put

$$\left.\begin{aligned} a_1' &= a_1 \cos^2 \delta \\ b_1' &= b_1 \cos^2 \delta \end{aligned}\right] \text{ semidiurnals} \qquad \left.\begin{aligned} a_2' &= a_2 \operatorname{sen} 2\delta \\ b_2' &= b_2 \operatorname{sen} 2\delta \end{aligned}\right] \text{ diurnals} \qquad (60)$$

the equation (59) becomes

$$H_T = (a_1' A_1 - b_1' B_1) + (a_2' A_2 - b_2' B_2)$$

Now, as a_1' , b_1' , a_2' , b_2' are known, then the unknowns A_1 , B_1 , A_2 , B_2 , can be obtained by using the filtering system, and the problem comes fully solved.

60. However, it must be taken into account that in the present circumstances, the filtering of any tidal wave is not really absolute, specially in the moon case, since we have considered a constant hourly speed of the body through out seven days. This is not actually so, because within this period, small variations in speed have ocurred.

Even so, we believe that the filtering of a wave is well within 98% instead of 100%; this is the real reason why we have taken a time span of only seven days, to avoid larger variations in the body speed.

Previous calculations performed on only five days have brought out excellent results too.

61. To start with the solution of the problem, the waves to be filtered (one by one) have been properly ordered; firstly those of the sun, which have the smaller variations in δ and ω , and then the lunar semidiurnal on account that $\cos^2 \delta$ represents smaller variations than sen 2δ as the time t is going on. The lunar diurnal constituent is the free or final wave and is obtained quite independently of all the others.

In a first attempt we obtain for every tidal component values of R and φ close to the real ones. A second step is performed by using the Δ's values in order to attain values of R and φ nearer to the actual ones.

62. Tables 3 and 4, corresponding to the schemes A and A' coming from Fig. 7, present all the necessary elements to enter in the computation. In each position three ordinates have been read on the record, and the

figures for a, b , and Q means:

$$a = \cos \omega (t - 1) + \cos \omega t + \cos \omega (t + 1)$$
$$b = \text{sen}\, \omega (t - 1) + \text{sen}\, \omega t + \text{sen}\, \omega (t + 1)$$
$$Q = q_{u-1} + q_u + q_{u+1}$$ (in units printed on the recording paper)

The separation from $(t - 1)$ to t, and from t to $(t + 1)$ is one solar hour; t, is at the middle point of every position, from [1] to [4•] . In the Tables 3 and 4, also stand the values Δ , which mean either $(a - a')$ or $(b - b')$. The proper use of Δ will be later explained in pag. 58.

63. Finally, the reader will be awared that continuous tidal records are not necessary in most of the cases, since their central part is not used at all.
So, discontinuous tidal records can also be useful if the computer deals with them in a profitable way.

64. The explanation given above, permite us to enter directly to the calculations.

SCHEMES A and A•

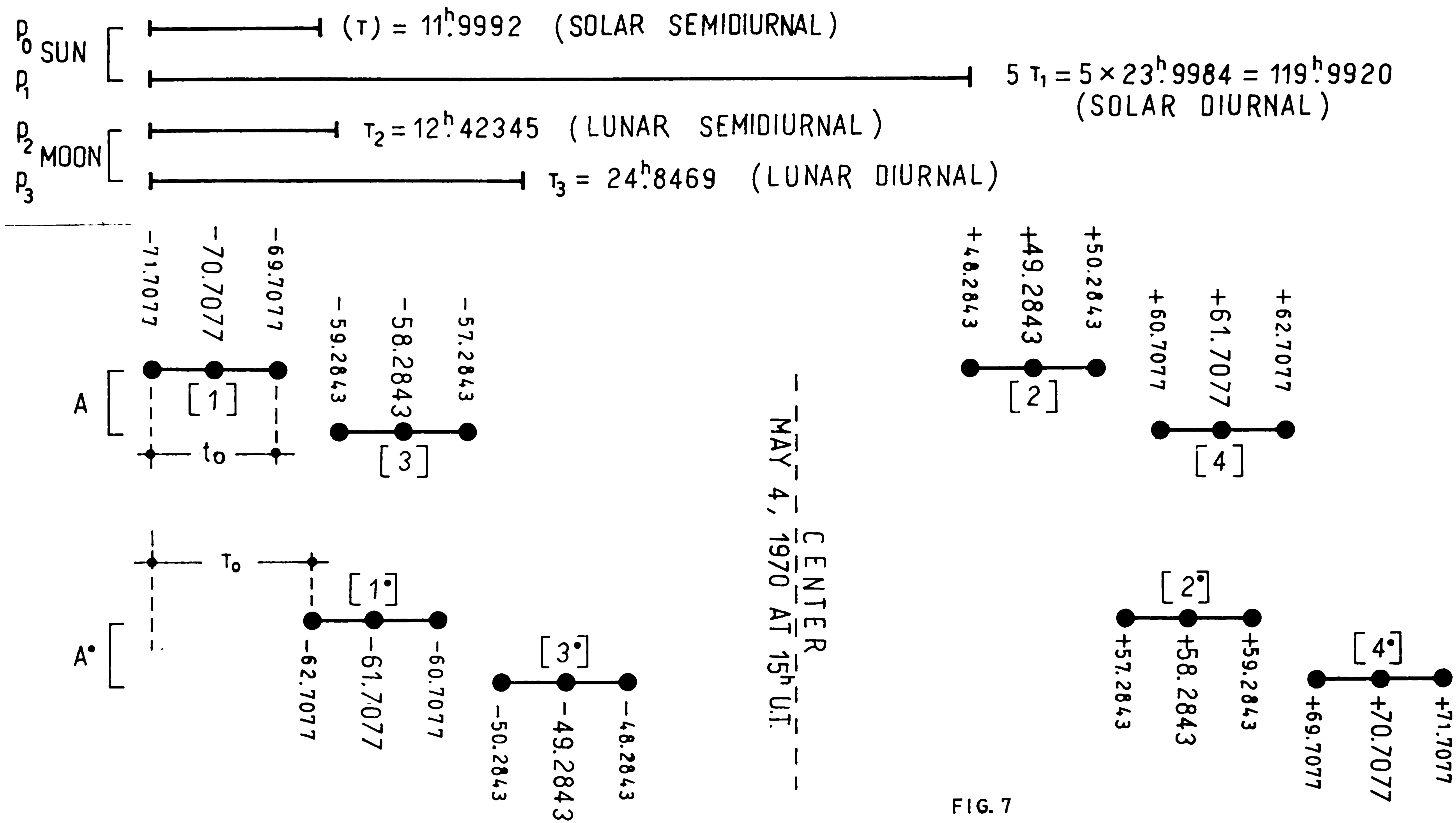

FIG. 7

TABLE 2

SCHEME	Posit.	t (average)	Date & Time (U.T.)	MOON δ	MOON $\cos^2\delta$	MOON sen 2δ	SUN δ	SUN $\cos^2\delta$	SUN sen 2δ
A	[1]	$-70^h.7077$	1970 May 1 at $16^h.2923$	$-3^\circ.2228$	0.9968	-0.1123	$+15^\circ.0894$	0.9322	+0.5027
	[2]	+49.2843	May 6 at 16.2841	+24.7519	0.8247	+0.7604	+16.5483	0.9189	+0.5460
	[3]	-58.2843	May 2 at 4.7157	+ 0.2444	1.0000	+0.0085	+15.2456	0.9308	+0.5074
	[4]	+61.7077	May 7 at 4.7077	+26.1731	0.8054	+0.7917	+16.6931	0.9175	+0.5503
A•	[1•]	-61.7077	May 2 at 1.2923	-0.7097	0.9998	-0.0248	+15.2030	0.9312	+0.5061
	[2•]	+58.2843	May 7 at 1.2843	+25.8511	0.8099	+0.7848	+16.6535	0.9179	+0.5491
	[3•]	-49.2843	May 2 at 13.7157	+ 2.7522	0.9977	+0.0959	+15.3578	0.9299	+0.5108
	[4•]	+70.7077	May 7 at 13.7077	+27.0311	0.7934	+0.8096	+16.7968	0.9165	+0.5533

TABLE 3

t (hours)	Position	Calculus' elements	SOLAR SEMIDIURNAL (T)=11ʰ.99920 ω_o=30°.00200			$\cos^2 \delta$	SOLAR DIURNAL T_1 =23ʰ.99840 ω_1= 15°.00100			sen 2δ
-71ʰ.7077		ωt	8°.6248	38°.6268	68°.6288	+0.9322	4°.3124	19°.3134	34°.3144	+0.5027
-70.7077	[1]	a b	+2.1345	+1.7052			+2.7669	+0.9695		
-69.7077		a' b'	+1.9898	+1.5896			+1.3909	+0.4874		
		Δa Δb	+0.1447	+0.1156			+1.3760	+0.4821		
+48ʰ.2843		ωt	8.6248	38.6268	68.6288	+0.9189	4.3124	19.3134	34.3144	+0.5460
+49.2843	[2]	a b	+2.1345	+1.7052			+2.7669	+0.9695		
+50.2843		a' b'	+1.9614	+1.5669			+1.5107	+0.5293		
		Δa Δb	+0.1731	+0.1383			+1.2562	+0.4402		
-59ʰ.2843		ωt	21.3532	51.3552	81.3572	+0.9309	190.6766	205.6776	220.6786	+0.5074
-58.2843	[3]	a b	+1.7059	+2.1340			-2.6422	-1.2705		
-57.2843		a' b'	+1.5880	+1.9865			-1.3407	-0.6447		
		Δa Δb	+0.1179	+0.1475			-1.3015	-0.6258		
+60ʰ.7077		ωt	21.3532	51.3552	81.3572	+0.9175	190.6766	205.6776	220.6786	+0.5503
+61.7077	[4]	a b	+1.7059	+2.1340			-2.6422	-1.2705		
+62.7077		a' b'	+1.5652	+1.9579			-1.4540	-0.6992		
		Δa Δb	+0.1407	+0.1761			-1.1882	-0.5713		

SCHEME A

LUNAR SEMIDIURNAL $T_2 = 12^h.42345$ $\omega_2 = 28^\circ.97746$	$\cos^2\delta$	LUNAR DIURNAL $T_3 = 24^h.84690$ $\omega_3 = 14^\circ.48873$	sen 2δ	q_u	Q
82.0923 111.0697 140.0472 -0.9884 +2.5658 -0.9852 +2.5576 -0.0032 +0.0082	+0.9968	41.0461 55.5349 70.0236 +1.6617 +2.4209 -0.1866 -0.2719 +1.8483 +2.6928	-0.1123	37.05 25.90 16.25	79.20
319.1556 348.1331 17.1106 +2.6909 -0.5654 +2.2192 -0.4663 +0.4717 -0.0991	+0.8247	339.5778 354.0666 8.5553 +2.9206 -0.3036 +2.2208 -0.2309 +0.6998 -0.0727	+0.7604	25.10 27.05 24.70	75.85
82.0923 111.0697 140.0472 -0.9884 +2.5658 -0.9884 +2.5658 0.0000 0.0000	+1.0000	221.0461 235.5349 250.0236 -1.6617 -2.4209 -0.0141 -0.0206 -1.6476 -2.4003	+0.0085	47.00 36.15 24.95	108.10
319.1556 348.1331 17.1106 +2.6909 -0.5654 +2.1672 -0.4554 +0.5237 -0.1100	+0.8054	159.5778 174.0666 188.5553 -2.9206 +0.3036 -2.3122 +0.2404 -0.6084 +0.0632	+0.7917	73.90 77.20 73.00	224.10

TABLE 4

t (hours)	Posit.	Calculus' elements	SOLAR SEMIDIURNAL $(T)=11^{h}.99920$ $\omega_0=30^{\circ}.00200$			$\cos^2\delta$	SOLAR DIURNAL $T_1=23^{h}.99840$ $\omega_1=15^{\circ}.00100$			sen 2δ
$-62^{h}.7077$	[1°]	ω t	$278^{\circ}.6428$	$308^{\circ}.6448$	$338^{\circ}.6468$	+0.9312	$139^{\circ}.3214$	$154^{\circ}.3224$	$169^{\circ}.3234$	+0.5061
-61.7077		a b	+1.7059	-2.1340			-2.6422	+1.2705		
-60.7077		a' b'	+1.5885	-1.9872			-1.3372	+0.6430		
		Δa Δb	+0.1174	-0.1468			-1.3050	+0.6275		
$+57^{h}.2843$	[2°]	ω t	278.6428	308.6448	338.6468	+0.9179	139.3214	154.3224	169.3234	+0.5491
+58.2843		a b	+1.7059	-2.1340			-2.6422	+1.2705		
+59.2843		a' b'	+1.5658	-1.9588			-1.4508	+0.6976		
		Δa Δb	+0.1401	-0.1752			-1.1914	+0.5729		
$-50^{h}.2843$	[3°]	ω t	291.3712	321.3732	351.3752	+0.9299	325.6856	340.6866	355.6876	+0.5108
-49.2843		a b	+2.1345	-1.7052			+2.7669	-0.9695		
-48.2843		a' b'	+1.9849	-1.5857			+1.4133	-0.4952		
		Δa Δb	+0.1496	-0.1195			+1.3536	-0.4743		
$+69^{h}.7077$	[4°]	ω t	291.3712	321.3732	351.3752	+0.9165	325.6856	340.6866	355.6876	+0.5533
+70.7077		a b	+2.1345	-1.7052			+2.7669	-0.9695		
+71.7077		a' b'	+1.9563	-1.5628			+1.5309	-0.5364		
		Δa Δb	+0.1782	-0.1424			+1.2360	-0.4331		

SCHEME A'

LUNAR SEMIDIURNAL $T_2 = 12^h.42345$ $\omega_2 = 28^\circ.97746$	$\cos^2\delta$	LUNAR DIURNAL $T_3 = 24^h.84690$ $\omega_3 = 14^\circ.48873$	sen 2δ	q_u°	Q°
342°.8894 11°.8669 40°.8423 +2.6909 +0.5654 +2.6904 +0.5653 +0.0005 +0.0001	+0.9998	171°.4447 185°.9334 200°.4222 -2.9206 -0.3036 +0.0724 +0.0075 -2.9930 -0.3111	-0.0248	50.75 56.45 56.65	163.85
219.9528 248.9302 277.9077 -0.9885 -2.5658 -0.8006 -2.0780 -0.1879 -0.4878	+0.8099	109.9764 124.4651 138.9539 -1.6617 +2.4209 -1.3041 +1.8999 -0.3576 +0.5210	+0.7848	28.80 44.20 59.50	132.50
342.8894 11.8669 40.8423 +2.6909 +0.5654 +2.6847 +0.5641 +0.0062 +0.0013	+0.9977	351.4447 5.9334 20.4222 +2.9206 +0.3036 +0.2801 +0.0291 +2.6405 +0.2745	+0.0959	42.00 45.15 42.50	129.65
219.9528 248.9302 277.9077 -0.9885 -2.5658 -0.7843 -2.0357 -0.2042 -0.5301	+0.7934	289.9764 304.4651 318.9539 +1.6617 -2.4209 +1.3453 -1.9600 +0.3164 -0.4609	+0.8096	6.85 11.05 16.85	34.75

N) NUMERICAL ANALYSIS I.

65. Schemes A and A• by using coefficients a' and b'.

WAVE ρ_3 - LUNAR DIURNAL

$$[4] - [3] - [2] + [1]$$
$$[4^\bullet] - [3^\bullet] - [2^\bullet] + [1^\bullet]$$

Equations

$$- 4.7055\ A_3 - 0.2200\ B_3 = + 119.35$$
$$+ 2.4417\ A_3 + 3.8815\ B_3 = - 63.55$$

Solution

$B_3 = - 0.43$ $\qquad R = 25.35$

$A_3 = - 25.34$ $\qquad \varphi = 181^\circ.0$

$Tg\ \varphi = + 0.01695$

WAVE ρ_2 - LUNAR SEMIDIURNAL

$$[4] - [3] + [2] - [1]$$
$$[4^\bullet] - [3^\bullet] + [2^\bullet] - [1^\bullet]$$

Equations

$$+ 6.3600\ A_2 + 6.0451\ B_2 = 112.65 - (0.1093\ A_3 - 0.3020\ B_3)$$
$$- 6.9600\ A_2 + 5.2431\ B_2 = -126.25 - (-0.3113\ A_3 + 0.0967\ B_3)$$

Solution

$B_2 = - 0.67$ $\qquad R = 18.78$

$A_2 = +18.76$ $\qquad \varphi = 358^\circ.0$

$Tg\ \varphi = - 0.03564$

WAVE p_1 - SOLAR DIURNAL

$$[4]+[3]-[2]-[1]$$

$$[4^\bullet]+[3^\bullet]-[2^\bullet]-[1^\bullet]$$

Equations

$$- 5.9663\ A_1 + 2.3606\ B_1 = 177.15 - (-4.3605\ A_3 - 0.7226\ B_3)$$
$$5.7322\ A_1 + 2.3722\ B_1 = -131.95 - (\ 2.8571\ A_3 + 3.8383\ B_3)$$

Solution

$B_1 = + 1.26$ $\qquad$ $R = 10.69$

$A_1 = - 10.62$ $\qquad$ $\varphi = 173^\circ.2$

$Tg\ \varphi = - 0.11840$

66. Schemes A and $A^\bullet$
Solutions by using coefficients a and b.

In every of the positions shown in Fig.7, the values of Q and $Q^\bullet$ depend on the actual declination either of the moon or of the sun. This has been the reason why we have used the coefficients a' and b'.

It is easily seen from Eq.(59), that if $\delta \equiv 0$ and then $\cos^2\delta \equiv 1$ or, if $\delta = 45^\circ$ and then sen $2\delta \equiv 1$, the Eq. (59) represents two waves, semidiurnal and diurnal respectively, with constant amplitudes produced by two bodies:one orbiting the equator, and another orbiting on declination 45°.

Waves with constant amplitude are of the type

$$P_i = a_i A_i - b_i B_i \qquad (62)$$

Waves with " actual " amplitude are of the type

$$P'_i = a'_i A_i - b'_i B_i \qquad (63)$$

In both equations above the values A_i and B_i are the same. From them we have already obtained the values R and φ for every tidal component shown in Fig. 7.

By sustracting equations (62) and (63), we have

$$\Delta P_i = (a_i - a_i') A_i - (b_i - b_i') B_i$$

$$= \Delta a_i \, . \, A_i - \Delta b_i \, . \, B_i \qquad (64)$$

which shows the shift of the ordinates in every position, when passing from the " actual " wave to the constant amplitude wave.

If we compute new values $\bar{Q}$ from

$$\bar{Q} = Q + \sum \Delta P_i \qquad (65)$$

then we can separate the tidal waves by using the coefficients a and b instead of the a' and b', and so to repeat the computations.

ΔP_3 of tidal wave p_3 (Eq. 64) - Lunar Diurnal

[1]	(+1.8483)	(-25.34)	-	(+2.6928)	(-0.43)	=	- 45.68
[2]	(+0.6998)	(")	-	(-0.0727)	(")	=	- 17.77
[3]	(-1.6476)	(")	-	(-2.4003)	(")	=	+ 40.73
[4]	(-0.6084)	(")	-	(+0.0632)	(")	=	+ 15.45
[1•]	(-2.9930)	(")	-	(-0.3111)	(")	=	+ 75.72
[2•]	(-0.3576)	(")	-	(+0.5210)	(")	=	+ 9.28
[3•]	(+2.6405)	(")	-	(+0.2745)	(")	=	- 66.80
[4•]	(+0.3164)	(")	-	(-0.4609)	(")	=	- 8.22

ΔP_2 of tidal wave p_2 - Lunar Semidiurnal

[1] (-0.0032)(+18.76) - (+0.0082)(-0.69) = -0.05

[2] (+0.4717)(") - (-0.0991)(") = +8.78

[3] (+0.0000)(") - (-0.0000)(") = 0.00

[4] (+0.5237)(") - (-0.1100)(") = + 9.76

[1•] (+0.0005)(") - (+0.0001)(") = +0.01

[2•] (-0.1879)(") - (-0.4878)(") = -3.86

[3•] (+0.0062)(") - (+0.0013)(") = +0.12

[4•] (-0.2042)(") - (-0.5301)(") = -4.18

ΔP_1 of tidal wave p_1 - Solar Diurnal

[1] (+1.3760)(-10.62) - (+0.4821)(+1.26) = -15.22

[2] (+1.2562)(") - (+0.4402)(") = -13.89

[3] (-1.3015)(") - (-0.6258)(") = +14.61

[4] (-1.8882)(") - (-0.5713)(") = +13.34

[1•] (-1.3050)(") - (+0.6275)(") = +13.07

[2•] (-1.1914)(") - (+0.5729)(") = +11.93

[3•] (+1.3536)(") - (-0.4743)(") = -13.77

[4•] (+1.2360)(") - (-0.4331)(") = -12.59

VALUES OF $\bar{Q}$ AND $\bar{Q}^{\bullet}$

	$Q/Q^{\bullet}$	ΔP_3	ΔP_2	ΔP_1		$\bar{Q}/\bar{Q}^{\bullet}$
[1]	79.20	-45.68	-0.05	-15.22	=	18.25
[2]	75.85	-17.77	+8.78	-13.89	=	52.97
[3]	108.10	+40.73	+0.00	+14.61	=	163.44
[4]	224.10	+15.45	+9.76	+13.34	=	262.65
[1•]	163.85	+75.72	+0.01	+13.07	=	252.65
[2•]	132.50	+ 9.28	-3.86	+11.93	=	149.85
[3•]	129.65	-66.80	+0.12	-13.77	=	49.20
[4•]	34.75	- 8.22	-4.18	-12.59	=	9.76

WAVE ρ_3 - Lunar Diurnal

[4] - [3] - [2] + [1]

[4•] - [3•] - [2•] + [1•]

Equations

$$-2.5178 \; A_3 - 5.4490 \; B_3 = +64.49$$

$$-2.5178 \; A_3 + 5.4490 \; B_3 = +63.36$$

Solution

$B_3 = -0.10$ $\qquad$ $R = 25.39$

$A_3 = -25.39$ $\qquad$ $\varphi = 180^{\circ}.2$

$\mathrm{Tg}\,\varphi = +0.00408$

WAVE p_2 - Lunar Semidiurnal

$$[4] - [3] + [2] - [1]$$

$$[4^\circ] - [3^\circ] + [2^\circ] - [1^\circ]$$

Equations

$$+7.3587\ A_2 + 6.2624\ B_2 = 133.93 - (0\ A_3 - 0\ B_3)$$

$$-7.3587\ A_2 + 6.2624\ B_2 = -142.24 - (0\ A_3 - 0\ B_3)$$

Solution

$B_2 = -0.66$ $\quad\quad$ $R = 18.78$

$A_2 = +18.76$ $\quad\quad$ $\varphi = 358^\circ.0$

$Tg\,\varphi = -0.03536$

WAVE p_1 - Solar Diurnal

$$[4] + [3] - [2] - [1]$$

$$[4^\circ] + [3^\circ] - [2^\circ] - [1^\circ]$$

Equations

$$-10.8182\ A_1 + 4.4800\ B_1 = 354.87 - (-9.1646\ A_3 + 4.2346\ B_3)$$

$$+10.8182\ A_1 + 4.4800\ B_1 = -343.54 - (+9.1646\ A_3 + 4.2346\ B_3)$$

Solution

$B_1 = +1.36$ $\quad\quad$ $R = 10.86$

$A_1 = -10.77$ $\quad\quad$ $\varphi = 172^\circ.8$

$Tg\,\varphi = -0.12650$

P) NUMERICAL ANALYSIS II

67. With the advantage of knowing by the preceding computation the values A_i and B_i of three tidal waves, namely the lunar diurnal, lunar semidiurnal and solar diurnal, we can now start solving another system (which is shown in Fig. 8) in the same way that we made with the former one, but this time beginning with the lunar waves, followed by the solar waves.

This new system, with the schemes B and $B^\bullet$, allow us to separate three tidal waves, among them the solar semidiurnal, which was not obtained before by the reason given in Sec. 56 and 52. It is for the same reason that the wave p_0 in Fig.8, it is to say the lunar semidiurnal, will not appear throughout the computations.

Tables 5, 6 and 7, corresponding to the schemes B and $B^\bullet$ have the same meaning as Tables 2 , 3 and 4, corresponding to the schemes A and $A^\bullet$.

Finally, as values A_i and B_i are formerly known in three out of the four waves, the coefficients a and b can be directly used, provided that before we compute the values ΔP_i , in order to obtain the values $\bar{Q}$ and $\bar{Q}^\bullet$.

SCHEMES B and B°

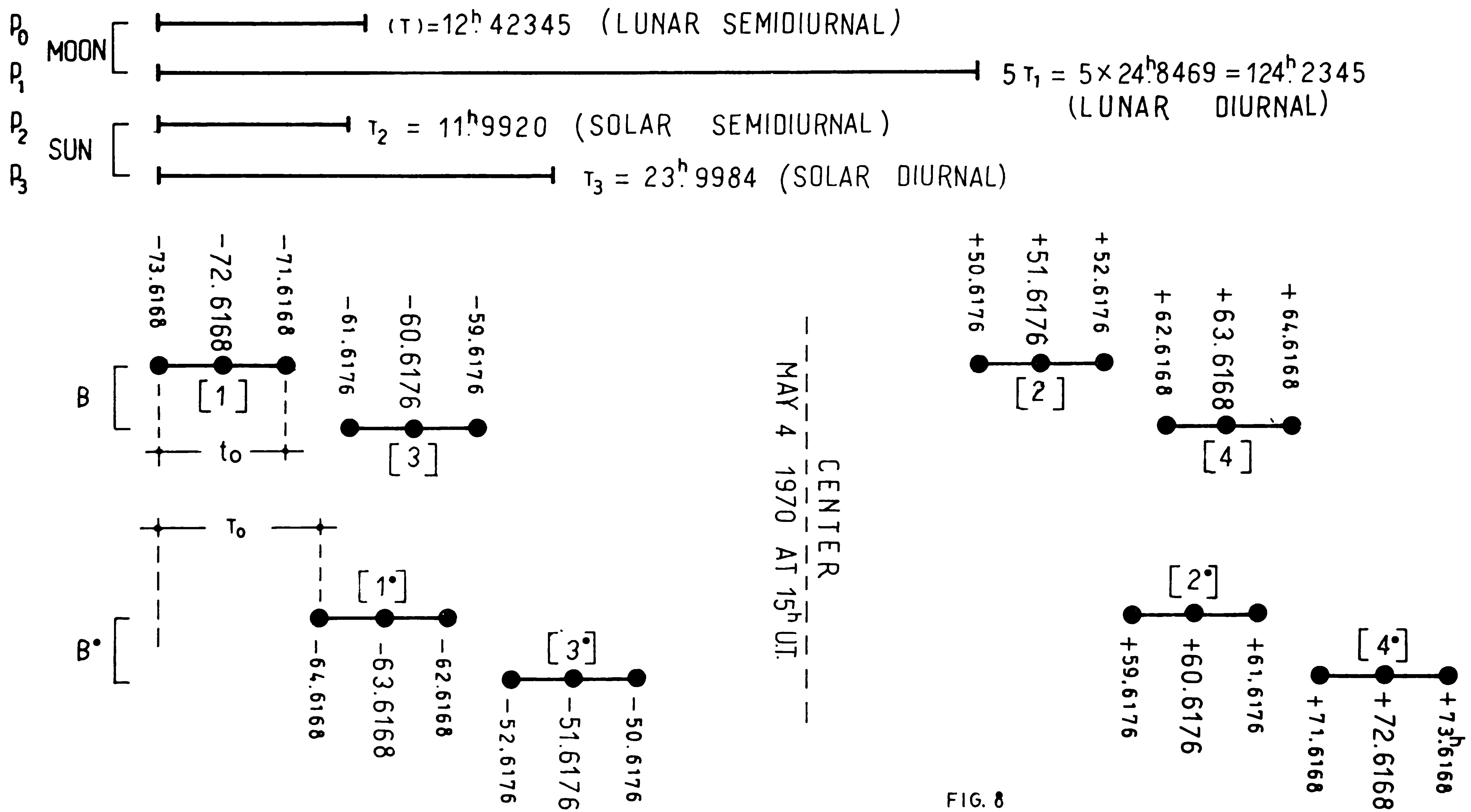

FIG. 8

TABLE 5

SCHEME	Posit.	t (average)	Date & Time (U.T.) 1970	MOON			SUN		
				δ	$\cos^2\delta$	sen 2δ	δ	$\cos^2\delta$	sen 2δ
B	[1]	$-72^h.6168$	May 1 at $14^h.3832$	-3.7570	0.9957	-0.1308	+15.0654	0.9324	+0.5020
	[2]	+51.6176	May 6 at 18.6176	+25.0561	0.8206	+0.7673	+16.5756	0.9186	+0.5468
	[3]	-60.6176	May 2 at 2.3824	-0.4077	1.0000	-0.0142	+15.2166	0.9311	+0.5063
	[4]	+63.6168	May 7 at 6.6168	+26.4061	0.8022	+0.7967	+16.7150	0.9173	+0.5509
B°	[1°]	-63.6168	May 1 at 23.3832	-1.2461	0.9995	-0.0435	+15.1791	0.9314	+0.5054
	[2°]	+60.6176	May 7 at 3.6176	+26.1028	0.8064	+0.7902	+16.6804	0.9176	+0.5499
	[3°]	-51.6176	May 2 at 11.3824	+ 2.1037	0.9986	+0.0734	+15.3287	0.9301	+0.5099
	[4°]	+72.6168	May 7 at 15.6168	+27.1768	0.7914	+0.8126	+16.8189	0.9163	+0.5539

TABLE 6

t (hours)	Posit.	Calculus' elements		LUNAR SEMIDIURNAL (T)=12.^{h}42345 ω_0=28°.97746			$\cos^2\delta$	LUNAR DIURNAL T_1=24.^{h}84690 ω_1=14°.48873			sen 2δ
		ωt		26°.7707	55°.7481	84°.7256		13°.3853	27°.8741	42°.3628	
-73.^{h}6168	[1]	a	b	+1.5476		+2.2728	+0.9957	+2.5957		+1.3729	-0.1308
-72.6168		a'	b'	+1.5409		+2.2630		-0.3395		-0.1796	
-71.6168		Δa	Δb	+0.0067		+0.0098		+2.9352		+1.5525	
		ωt		26.7707	55.7481	84.7256		13.3853	27.8741	42.3628	
+50.^{h}6176	[2]	a	b	+1.5476		+2.2728	+0.8206	+2.5957		+1.3729	+0.7673
+51.6176		a'	b'	+1.2700		+1.8651		+1.9917		+1.0534	
+52.6176		Δa	Δb	+0.2776		+0.4077		+0.6040		+0.3195	
		ωt		14.4770	43.4545	72.4319		187.2385	201.7272	216.2160	
-61.^{h}6176	[3]	a	b	+1.9960		+1.8911	+1.0000	-2.7278		-1.0870	-0.0142
-60.6176		a'	b'	+1.9960		+1.8911		+ 0.0387		+0.0154	
-59.6176		Δa	Δb	+0.0000		-0.0000		-2.7665		-1.1024	
		ωt		14.4770	43.4545	72.4319		187.2385	201.7272	216.2160	
+62.^{h}6168	[4]	a	b	+1.9960		+1.8911	+0.8022	-2.7278		-1.0870	+0.7967
+63.6168		a'	b'	+1.6012		+1.5170		-2.1732		-0.8660	
+64.6168		Δa	Δb	+0.3948		+0.3741		-0.5546		-0.2210	

SOLAR SEMIDIURNAL $T_2 = 11^h.99920$ $\omega_2 = 30^\circ.00200$	$\cos^2\delta$	SOLAR DIURNAL $T_3 = 23^h.99840$ $\omega_3 = 15^\circ.00100$	sen 2δ	q_u	Q
311°.3473 341°.3493 11°.3513 +2.5885 -0.8737 +2.4135 -0.8146 +0.1750 -0.0591	+0.9324	335°.6736 350°.6746 5°.6756 +2.8931 -0.4751 +1.4523 -0.2385 +1.4408 -0.2366	+0.5020	49.45 45.15 36.50	131.10
78.6307 108.6327 138.6347 -0.8729 +2.5888 -0.8018 +2.3781 -0.0711 +0.2107	+0.9186	39.3154 54.3164 69.3174 +1.7102 +2.3814 +0.9351 +1.3021 +0.7751 +1.0793	+0.5468	23.40 17.30 10.30	51.00
311.3473 341.3493 11.3513 +2.5885 -0.8737 +2.4102 -0.8135 +0.1783 -0.0602	+0.9311	155.6736 170.6746 185.6756 -2.8931 +0.4751 -1.4654 +0.2406 -1.4277 +0.2345	+0.5065	56.20 56.00 49.95	162.15
78.6307 108.6327 138.6347 -0.8729 +2.5888 -0.8007 +2.3747 -0.0722 +0.2141	+0.9173	219.3154 234.3164 249.3174 -1.7102 -2.3814 -0.9421 -1.3119 -0.7681 -1.0695	+0.5509	73.65 64.10 50.50	188.25

SCHEME B

TABLE 7

t (hours)	Posit.	Calculus' elements	LUNAR SEMIDIURNAL (T)=12ʰ.42345 ω_0=28°.97746			$\cos^2\delta$	LUNAR DIURNAL T_1=24ʰ.84690 ω_1=14°.48873			sen 2δ
-64ʰ.6168	[1°]	ωt	287°.5678	316°.5453	345°.5227	+0.9995	143°.7839	158°.2726	172°.7614	-0.0435
-63.6168		a b	+1.9960	-1.8911			-2.7278	+1.0870		
-62.6168		a' b'	+1.9950	-1.8902			+0.1187	-0.0473		
		Δa Δb	+0.0010	-0.0009			-2.8465	+1.1343		
+59ʰ.6176	[2°]	ωt	287.5678	316.5453	345.5227	+0.8064	143.7839	158.2726	172.7614	+0.7902
+60.6176		a b	+1.9960	-1.8911			-2.7278	+1.0870		
+61.6176		a' b'	+1.6096	-1.5250			-2.1555	+0.8589		
		Δa Δb	+0.3864	-0. 3661			-0.5723	+0.2281		
-52ʰ.6176	[3°]	ωt	275.2742	304.2516	333.2291	+0.9986	317.6371	332.1258	346.6145	+0.0734
-51.6176		a b	+1.5476	-2.2728			+2.5957	-1.3729		
-50.6176		a' b'	+1.5454	-2.2696			+0.1905	-0.1008		
		Δa Δb	+0.0022	-0.0032			+2.4052	-1.2721		
+71ʰ.6168	[4°]	ωt	275.2742	304.2516	333.2291	+0.7914	317.6371	332.1258	346.6145	+0.8126
+72.6168		a b	+1.5476	-2.2728			+2.5957	-1.3729		
+73.6168		a' b'	+1.2248	-1.7987			+2.1093	-1.1156		
		Δa Δb	+0.3228	-0.4741			+0.4864	-0.2573		

SCHEME B°

SOLAR SEMIDIURNAL T_2=11.99920 ω_2=30.00200	$\cos^2\delta$	SOLAR DIURNAL T_3=23.99840 ω_3=15.00100	sen 2δ	q_u°	Q°
221°.3653 251°.3673 281°.3693 -0.8729 -2.5888 -0.8130 -2.4112 -0.0599 -0.1776	+0.9314	110°.6826 125°.6836 140°.6846 -1.7102 +2.3814 -0.8643 +1.2036 -0.8459 +1.1778	+0.5054	28.15 40.60 50.55	119.30
348.6487 18.6507 48.6527 +2.5885 +0.8737 +2.3752 +0.8017 +0.2133 +0.0720	+0.9176	174.3244 189.3254 204.3264 -2.8931 -0.4751 -1.5909 -0.2613 -1.3022 -0.2138	+0.5499	63.10 73.00 77.00	213.10
221.3653 251.3673 281.3693 -0.8729 -2.5888 -0.8119 -2.4078 -0.0610 -0.1810	+0.9301	290.6826 305-6836 320.6846 +1.7102 -2.3814 +0.8720 -1.2143 +0.8382 -1.1671	+0.5099	22.50 32.00 40.35	94.85
348.6487 18.6507 48.6527 +2.5885 +0.8737 +2.3718 +0.8006 +0.2167 +0.0731	+0.9163	354.3244 9.3254 24.3264 +2.8931 +0.4751 +1.6025 +0.2632 +1.2906 +0.2119	+0.5539	16.50 21.25 23.60	61.35

SCHEMES B and B•

ΔP_0 of tidal wave p_0 (Eq. 64) - Lunar Semidiurnal

[1]	(+0.0067)(+18.76)	-	(+0.0098)(-0.66)	= +0.14
[2]	(+0.2776)(")	-	(+0.4077)(")	= +5.48
[3]	(+0.0000)(")	-	(+0.0000)(")	= +0.00
[4]	(+0.3948)(")	-	(+0.3741)(")	= +7.66
[1•]	(+0.0010)(")	-	(-0.0009)(")	= +0.02
[2•]	(+0.3864)(")	-	(-0.3661)(")	= +7.01
[3•]	(+0.0022)(")	-	(-0.0032)(")	= +0.04
[4•]	(+0.3228)(")	-	(-0.4741)(")	= +5.75

ΔP_1 of tidal wave p_1 - Lunar Diurnal

[1]	(+2.9352)(-25.39)	-	(+1.5525)(-0.10)	= -74.36
[2]	(+0.6040)(")	-	(+0.3195)(")	= -15.31
[3]	(-2.7665)(")	-	(-1.1024)(")	= +70.13
[4]	(-0.5546)(")	-	(-0.2210)(")	= +14.06
[1•]	(-2.8465)(")	-	(+1.1343)(")	= +72.39
[2•]	(-0.5723)(")	-	(+0.2281)(")	= +14.55
[3•]	(+2.4052)(")	-	(-1.2721)(")	= -61.20
[4•]	(+0.4864)(")	-	(-0.2573)(")	= -12.38

ΔP_3 of tidal wave p_3 - Solar Diurnal

[1]	(+1.4408)(-10.77)	-	(-0.2366)(+1.36)	= -15.20
[2]	(+0.7741)(")	-	(+1.0793)(")	= - 9.81
[3]	(-1.4277)(")	-	(+0.2345)(")	= +15.06
[4]	(-0.7681)(")	-	(-1.0695)(")	= + 9.73
[1•]	(-0.8459)(")	-	(+1.1778)(")	= + 7.51
[2•]	(-1.3022)(")	-	(-0.2138)(")	= +14.32
[3•]	(+0.8382)(")	-	(-1.1671)(")	= - 7.44
[4•]	(+1.2906)(")	-	(+0.2119)(")	= -14.19

PREVIOUS VALUES OF $\bar{Q}$ AND $\bar{Q}^{\bullet}$

	$Q/Q^{\bullet}$		ΔP_0		ΔP_1		ΔP_3		$\bar{Q}/\bar{Q}^{\bullet}$
[1]	131.10	+	0.14	-	74.36	-	15.20	=	41.68
[2]	51.00	+	5.48	-	15.31	-	9.81	=	31.36
[3]	162.15	+	0.00	+	70.13	+	15.06	=	247.34
[4]	188.25	+	7.66	+	14.06	+	9.73	=	219.70
[1•]	119.30	+	0.02	+	72.39	+	7.51	=	199.22
[2•]	213.10	+	7.01	+	14.55	+	14.32	=	248.98
[3•]	94.85	+	0.04	-	61.20	-	7.44	=	26.25
[4•]	61.35	+	5.75	-	12.38	-	14.19	=	40.53

NOTE . Since there already are three tidal waves compensated with the ΔP computated above, the other remaining wave, the solar semidiurnal p_2 , can be solved by using coefficients a' and b' on virtue that the coefficients of A_3 and B_3 in their equations, are zero. As soon as the values A_2 and B_2 be known, we can go back to computate the respective ΔP_2 , and finally obtain the real values of $\bar{Q}$ and $\bar{Q}^{\bullet}$. With them we resolve the three tidal waves p_1 , p_2 and p_3 by using the coefficients a and b .

WAVE ρ_2 - Solar Semidiurnal

(Previous computation by using coefficients a' and b')

$$[4]-[3]+[2]-[1]$$
$$[4^{\bullet}]-[3^{\bullet}]+[2^{\bullet}]-[1^{\bullet}]$$

Equations

$$-6.4292\ A_2 - 6.3809\ B_2 = -\ 37.96 - (0\ A_3 - 0\ B_3)$$
$$+6.3719\ A_2 - 6.4213\ B_2 = +\ 64.04 - (0\ A_3 - 0\ B_3)$$

Solution

$B_2 = -\ 2.07$ $\qquad R = 8.23$

$A_2 = +\ 7.96$ $\qquad \varphi = 345.^{\circ}4$

$Tg\ \varphi = -\ 0.26006$

ΔP_2 of tidal wave ρ_2 - Solar Semidiurnal

[1] (+0.1750)(+7.96) - (-0.0591)(-2.07) = + 1.28

[2] (-0.0711)(") - (+0.2107)(") = - 0.13

[3] (+0.1783)(") - (-0.0602)(") = + 1.30

[4] (-0.0722)(") - (+0.2141)(") = - 0.14

[1•] (-0.0599)(") - (-0.1776)(") = - 0.85

[2•] (+0.2133)(") - (+0.0720)(") = + 1.85

[3•] (-0.0610)(") - (-0.1810)(") = - 0.86

[4•] (+0.2167)(") - (+0.0731)(") = + 1.88

VALUES OF $\bar{Q}$ AND $\bar{Q}^{\bullet}$

With the previous values of $\bar{Q}$ and $\bar{Q}^{\bullet}$, and the ΔP_2 computed, we find the following final values for $\bar{Q}$ and $\bar{Q}^{\bullet}$.

		ΔP_2	$\bar{Q}/\bar{Q}^{\bullet}$
[1]	41.68	+ 1.28 =	42.96
[2]	31.36	- 0.13 =	31.23
[3]	247.34	+ 1.30 =	248.64
[4]	219.70	- 0.14 =	219.56
[1•]	199.22	- 0.85 =	198.37
[2•]	248.98	+ 1.85 =	250.83
[3•]	26.25	- 0.86 =	25.39
[4•]	40.53	+ 1.88 =	42.41

68. SCHEMES B AND B•

Solutions by using coefficients a and b

WAVE p_3 - Solar Diurnal

[4] - [3] - [2] + [1]

[4•] - [3•] - [2•] + [1•]

Equations

$$+2.3658\ A_3 + 5.7130\ B_3 = -17.35$$

$$+2.3658\ A_3 - 5.7130\ B_3 = -35.44$$

Solution

$B_3 = +1.58$ $\qquad$ $R = 11.27$

$A_3 = -11.16$ $\qquad$ $\varphi = 171^\circ.9$

$Tg\varphi = -0.14191$

WAVE p_2 - Solar Semidiurnal

$$[4]-[3]+[2]-[1]$$
$$[4^\bullet]-[3^\bullet]+[2^\bullet]-[1^\bullet]$$

Equations

$$-6.9228\,A_2 - 6.9250\,B_2 = -40.81 - (0\,A_3 - 0\,B_3)$$
$$+6.9228\,A_2 - 6.9250\,B_2 = +69.48 - (0\,A_3 - 0\,B_3)$$

Solutions

$B_2 = -2.07$ $\qquad$ $R = 8.23$

$A_2 = +7.97$ $\qquad$ $\varphi = 345.^\circ4$

$Tg\,\varphi = -0.25987$

WAVE p_1 - Lunar Diurnal

$$[4]+[3]-[2]-[1]$$
$$[4^\bullet]+[3^\bullet]-[2^\bullet]-[1^\bullet]$$

Equations

$$-10.6470\,A_1 + 4.9198\,B_1 = +394.01 - (-9.2066\,A_3 + 3.8126\,B_3)$$
$$+10.6470\,A_1 + 4.9198\,B_1 = -381.40 - (+9.2066\,A_3 + 3.8126\,B_3)$$

Solutions

$B_1 = +0.05$ $\qquad$ $R = 26.77$

$A_1 = -26.77$ $\qquad$ $\varphi = 179.^\circ9$

$Tg\,\varphi = -0.00204$

69. RESUME

CLASS	SCHEMES A and A'		SCHEMES B and B'	
	R	φ	R	φ
Lunar Diurnal	25.39	180.°2	26.77	179.°9
Lunar Semidiurnal	18.78	358.°0		
Solar Diurnal	10.86	172.°8	11.27	171.°9
Solar Semidiurnal			8.23	345.°4

70. Explicit Results

(From Secs. 54, 55 and 69)

Lunar Diurnal

$$26.08 \cos (14.^\circ 48873\ t + 180.^\circ 1) \text{ sen } 2\ \delta_L \qquad (66)$$

Lunar Semidiurnal

$$18.78 \cos (28.^\circ 97746\ t + 358.^\circ 0) \cos^2 \delta_L \qquad (67)$$

Solar Diurnal

$$11.07 \cos (15.^\circ 00100\ t + 172.^\circ 4) \text{ sen } 2\ \delta_S \qquad (68)$$

Solar Semidiurnal

$$8.23 \cos (30.^\circ 00200\ t + 345.^\circ 4) \cos^2 \delta_S \qquad (69)$$

Q) CONSIDERATIONS ABOUT THE RESULTS

71. At La Plata latitude ($\emptyset = -34° 54'$), the semiamplitude ratio between the diurnal and semidiurnal waves (either of the sun or of the moon) must be

$$\frac{\text{sen } 2\emptyset}{\cos^2 \emptyset} = 1.40$$

We have got:

For the moon, $\frac{26.08}{18.78} = 1.39$

For the Sun, $\frac{11.07}{8.23} = 1.35$

that can be considered as excellent.

72. The initial phase angle of a diurnal wave must be one half of that of the semidiurnal wave. The following table shows the goodness of both, the results and the filtering system to get them.

Body	Initial Phase Angles		
	Diurnal wave	Corresponding Semidiurnal Wave	Computed Semidiurnal Wave
MOON	358.°0	179.°0	180.°1
SUN	345.°4	172.°7	172.°4

73. If both, the sun and the moon, were acting at their mean distance, either at equator ($\cos^2 \delta_S = \cos^2 \delta_L = 1$) or at declination 45° ($\text{sen } 2\,\delta_S = \text{sen } 2\,\delta_L = 1$), then the semiamplitudes ratio should be, (sec.58, Eq. 55)

$$r = \frac{R_S}{R_L} = 0.46$$

At the date of the record readings, the moon was nearer to the Earth than its mean distance. Then, the amplitudes computed for the diurnal and semidiurnal waves, are greater by a factor $\left(\frac{\rho_m}{\rho_m}\right)^3 = 1.046$, so that, the amplitudes at the mean distance of the moon, would be:

Diurnal wave, $\frac{26.08}{1.046} = 25.08$

Semidiurnal wave, $\frac{18.78}{1.046} = 18.06$

In the case of the sun, also at the same date, the body was farther from the Earth than its mean distance, which in turn means that the amplitudes obtained are smaller by a factor $\left(\frac{\rho_m}{\rho_m}\right)^3 = 1.024$. Therefore, the amplitudes at the mean distance will become:

Diurnal wave : $11.07 \times 1.024 = 11.34$

Semidiurnal wave : $8.23 \times 1.024 = 8.43$

With those figures, the resulting r ratios, are

Diurnal waves $r = \frac{11.34}{25.08} = 0.45$

Semidiurnal waves $r = \frac{8.43}{18.06} = 0.47$

which fit well with the theoretical value of $r = 0.46$

R) SEPARATION OF THE WAVES M_2, N_2 AND $[K_2]_L$

74. These are the principal constituents of the already known lunar semidiurnal wave (Eq. 67).

It must be:

$$Q = R\cos(\omega t + \varphi)\cos^2\delta_L\left(\frac{\rho_m}{\rho_a}\right)^3 = R_1\cos(\omega_1 t + \varphi_1) + R_2\cos(\omega_2 t + \varphi_2) + R_3\cos(\omega_3 t + \varphi_3) + \ldots$$

where

$$R = 18.78$$

$$\omega = 28^\circ.97746$$

$$\varphi = 358^\circ.0$$

$M_2 \longrightarrow \omega_1 = 28.98411 \qquad (T_1 = 12^h.42060)$

$N_2 \longrightarrow \omega_2 = 28.43973 \qquad (T_2 = 12.65835)$

$[K_2]_L \longrightarrow \omega_3 = 30.08214 \qquad (T_3 = 11.96724)$

The (R_1, φ_1), (R_2, φ_2) and (R_3, φ_3) are the semiamplitudes and initial phase angles of the respective tidal components M_2, N_2 and $[K_2]_L$

We can also say that

$$R_1 = R \cdot C^{\bullet}_{M_2}$$

$$R_2 = R \cdot C^{\bullet}_{N_2}$$

$$R_3 = R \cdot C^{\bullet}_{[K_2]_L}$$

where $C^{\bullet}_{M_2}$, $C^{\bullet}_{N_2}$ and $C^{\bullet}_{[K_2]_L}$ respectively are the " actual" coefficients of those waves, which are related to the mean coefficients C_{M_2}, C_{N_2} and $C_{[K_2]_L}$ by

$$C_{M_2} = f \cdot C^{\bullet}_{M_2}$$

$$C_{N_2} = f \cdot C^{\bullet}_{N_2}$$

$$C_{[K_2]_L} = f \cdot C^{\bullet}_{[K_2]_L}$$

where " f " is the node factor for every tidal wave.

Since Q and $Q^{\bullet}$ are theoretically obtained in every position $([1],[2]...\text{etc.})$ it is not necessary to use the tidal record.

To perform the computations, the variation of the moon distance must be taken into account, to get the factor $\left(\frac{\rho_m}{\rho_a}\right)^3$, where $\rho^{\bullet}_m$ is the " average " distance at the date of the computations. Fig. 9 shows the path of the moon, where distance changes are evident.

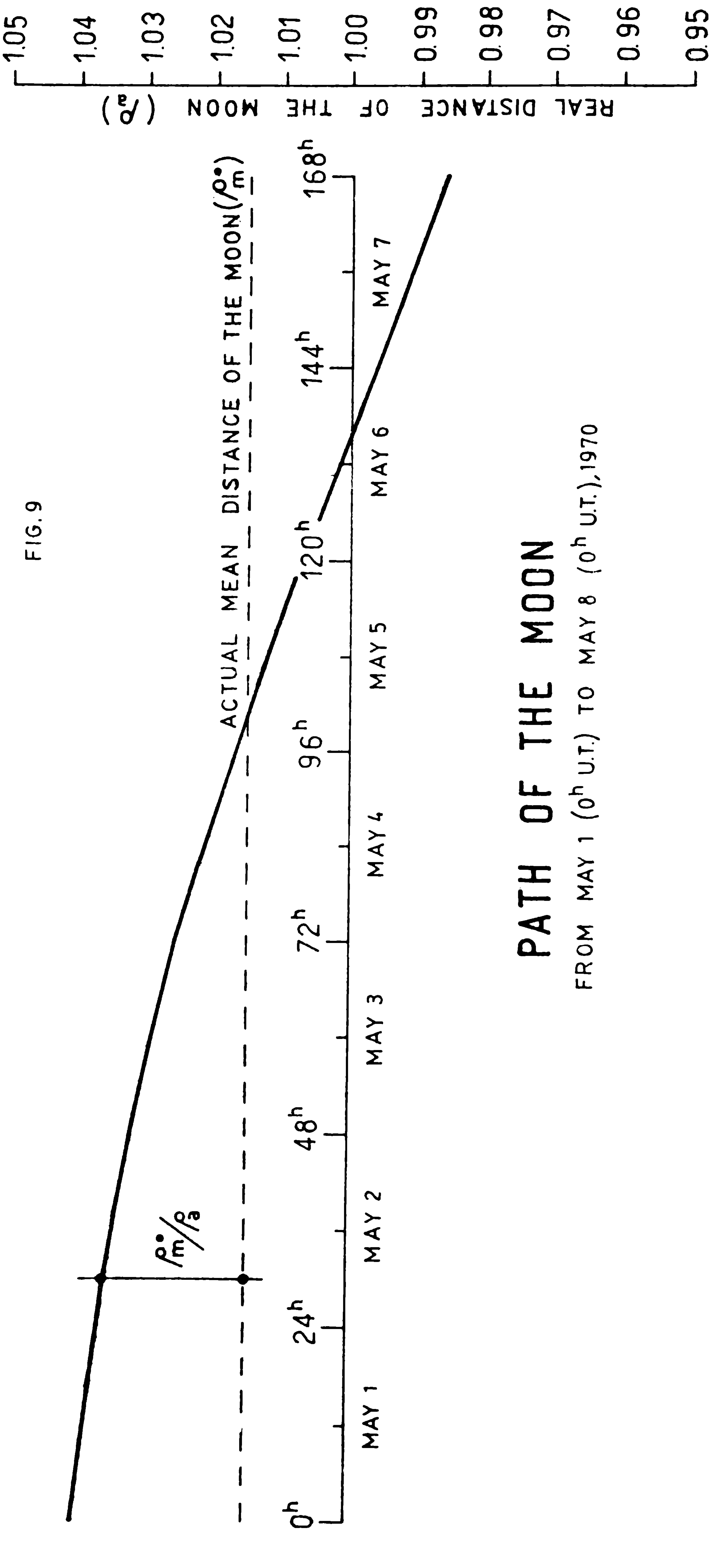

FIG. 9

PATH OF THE MOON

FROM MAY 1 (0^h U.T.) TO MAY 8 (0^h U.T.), 1970

Every tidal component will be obtained quite independently. In this way better results can be reached; the increasing in work computations is only apparent.

Being that so, we have prepared the following filtering schemes and Tables:

For wave M_2 schemes in Fig. 10 and Table 8.
" " N_2 " " Fig. 11 " Table 9.
" " $[K_2]_L$ " " Fig. 12 " Table 10.

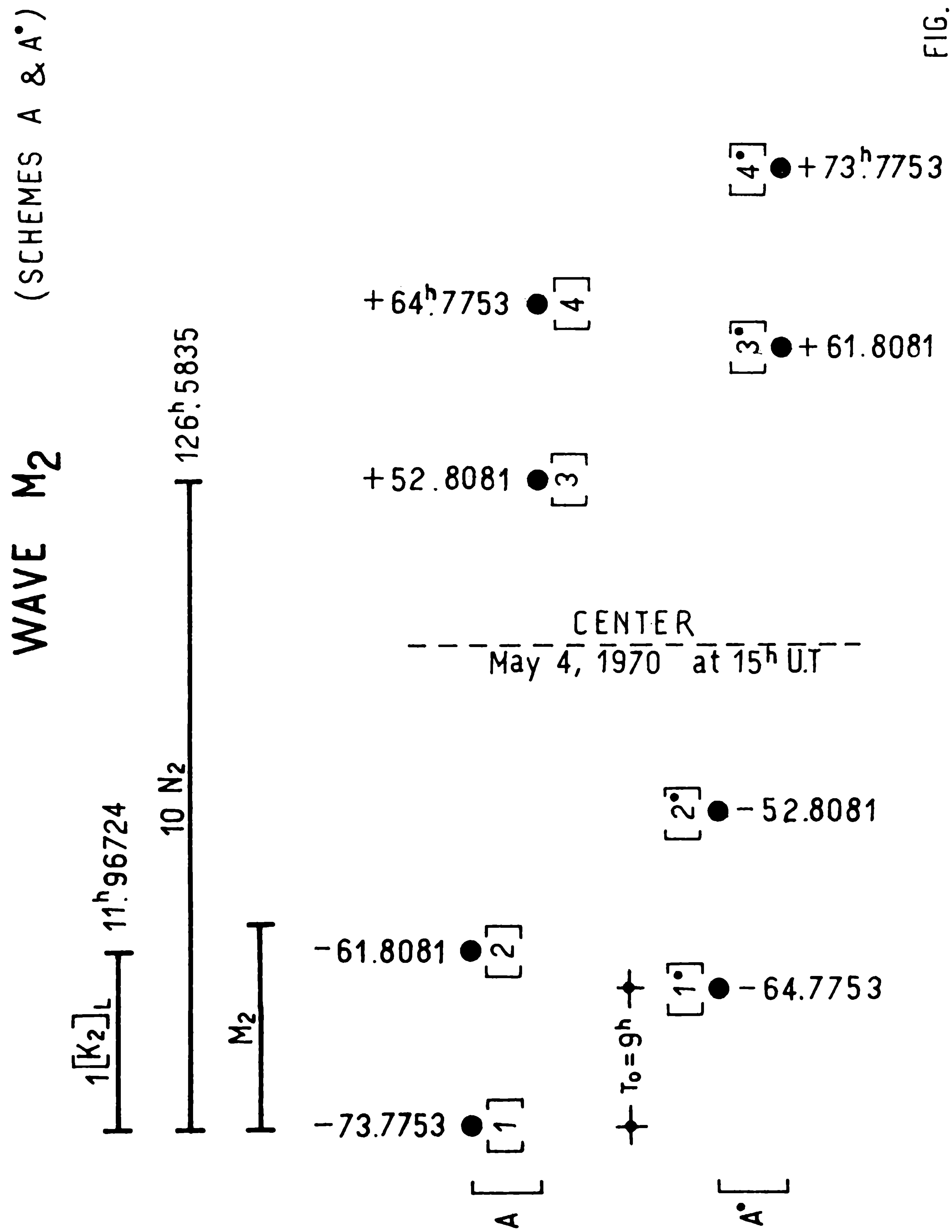

FIG. 10

TABLE 8

$t_{(h)}$	Posit.	$[K_2]_L$ & N_2	M_2 cos ωt	M_2 sen ωt	Q
-73.7753	[1]		21°69 +0.9292	+0.3696	+18.838
-61.8081	[2]		8°55 +0.9889	+0.1486	+19.888
+52.8081	[3]		90°60 -0.0104	+1.0000	+ 0.454
+64.7753	[4]		77°46 +0.2172	+0.9761	+ 3.658

$t_{(h)}$	Posit.	$[K_2]_L$ & N_2	M_2 cos ωt	M_2 sen ωt	Q•
-64.7753	[1•]		282°54 +0.2172	-0.9761	+ 3.813
-52.8081	[2•]		269°40 -0.0104	-1.0000	- 0.788
+61.8081	[3•]		351°45 +0.9889	-0.1486	+14.005
+73.7753	[4•]		338°31 +0.9292	-0.3696	+12.548

WAVE M_2

[4] - [3] - [2] + [1]

[4•] - [3•] - [2•] + [1•]

Equations

$$+ 0.1679 \ A - 0.1971 \ B = + 2.154$$

$$+ 0.1679 \ A + 0.1971 \ B = + 3.144$$

Solutions

$B = + 2.51$ $\qquad$ $R_{M_2} = 15.98 = R \cdot C^{\bullet}_{M_2}$

$A = + 15.78$ $\qquad$ $\varphi = 9.°0$

$Tg\ \varphi = + 0.15918$

f = Node factor = 0.966 $\qquad$ $C_{M_2} = 0.881$

WAVE N_2 (SCHEMES A & A°)

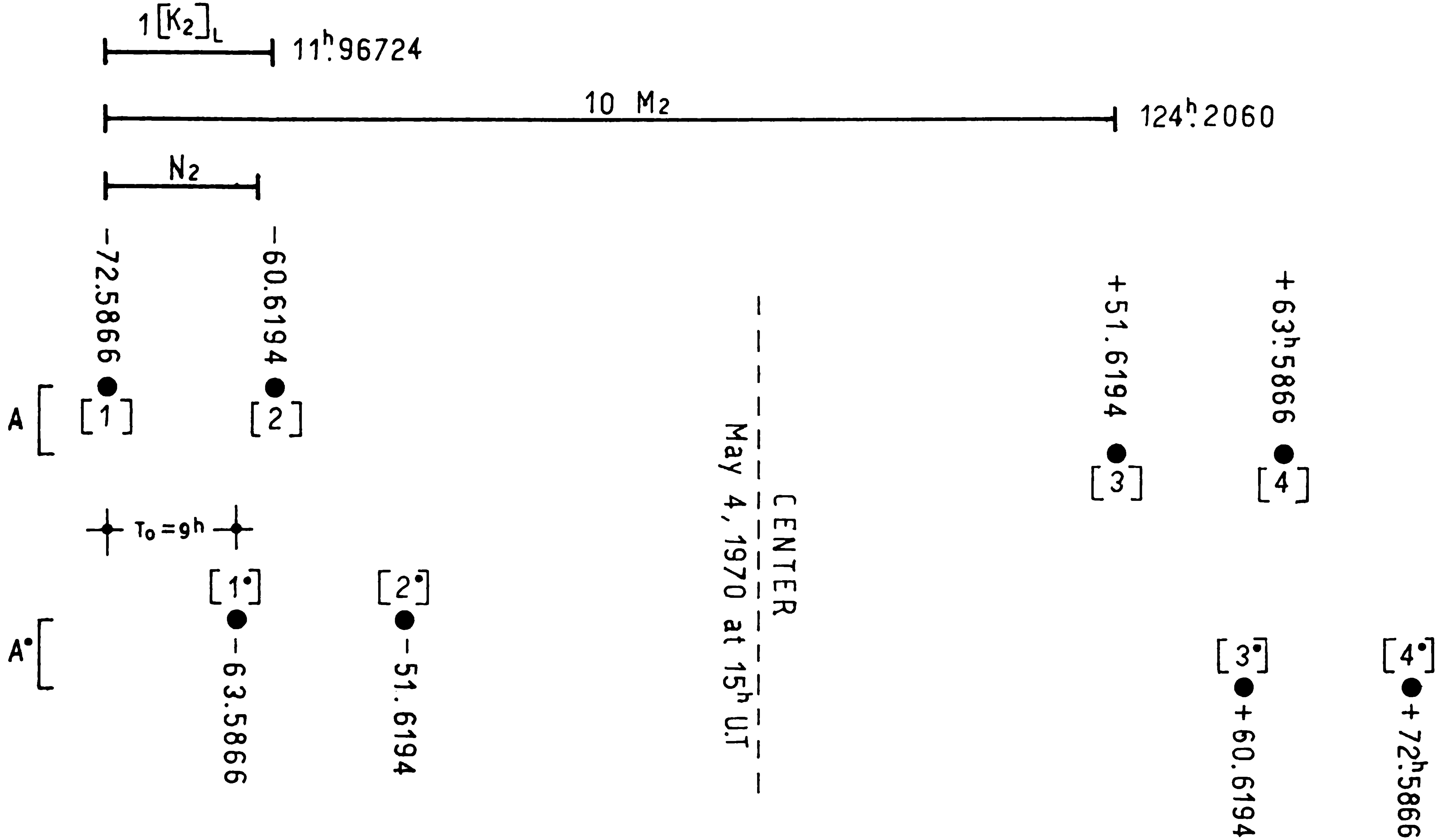

FIG. 11

TABLE 9

$t_{(h)}$	Posit.	$[K_2]_L$ & M_2	N_2 cos ωt	N_2 sen ωt	Q
-72.5866	[1]		95°.66 -0.0986	±0.9951	±11.612
-60.6194	[2]		76°.00 +0.2419	+0.9703	+15.021
+51.6194	[3]		28°.04 +0.8826	+0.4701	+ 8.688
+63.5866	[4]		8°.39 +0.9893	+0.1458	+10.809

$t_{(h)}$	Posit.	$[K_2]_L$ & M_2	N cos ω t	N sen ω t	$Q^{\bullet}$
-63.5866	[1•]		351°.61 +0.9893	-0.1458	+14.263
-51.6194	[2•]		331°.96 +0.8826	-0.4701	+10.597
+60.6194	[3•]		284°.00 +0.2419	-0.9703	+10.023
+72.5866	[4•]		264°.34 -0.0986	-0.9951	+ 7.171

WAVE N_2

[4] - [3] - [2] + [1]

[4•] - [3•] - [2•] + [1•]

Equations

$$-0.2338\ A + 0.2995\ B = -1.288$$

$$-0.2338\ A - 0.2995\ B = +0.814$$

Solutions

$B = -3.51$

$A = +1.01$

$Tg\,\varphi = -3.46180$

$R_{N_2} = 3.65 = R \cdot C^{\bullet}_{N_2}$

$\varphi = 286°.1$

f = Node factor = 0.966

$C_{N_2} = 0.201$

WAVE $[K_2]_L$ (SCHEMES A & A°)

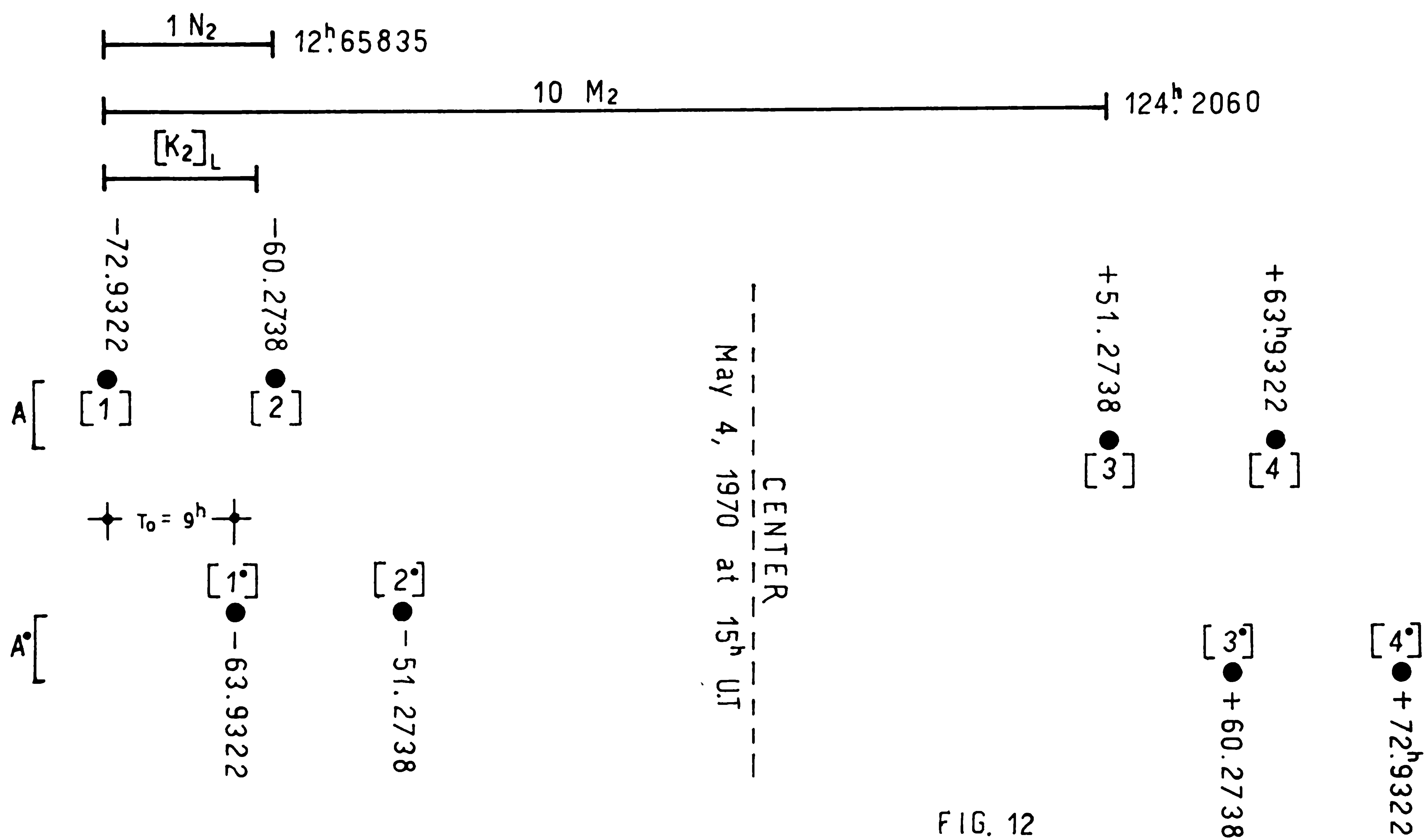

FIG. 12

TABLE 10

$t_{(h)}$	Posit.	N_2 & M_2	$[K_2]_L$ cos ω t	$[K_2]_L$ sen ω t	Q
-72.9322	[1]		326°05 +0.8295	-0.5586	+14.271
-60.2738	[2]		346°84 +0.9737	-0.2278	+12.467
+51.2738	[3]		102°43 -0.2152	+0.9766	+10.650
+63.9322	[4]		123°22 -0.5478	+0.8366	+ 8.970

$t_{(h)}$	Posit.	N_2 & M_2	$[K_2]_L$ cos ω t	$[K_2]_L$ sen ω t	Q•
-63.9322	[1•]		236°78 -0.5478	-0.8366	+11.597
-51.2738	[2•]		257°57 -0.2152	-0.9766	+13.383
+60.2738	[3•]		13°16 +0.9737	+0.2278	+ 8.123
+72.9322	[4•]		33°95 +0.8295	+0.5586	+ 9.101

WAVE $[K_2]_L$

[4] - [3] - [2] + [1]

[4•] - [3•] - [2•] + [1•]

Equations

$$-0.4768\ A + 0.4708\ B = +0.124$$

$$-0.4768\ A - 0.4708\ B = -0.808$$

Solutions

$B = +0.99$

$A = +0.72$

$Tg\ \varphi = +1.37994$

$R_{[K_2]_L} = 1.22 = R.C^{\bullet}_{[K_2]_L}$

$\varphi = 54°1$

f = Node factor = 1.289

$C_{[K_2]_L} = 0.050$

S) SEPARATION OF THE WAVES S_2 AND $[K_2]_S$

75. These are the main components of the already known solar semidiurnal wave, (Eq. 69).
It must be,

$$Q = R \cos(\omega t + \varphi) \cos^2 \delta_S \left(\frac{\rho_m}{\rho_a}\right)^3 = R_1 \cos(\omega_1 t + \varphi_1) + R_2 \cos(\omega_2 t + \varphi_2) + \ldots$$

where

$$R = 8.23$$

$$\omega = 30^{\circ}.00200$$

$$\varphi = 345^{\circ}.4$$

$S_2 \longrightarrow \omega_1 = 30.00000$ ($T_1 = 12^h.00000$)

$[K_2]_S \longrightarrow \omega_2 = 30.08214$ ($T_2 = 11^h.96724$)

As the distance variations of the sun have been very small, the factor $\left(\frac{\rho_m}{\rho_a}\right)^3$ should be taken as the unity.

The (R_1 , φ_1) and (R_2 , φ_2) are the semiamplitudes and initial phase angles of the waves S_2 and $[K_2]_S$

The "actual" coefficients of these waves, ($\dot{C}_{S_2}$ and $\dot{C}_{[K_2]_S}$) can be obtained from

$$R_1 = R \,.\, \dot{C}_{S_2}$$

$$R_2 = R \,.\, \dot{C}_{[K_2]_S}$$

It must be pointed out that the tidal waves formula takes into account the moon and sun joint effects. In order to use only one coefficient, namely that corresponding to the moon, it is necessary to multiply the coefficients of the sun component waves by the factor $r = 0.46$ (Sec.58, Eq.55).

Now, the mean coefficients C_{S_2} and $C_{[K_2]_S}$ are related to the " actual" coefficients by

$$C_{S_2} = r \,.\, \dot{C}_{S_2}$$

$$C_{[K_2]_S} = r \,.\, \dot{C}_{[K_2]_S}$$

To separate both waves from the solar semidiurnal wave, we have prepared the schemes shown in Fig. 13 as well as the Table 11. The computation follows the same steps as usual.

WAVES $[K_2]_S$ & S_2 (SCHEMES A & A°)

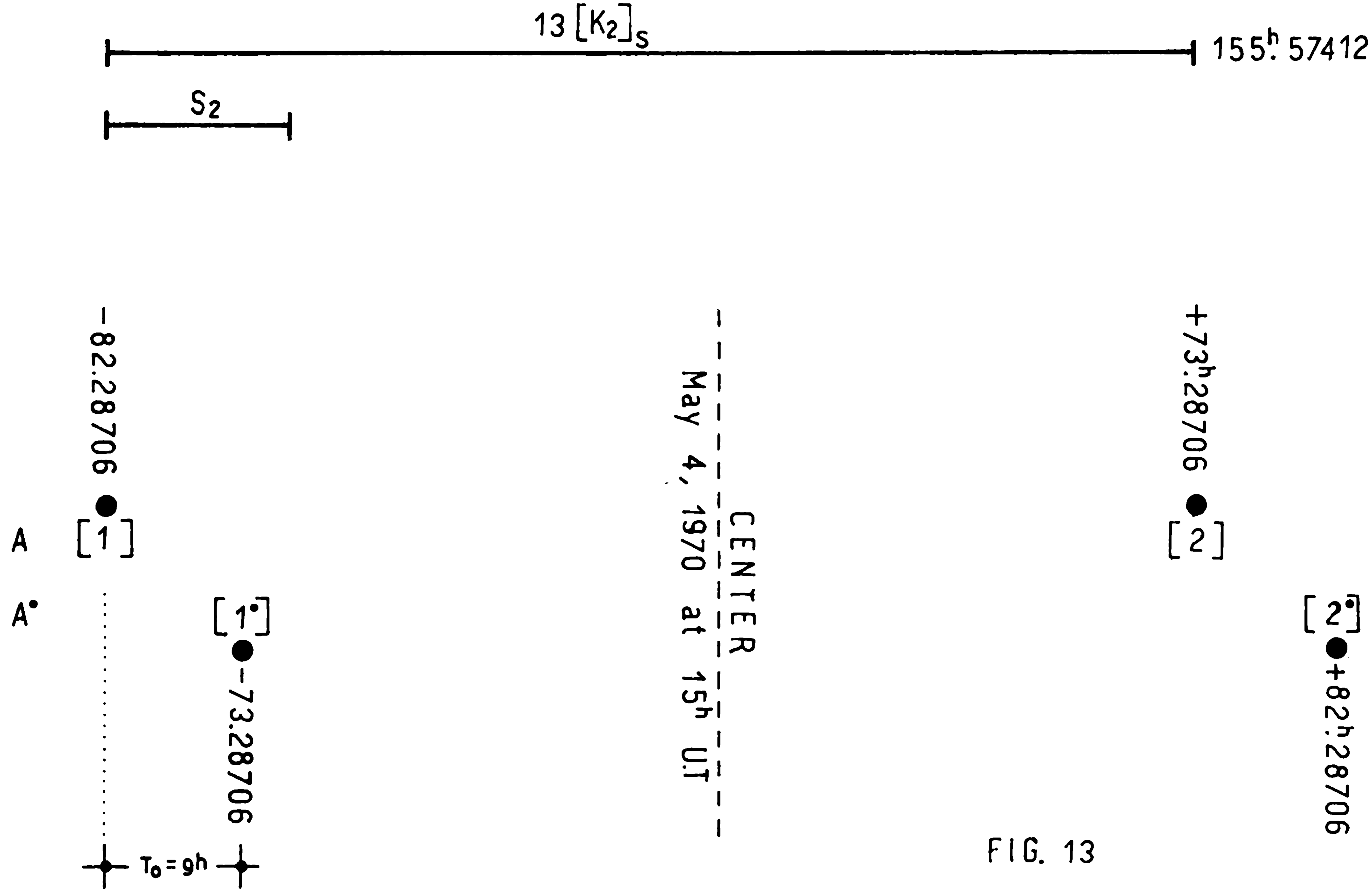

FIG. 13

TABLE 11

t (h)	Posit.	$[K_2]_S$		S_2		Q
		cos $\omega_2 t$	sen $\omega_2 t$	cos $\omega_1 t$	sen $\omega_1 t$	
- 82.28706	[1]	44°.629 +0.7117	+0.7025	51°.388 +0.6240	+0.7814	+6.163
+ 73.28706	[2]	44°.632 +0.7116	+0.7026	38°.612 +0.7814	+0.6240	+6.898

t (h)	Posit.	$[K_2]_S$		S_2		Q•
		cos $\omega_2 t$	sen $\omega_2 t$	cos $\omega_1 t$	sen $\omega_1 t$	
-73.28706	[1°]	315°.368 +0.7116	-0.7026	321°.388 +0.7814	-0.6240	+4.584
+82.28706	[2°]	315°.371 +0.7117	-0.7025	308°.612 +0.6240	-0.7814	+3.088

WAVE S_2

$$[2]-[1]$$
$$[2^{\bullet}]-[1^{\bullet}]$$

Equations

$$+\ 0.1574\ A\ +\ 0.1574\ B\ =\ +\ 0.735$$

$$-\ 0.1574\ A\ +\ 0.1574\ B\ =\ -\ 1.496$$

Solution

$B = -\ 2.42$	$R = 7.49$
$A = +\ 7.09$	$\varphi = 341.^{\circ}17$
$Tg\varphi = -0.34110$	$C_{S_2} = 0.419$

WAVE $[K_2]_S$

$$[2]+[1]$$
$$[2^{\bullet}]+[1^{\bullet}]$$

Equations

$$+1.4233\ A - 1.4051\ B + (1.4054\ A_{S_2} - 1.4054\ B_{S_2}) = 13.061$$

$$+1.4233\ A - 1.4051\ B + (1.4054\ A_{S_2} + 1.4054\ B_{S_2}) = 7.672$$

Solution

$B = +\ 0.50$	$R = 0.58$
$A = +\ 0.28$	$\varphi = 60.^{\circ}7$
$Tg\varphi = +1.77945$	$C_{[K_2]_S} = 0.032$

T) SEPARATION OF THE WAVES $[K_1]_L$, O_1 and Q_1

76. These waves are the principal components of the already known lunar diurnal tidal wave. (Eq. 66).

It must be accomplished that

$$Q = R\cos(\omega t+\varphi)\ \text{sen}\ 2\delta_L \cdot \left(\frac{\rho_m^{\bullet}}{\rho_a}\right)^3 = R_1\cos(\omega_1 t+\varphi_1) + R_2\cos(\omega_2 t+\varphi_2) + R_3\cos(\omega_3 t+\varphi_3) + \ldots$$

wherein

$R = 26.08$

$\omega = 14^{\circ}.48873$

$\varphi = 180^{\circ}.1$

$[K_1] \longrightarrow \omega_1 = 15^{\circ}.041069 \qquad (T_1 = 23^h.93447)$

$O_1 \longrightarrow \omega_2 = 13^{\circ}.943036 \qquad (T_2 = 25^h.81935)$

$Q_1 \longrightarrow \omega_3 = 13^{\circ}.398661 \qquad (T_3 = 26^h.86836)$

Moreover, the (R_1, φ_1), (R_2, φ_2) and (R_3, φ_3) are the semi-amplitudes and initial phase angles of the respective tidal components $[K_1]_L$, O_1 and Q_1

It also follows that

$$R_1 = R \cdot C^{\bullet}_{[K_1]_L}$$
$$R_2 = R \cdot C^{\bullet}_{O_1}$$
$$R_3 = R \cdot C^{\bullet}_{Q_1}$$

where $C^{\bullet}_{[K_1]_L}$, $C^{\bullet}_{O_1}$ and $C^{\bullet}_{Q_1}$ respectively are the " actual " coefficients of those waves, related in turn to the mean coefficients $C_{[K_1]_L}$, C_{O_1} and C_{Q_1} by

$$C_{[K_1]_L} = f \cdot C^{\bullet}_{[K_1]_L}$$
$$C_{O_1} = f \cdot C^{\bullet}_{O_1}$$
$$C_{Q_1} = f \cdot C^{\bullet}_{Q_1}$$

where " f " is the node factor

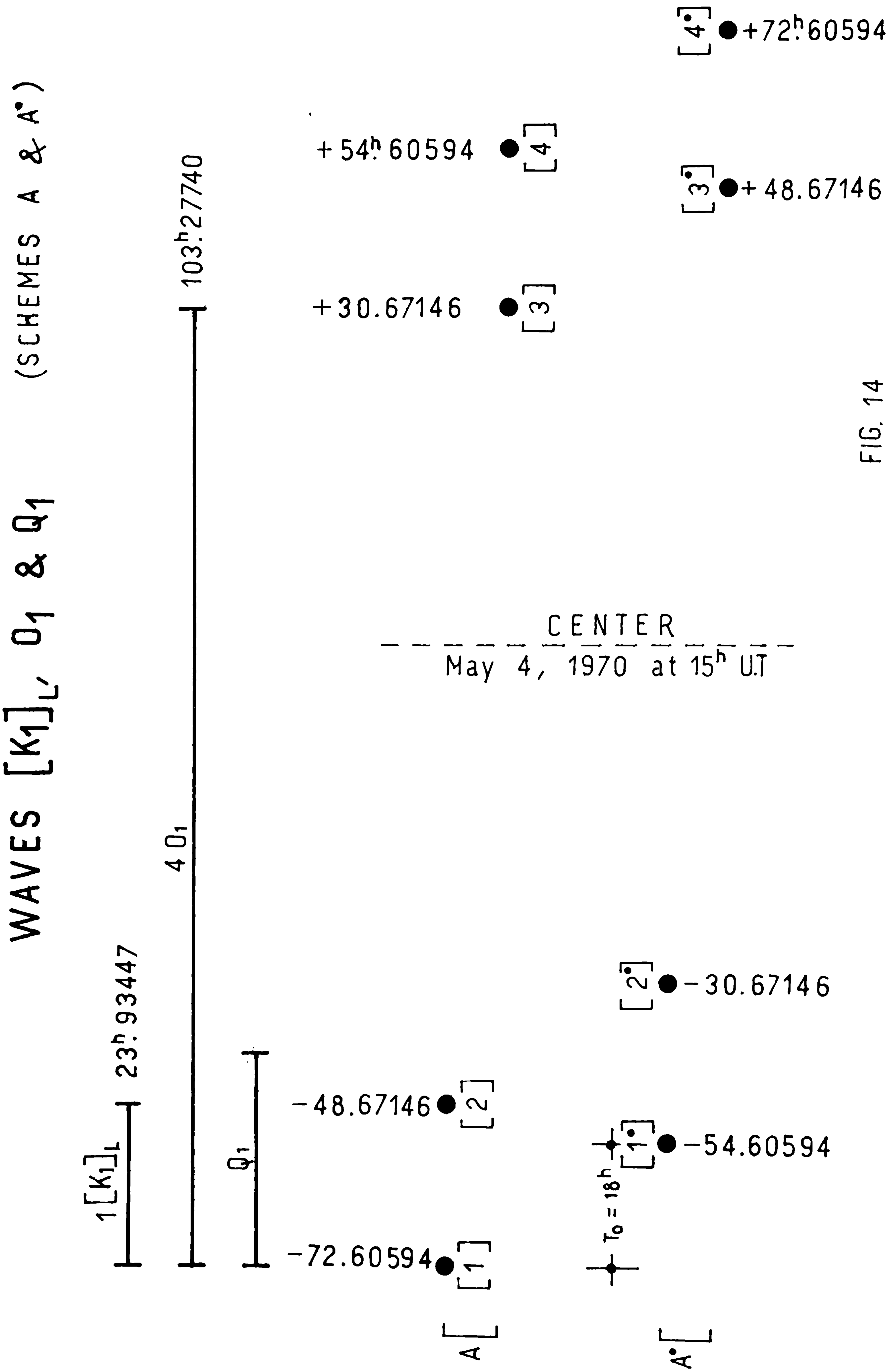
WAVES [K1]L, O1 & Q1 (SCHEMES A & A')
1[K1]L
23h.93447
4 O1
103h.27740
Q1
-72.60594
[1]
-48.67146
[2]
+30.67146
[3]
+54h.60594
[4]
CENTER
May 4, 1970 at 15h U.T
To = 18h
[1']
-54.60594
[2']
-30.67146
[3']
+48.67146
[4']
+72h.60594
A
A'

FIG. 14

To separate the named tidal components waves from the lunar diurnal tidal wave, expressed by Eq. 66, (and owing to the fact that a short time record have been used), we may avoid to introduce into computations the factor $\left(\frac{\rho_{m^\bullet}}{\rho_a}\right)^3$, then considered as the unity.

Fig. 14 shows the schemes A and A$^\bullet$, and Table 12 have the elements to enter, to the necessary computations in order to separate the proposed tidal component waves. As the steps to bring out the solutions are as always, any further commentary is needed.

TABLE 12

Scheme A

$t_{(h)}$	Posit.	$[K_1]_L$ cos $\omega_1 t$	$[K_1]_L$ sen $\omega_1 t$	O_1 cos $\omega_2 t$	O_1 sen $\omega_2 t$	Q_1 cos $\omega_3 t$	Q_1 sen $\omega_3 t$	Q
-72.60594	[1]	347°.93 +0.9779	-0.2091	67°.65 +0.3803	+0.9249	107°.18 -0.2954	+0.9554	+3.005
-48.67146	[2]	347°.93 +0.9779	-0.2091	41°.37 +0.7505	+0.6609	67°.87 +0.3767	+0.9263	-2.587
+30.67146	[3]	101°.33 -0.1965	+0.9805	67°.65 +0.3803	+0.9249	50°.96 +0.6299	+0.7767	-1.744
+54.60594	[4]	101°.33 -0.1965	+0.9805	41°.37 +0.7505	+0.6609	11°.65 +0.9794	+0.2019	-6.507

Scheme A•

$t_{(h)}$	Posit.	$[K_1]_L$ cos $\omega_1 t$	$[K_1]_L$ sen $\omega_1 t$	O_1 cos $\omega_2 t$	O_1 sen $\omega_2 t$	Q_1 cos $\omega_3 t$	Q_1 sen $\omega_3 t$	Q•
-54.60594	[1•]	258°.67 -0.1965	-0.9805	318°.63 +0.7505	-0.6609	348°.35 +0.9794	-0.2019	-0.456
-30.67146	[2•]	258°.67 -0.1965	-0.9805	292°.35 +0.3803	-0.9249	309°.04 +0.6299	-0.7767	-0.696
+48.67146	[3•]	12°.07 +0.9779	+0.2091	318°.63 +0.7505	-0.6609	292°.13 +0.3767	-0.9263	-19.132
+72.60594	[4•]	12°.07 +0.9779	+0.2091	292°.35 +0.3803	-0.9249	252°.82 -0.2954	-0.9554	-18.718

WAVE Q_1

$$[4]-[3]-[2]+[1]$$
$$[4^{\bullet}]-[3^{\bullet}]-[2^{\bullet}]+[1^{\bullet}]$$

Equations

$$-0.3226\ A + 0.5457\ B = +0.829$$

$$-0.3226\ A - 0.5457\ B = +0.654$$

Solution

$B = +0.16$ $\qquad$ $R = 2.30$

$A = -2.30$ $\qquad$ $\varphi = 176^{\circ}.0$

$Tg\varphi = -0.06976$

f = node factor = 1.17 $\qquad$ $C_{Q_1} = 0.075$

WAVE O_1

$$[4]-[3]+[2]-[1]$$
$$[4^{\bullet}]-[3^{\bullet}]+[2^{\bullet}]-[1^{\bullet}]$$

Equations

$$+0.7404\ A + 0.5280\ B = -10.355 - (+1.0216\ A_{Q_1} + 0.6039\ B_{Q_1})$$

$$-0.7404\ A + 0.5280\ B = +0.174 - (-1.0216\ A_{Q_1} + 0.6039\ B_{Q_1})$$

Solution

$B = -9.82$ $\qquad$ $R = 10.58$

$A = -3.94$ $\qquad$ $\varphi = 248^{\circ}.2$

$Tg\varphi = +2.49418$

f = node factor = 1.17 $\qquad$ $C_{O_1} = 0.35$

WAVE $[K_1]_L$

$$[4]+[3]-[2]-[1]$$

$$[4^\bullet]+[3^\bullet]-[2^\bullet]-[1^\bullet]$$

Equations

$$-2.3488\ A - 2.3792\ B = -8.669 - (+1.5280\ A_{Q_1} + 0.9031\ B_{Q_1})$$

$$+2.3488\ A - 2.3792\ B = -36.700 - (-1.5280\ A_{Q_1} + 0.9031\ B_{Q_1})$$

Solution

$B = +9.60$ $\quad\quad$ $R = 12.20$

$A = -7.46$ $\quad\quad$ $\varphi = 127^\circ.9$

$Tg\varphi = -1.28585$

$f = \text{node factor} = 1.105$ $\quad\quad$ $C_{[K_1]_L} = 0.42$*

* This coefficient really involves not only that of the wave $[K_1]_L$ but also those of the waves M_1 and J_1, whose average period and instantaneous average phase angle are exactly the same of the wave $[K_1]$

At the date of the present example (May 1 - 8, 1970) the mixed semiamplitude incorporated by the wave M_1 and J_1, is within the order of 0.05; whence, the semiamplitude of $[K_1]_L$ alone, must be within the order of

$$C_{[K_1]_L} = 0.37$$

U) SEPARATION OF THE WAVES $[K_1]_S$ and P_1

77. These are the main tidal components, which can be obtained directly from the already known solar diurnal tidal wave, (Eq. 68)

It must be

$$Q = R\cos(\omega t + \varphi)\text{sen}\ 2\delta_S = R_1 \cos(\omega_1 t \quad \varphi_1) +$$

$$+ R_2 \cos(\omega_2 t \quad \varphi_2) + \ldots$$

where

$$R = 11.07$$

$$\omega = 15^\circ.00100$$

$$\varphi = 172^\circ.4$$

$[K_1]_S \longrightarrow \omega_1 = 15.04107$ ($T_1 = 23.93447$)

$P_1 \longrightarrow \omega_2 = 14.95893$ ($T_2 = 24.06589$)

The (R_1 , φ_1) and (R_2 , φ_2) are the semiamplitudes and initial phase angles of the respective tidal components $[K_1]_S$ and P_1.

It is easily seen that

$$R_1 = R \cdot C^{\bullet}_{[K_1]_S}$$

$$R_2 = R \cdot C^{\bullet}_{P_1}$$

where $C^{\bullet}_{[K_1]_S}$ and $C^{\bullet}_{P_1}$ are the " actual " coefficients of these waves. By the reasons given in Sec 75 , the mean coefficients come from

$$C_{[K_1]_S} = r \cdot C^{\bullet}_{[K_1]_S}$$

$$C_{P_1} = r \cdot C^{\bullet}_{P_1}$$

The factor $r = 0.46$ has the meaning already explained by Eq. 55.

In order to separate the proposed tidal waves, we have prepared the schemes shown in Fig. 15, and also the Table 13, dealing with them in the same way as always.

WAVES $[K_1]_S$ & P_1 (SCHEMES A & A$^\bullet$)

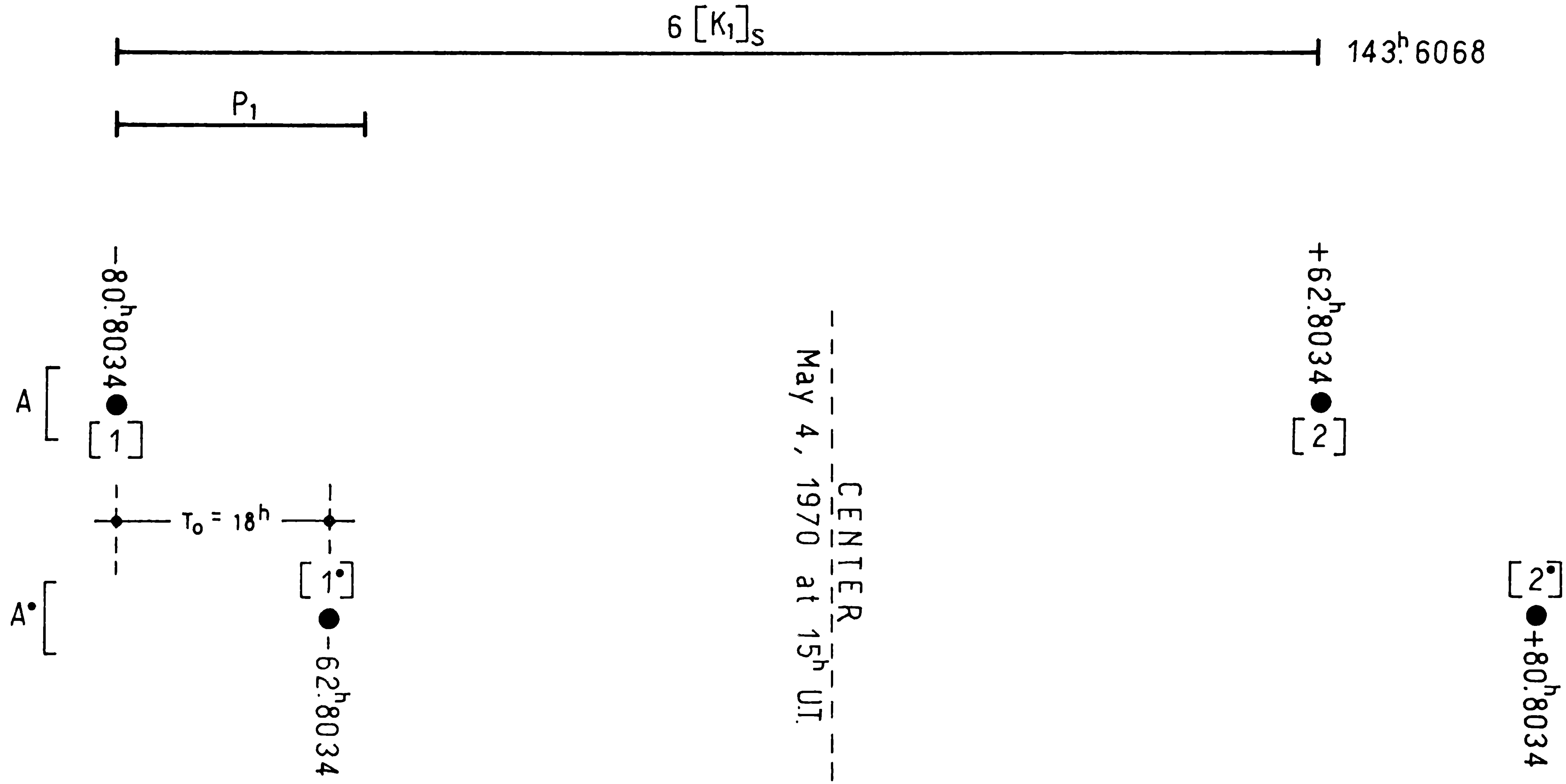

FIG. 15

TABLE 13

t (h)		$[K_1]_S$ $\cos\omega_1 t$	$\text{sen}\,\omega_1 t$	P_1 $\cos\omega_2 t$	$\text{sen}\,\omega_2 t$	Q
-80.8034	[1]	224.63 -0.7117	-0.7025	231.27 -0.6257	-0.7801	+ 4.217
+62.8034	[2]	224.63 -0.7117	-0.7025	219.47 -0. 7719	-0.6357	+ 5.034

t (h)		$[K_1]_S$ $\cos\omega_1 t$	$\text{sen}\,\omega_1 t$	P_1 $\cos\omega_2 t$	$\text{sen}\,\omega_2 t$	$Q^\bullet$
-62.8034	[1•]	135.37 -0.7117	+0.7025	140.53 -0.7719	+0.6357	+ 3.617
+80.8034	[2•]	135.37 -0.7117	+0.7025	128.73 -0.6257	+0.7801	+ 3.490

WAVE P_1

[2] - [1]

[2•] - [1•]

Equations

$$-0.1462\ A - 0.1444\ B = +\ 0.817$$

$$+\ 0.1462\ A - 0.1444\ B = -\ 0.127$$

Solution

$B = -2.39$ $\qquad R = 4.01$

$A = -3.23$ $\qquad \varphi = 216^\circ.\ 5$

$\text{Tg}\,\varphi = +\ 0.74003$

$C_{P_1} = 0.17$

WAVE $[K_1]_S$

$$[2] + [1]$$

$$[2^{\bullet}] + [1^{\bullet}]$$

Equations

$$-1.4234\ A + 1.4050\ B = +9.251 - (-1.3976\ A_{P_1} + 1.4158\ B_{P_1})$$

$$-1.4234\ A - 1.4050\ B = +7.107 - (-1.3976\ A_{P_1} - 1.4158\ B_{P_1})$$

Solution

$B = +3.14$ $\qquad R = 4.03$

$A = -2.53$ $\qquad \varphi = 128^{\circ}.9$

$Tg\varphi = -1.23952$

$C_{[K_1]_S} = 0.17$

V) FINAL RESULTS

78. In the following Table 14, there stand the values of the set of coefficients to which we have arrived after all the computations.

By comparison with the theoretical values it is seen that the greater the coefficients are, the better the internal accordance is.

Besides, this is quite natural.

Regarding the wave components $[K_1]$ and $[K_2]$ we have the chance of comparing the initial phase angles obtained for $[K_1]_L$ and $[K_1]_S$ on the one hand, and $[K_2]_L$ and $[K_2]_S$ on the other hand.

In the first case it has been obtained 127°.9 and 128°.9 for $[K_1]_L$ and $[K_1]_S$ respectively, which fit better than expected.

In the second case, for waves $[K_2]_L$ and $[K_2]_S$ we have obtained 54°.1 and 60°.7, which also represent a good internal agreement, keeping in mind the smallness of their coefficients.

It has not been our intention to achieve the wave component coefficients with a greater accuracy, because there are many other wave components that we have put aside on account of the smallness of their coefficients.

These waves mix up, of course, with the principal tidal constituents we have considered. We have been only depending on a seven days long tidal record.

TABLE 14

CLASS	SYMBOL	NAME	COEFFICIENT		Δ
			REAL	COMP.	
SEMI DIURNAL	M_2	PRINCIPAL LUNAR	0.908	0.88	-0.03
	N_2	LUNAR ELLIPTICITY	0.176	0.20	+0.02
	S_2	PRINCIPAL SOLAR	0.423	0.42	0.00
	K_2	LUNI - SOLAR	0.115	0.08	-0.04
DIURNAL	O_1	LUNAR DECLINATION	0.377	0.35	-0.03
	Q_1	LUNAR DIURNAL	0.073	0.075	0.00
	P_1	SOLAR DECLINATION	0.176	0.17	-0.01
	K_1	LUNI - SOLAR	0.530	0.54	+0.01

$$K_2 = [K_2]_L + [K_2]_S$$

$$K_1 = [K_1]_L + [K_1]_S$$

W' COMPUTED DIURNAL AND SEMIDIURNAL WAVES, VERSUS TIDAL RECORD.

Figs. 16 a, 16 b and 16 c present a comparison between the tidal wave (solid line), basically used for computations, and the results obtained (black dots) by adding the expressions 66, 67, 68 and 69 given in Sec. 70.

There are also plotted (white dots) the values which are obtained by adding the eight wave components ($M_2 + N_2 + S_2 + K_2 + O_1 + P_1 + Q_1 + K_1$) separeted from the lunar and solar diurnal and semidiurnal waves, all along the method and computations presented in the text.

The goal of the author was to produce a suitable method to separate tidal component waves from a short period tidal record.

Figs. 16 a, b, c, which only cover a time span of 48 hours (from May 3/1970 at 15 H.U.T. to May 5/1970 at 15 H.U.T.) show how the results fit with the original tidal wave record. The accordance is as good as expected.

In the case of the "black dots" , the variation in distance of the moon has not been taken into account. When introduced, the accordance is improved.

In the case of the "white dots", it must be kept in mind that only eight tidal components have been separated, while a lot of others, with smaller amplitudes, are set aside, provided that owing to their different initial phase angles the waves mix up to almost a self compensation.

The value z is easily obtained if formulas (11) and (12) are used.

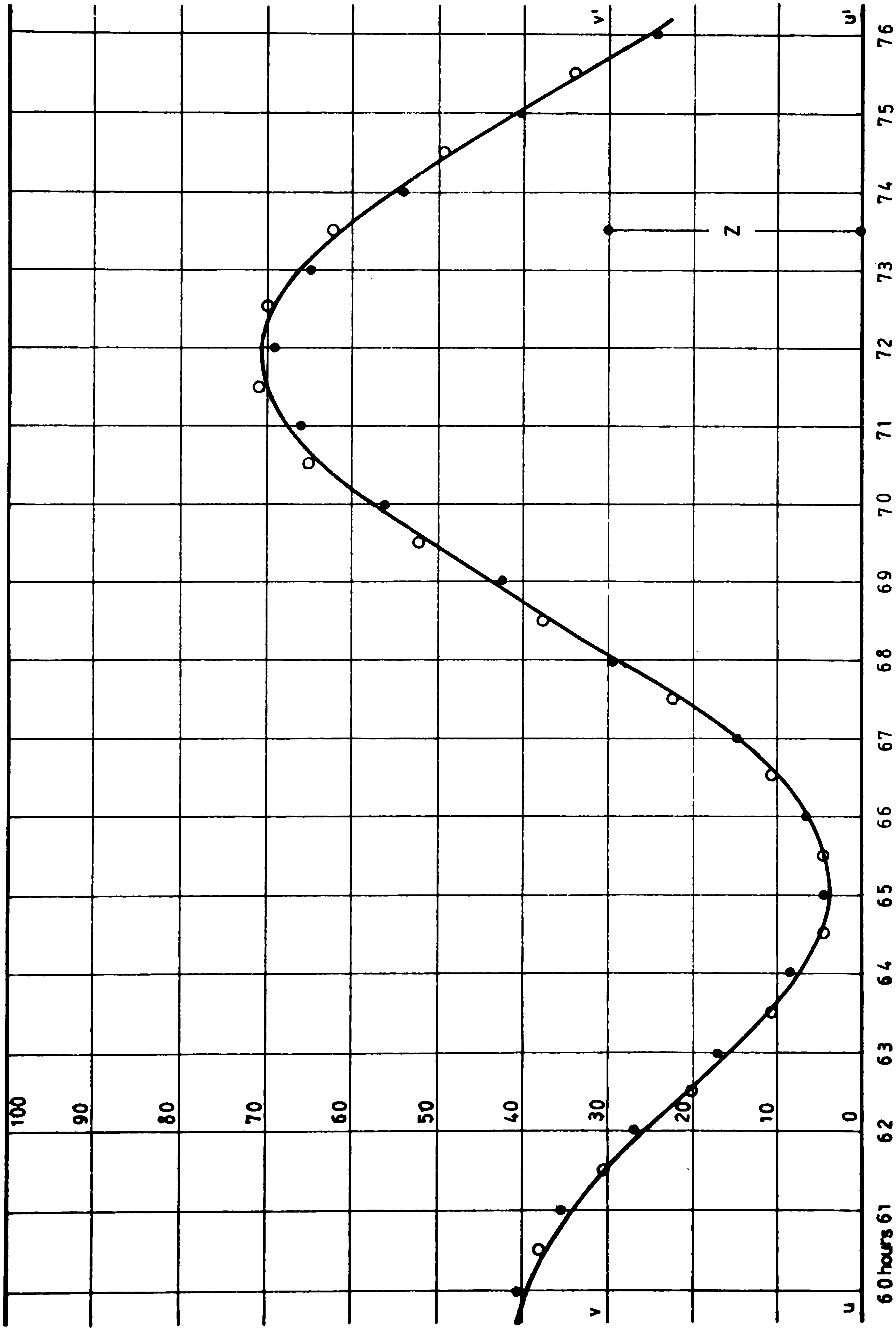
60 hours 61
62
63
64
65
66
67
68
69
70
71
72
73
74
75
76
0
10
20
30
40
50
60
70
80
90
100
v
v'
u
u'
Z

FIG.16 a

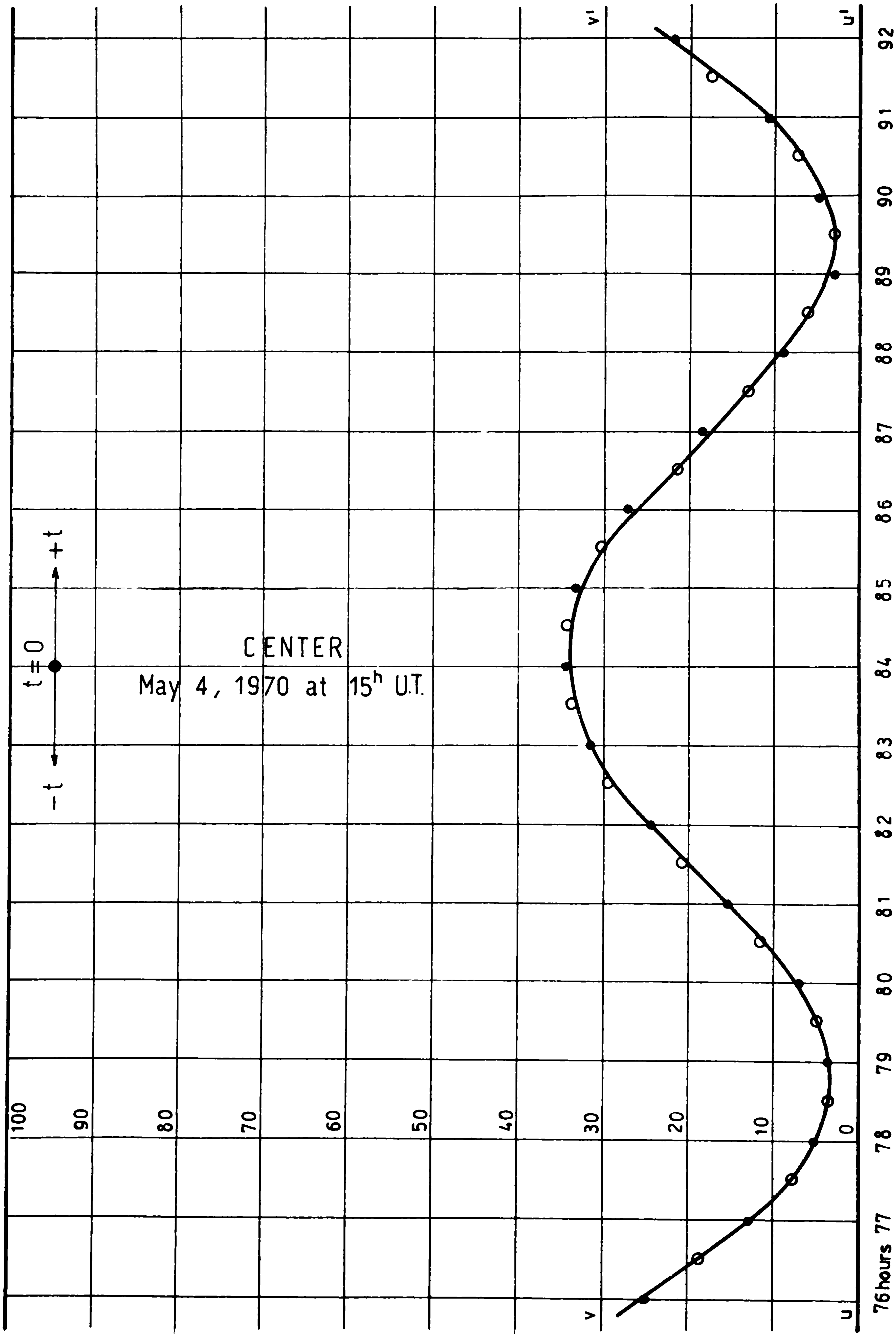
CENTER
May 4, 1970 at 15h U.T.
t=0
−t
+t
v
v'
u
u'
100
90
80
70
60
50
40
30
20
10
0
76hours 77
78
79
80
81
82
83
84
85
86
87
88
89
90
91
92

FIG.16b

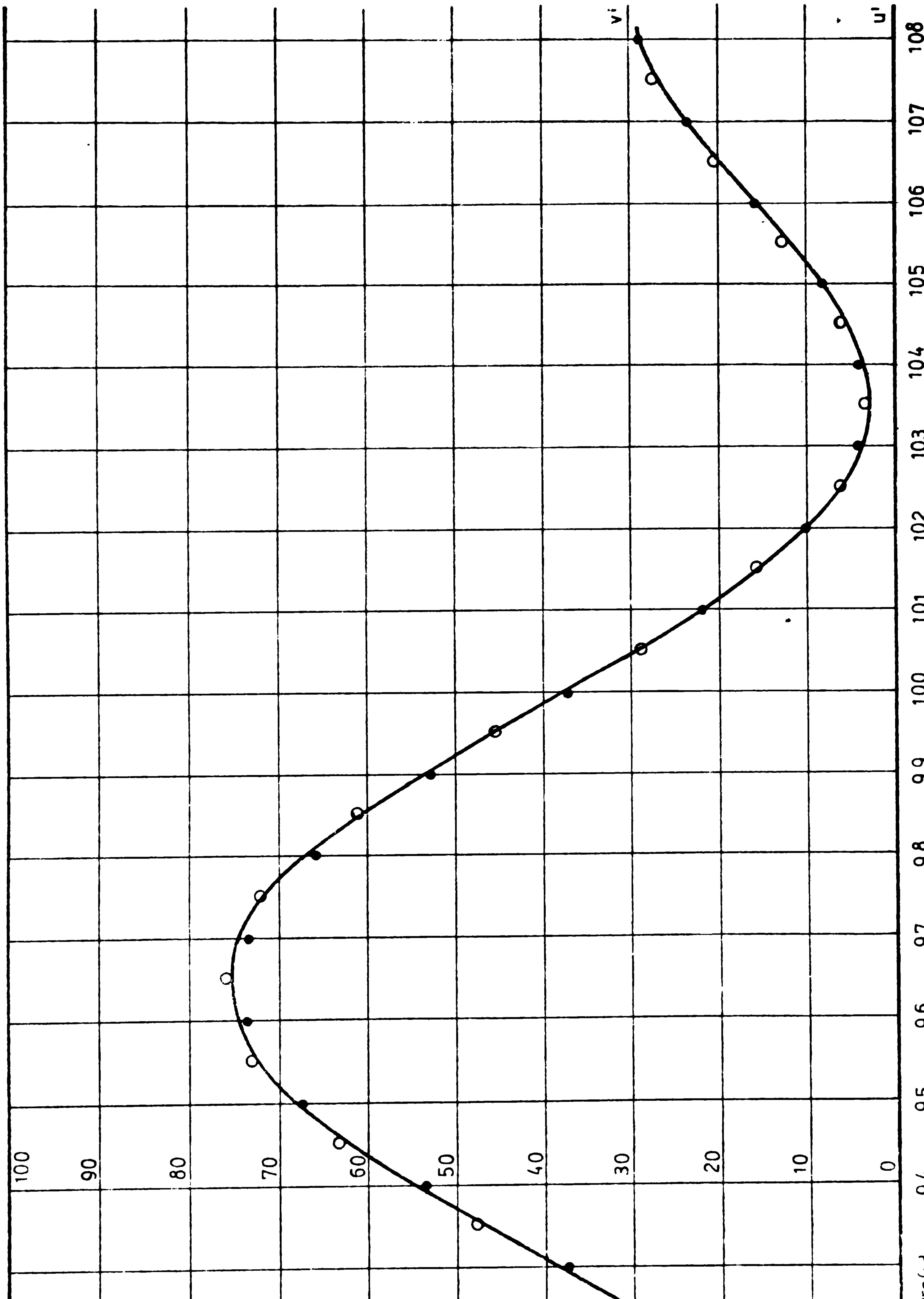
v
u
v'
u'
0
10
20
30
40
50
60
70
80
90
100
92 hours 93
94
95
96
97
98
99
100
101
102
103
104
105
106
107
108

FIG. 16 C