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Dynamics of damage spreading in the two-dimensional Ising magnet at criticality

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Abstract

The spreading dynamics of an initially small damage is studied for the two-dimensional Ising model at criticality using the Glauber dynamics. The number of damaged sites, $N_d(t)$, the survival probability of the damage, $P(t)$, and the mean square distance over which the damage spreads, $R^2(t)$, obey a simple power law behavior with critical exponents $\eta \simeq 1.11 \pm 0.03$, $\delta \simeq 0.58 \pm 0.03$ and $z^* \simeq 1.19 \pm 0.03$, respectively. It is found that the scaling relation $d_f = 2\eta/z^*$ gives the fractal dimension of the Ising droplets.

Damage spreading is a very useful method for extracting thermal properties from Monte Carlo simulations of Ising magnets (see Refs. [1-11], for a review see also Ref. [12]). Furthermore this numerical technique has very recently been employed for the study of irreversible phase transitions in reaction systems [13-15].

The damage spreading problem consists, first, in taking a steady state configuration of the system, $\{\sigma^A\}$, and to create, at $t = 0$, an initial damage $D(0)$ in that configuration (this procedure gives a second configuration $\{\sigma^B\}$). Then, one investigates the time evolution of both configurations using the same dynamics calculating their Hamming distance, defined by

$$D(t) = \frac{1}{N} \sum_{i=1}^{N} | \sigma_i^A(t) - \sigma_i^B(t) |, \qquad (1)$$

where N is the number of sites of the system. Physically $D(t)$ measures the fraction of sites for which both configurations are different. Starting with a small $D(0)$ value, $D(t)$ will go asymptotically to zero in the "frozen phase", whereas it will tend to a finite value different from zero in the "chaotic phase".

The aim of this work is to report results obtained studying the dynamics of damage spreading in the 2D Ising model using the Glauber dynamics.

The 2D Ising model is simulated in a square lattice of side L assuming periodic boundary conditions. The spin σ_i associated to the node i of the lattice takes either of the two values ± 1. Interactions between spins are described through the Hamiltonian H given by

$$H = -J \sum_{\langle i, j \rangle} \sigma_i \sigma_j, \qquad (2)$$

where $\langle i, j \rangle$ indicates nearest neighbor nodes and $J > 0$ is the coupling constant of the ferromagnet. The model is simulated using the Glauber dynamics,

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so a randomly selected spin is flipped with a probability $P(\text{flip})$ given by [12]

$$P(\text{flip}) = \exp(-\beta \Delta H)/[1 + \exp(-\beta \Delta H)], \quad (3)$$

where ΔH is the difference between the energy of the would-be new configuration and the old configuration. Reported results are obtained at the critical temperature T_c, so $\beta = 1/kT_c$.

In order to study the dynamics of damage spreading an equilibrium configuration $\{\sigma^A\}$ is generated and replicated identically in a second system $\{\sigma^B\}$, except for the central spin which takes opposite directions in both configurations. Then according to Eq. (1) the initial damage is $D(0) = 1/L^2$. Then the Monte Carlo procedure is implemented in the standard sequential manner but equivalent sites in both configurations are visited randomly at the same time and the same random number is employed in order to update the systems according to Eq. (3). This procedure assures that the configurations σ^A and σ^B undergo the same dynamics. The Monte Carlo time unit involves L^2 trials. Runs are performed using a multitransputer system with five T-805 processors working in parallel. Results are averaged over 2×10^5 (2×10^4) different configurations for $L = 13$ ($L = 100$), respectively. The data shown for $L = 100$ only have required about one month of CPU time.

Starting from a damaged spin localized at the center of the sample at $t = 0$, the following dynamic properties are evaluated: the survival probability of the damage $\{P(t)\}$, the average number of damaged spins per lattice site $\{N_d(t)\}$, and the mean square distance over which the damage spreads $\{R^2(t)\}$. Both $N_d(t)$ and $R^2(t)$ are evaluated only for those samples having a nonvanishing damage at time t.

It is expected that at criticality, the measured properties would exhibit power law behavior according to

$$N_d(t) \propto t^{\eta}, \quad (4a)$$

$$P(t) \propto t^{-\delta}, \quad (4b)$$

and

$$R^2 \propto t^{z^*}, \quad (4c)$$

where η, δ and z^* are critical exponents.

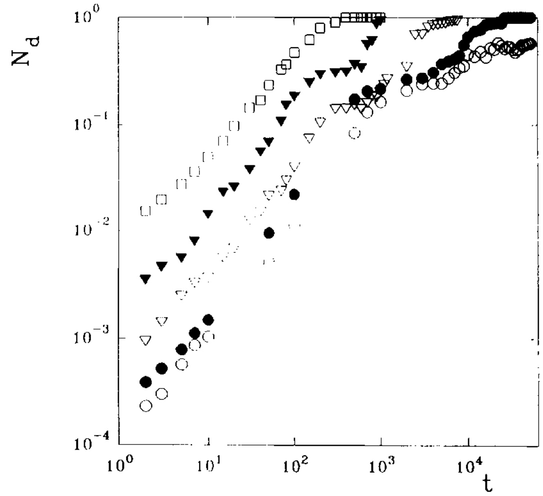

Fig. 1. Log-log plot of $N_d(t)$ versus t for lattices of different sizes. (○) $L = 12$, (●) $L = 25$, (▿) $L = 50$, (▾) $L = 75$, (□) $L = 100$.

Fig. 1 shows log-log plots of N_d versus t obtained using lattices of different sizes. It is observed that N_d grows from the initial value $N_d(0) = 1/L^2$ and then reaches a plateau $N_d(t \geqslant \tau) = 1$, where at time τ one has the maximum (unitary) damage. The obtained unitary damage for $t \geqslant \tau$ implies that the reference configuration σ^A is the mirror image of the initially damaged configuration σ^B. This is a dramatic evidence of a macroscopic effect that can be caused by a microscopical initial damage. This effect can be understood considering that for finite lattices the order parameter assumes a finite value even at criticality (i.e. the spontaneous magnetization takes a value $|M_{sp}(L)| > 0$ for $T = T_c$). For a finite system, there is always a nonzero probability that the system may pass from a state near $+|M_{sp}(L)|$ to a state near $-|M_{sp}(L)|$, as well in the opposite direction. So, for $T > T_c$ the order parameter probability distribution can be described by a single peaked Gaussian, while for $T < T_c$ one has a double peaked Gaussian at $\pm |M_{sp}(L)|$. At criticality both peaks approach each other and stay at a distance of the order of $L^{-\beta/\nu}$ [16]. Accordingly the initial damage may induce the damaged configuration σ^B to evolve towards the mirror image of the reference configuration σ^A. After that the damage remains unitary due the symmetry of the Glauber dynamics given by Eq.

(3). Obviously, this behavior cannot be observed using other dynamics, e.g. the heat bath dynamics.

The ergodic time τ necessary to observe the excursion of the order parameter from a state near $+|M_{sp}(L)|$ to a state near $-|M_{sp}(L)|$, as well as in the opposite direction behaves as [17]

$$\tau \propto \xi^2 \propto |1 - T/T_c|^{-\nu z}, \quad (5)$$

where $\xi \propto |-T/T_c|^{-\nu}$ is the correlation length, ν the correlation length exponent and z the standard dynamic exponent. However, just at criticality and in a finite system the correlation length is of the order of the lattice size L, so $\xi \simeq L$ and from Eq. (5) it follows that

$$\tau \propto L^z. \quad (6)$$

This behavior agrees qualitatively with results shown in Fig. 1. In fact the time required for a complete inversion of the spins in both configurations depends on L. Furthermore, a log-log plot of τ versus L (Fig. 2) shows that the obtained data is consistent with Eq. (6) and $z \simeq 2.16 \pm 0.02$ [18]. Note that in Fig. 2 we have also included the L-dependent times at which the damage spreads over half of the lattice sites.

Based on the already discussed results one may expect that log-log plots of N_d versus the scaled time t/τ would exhibit data collapsing. In fact this behavior is nicely observed in Fig. 3. A least-squares fit of the observed straight line for $t < \tau$ according to Eq. (4a) gives $\eta \simeq 1.11 \pm 0.03$.

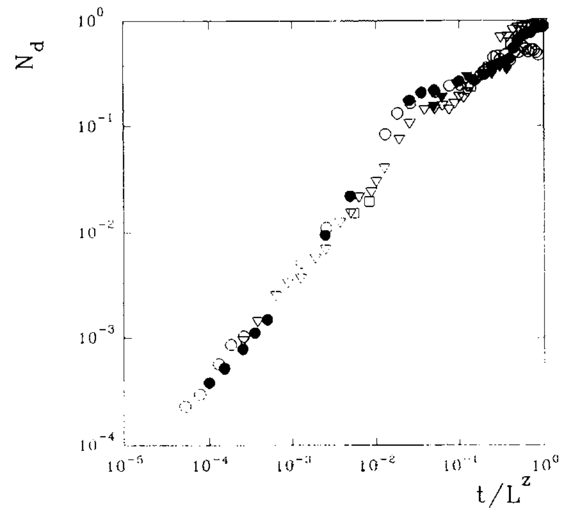

Fig. 3. Log-log plot of $N_d(t)$ versus the scaled time t/L^z for the same lattices as shown in Fig. 1.

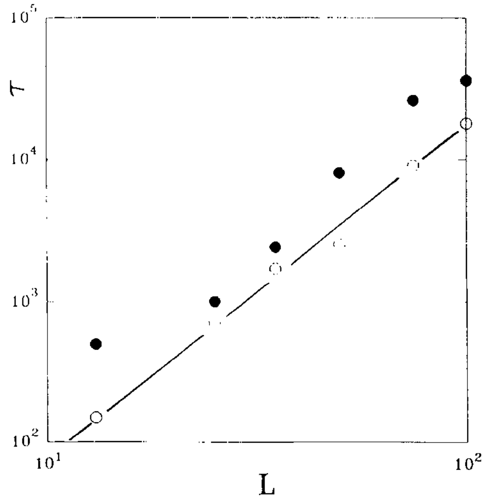

Fig. 2. Log-log plot of τ versus L. (●), (○) correspond to the times required for the damage to spread over half and the whole lattice, respectively. The straight line has slope $z = 2.16$ and has been drawn for comparison.

Fig. 4a shows log-log plots of the survival probability of the damage versus t. In agreement with the behavior already discussed for $N_d(t)$ versus t, one has that at early times $P(t)$ decreases and for $t > \tau$ reaches a constant value ($P_s(t \to \infty)$) independent of t since mirror configurations will remain so forever. A least-squares fit of the data for $t \ll \tau$ gives $\delta \simeq 0.58 \pm 0.03$. As it follows from Fig. 4a P_s depends on the lattice size. This behavior is shown in Fig. 4b, where a simple power law dependence of the type $P_s \propto L^{-x}$, with $x \simeq 0.93 \pm 0.05$, is found.

Since N_d gives the damage of the surviving samples, the average damage of all samples ($D(t)$) can be written as $D(t) = N_d(t)P(t)$ and therefore $D(t)$ increases according to a power law behavior given by t^γ, with $\gamma = \eta - \delta \simeq 0.53 \pm 0.06$. This results shows that at criticality one has damage spreading. In contrast, we have observed damage healing slightly below T_c.

Fig. 5 shows log-log plots of R^2 versus t. The observed plateau for $t > \tau$ is consistent with the already discussed inversion of the spins. A least-squares fit of the data for $t < \tau$ gives $z^* \simeq 1.19 \pm 0.03$.

The fractal dimension of the damaged cloud (d_f) can be defined as

$$N_d(t) \propto R^{d_f} \tag{7}$$

so using Eqs. (4a) and (4c) one has $d_f = 2\eta/z^*$. Now replacing the obtained values of the exponents it follows that $d_f \simeq 1.87$. This figure is in agreement, within error bars, with the fractal dimension of Ising droplets at criticality given by $d_f = d - \beta/\nu = 15/8 = 1.875$. Note that Ising droplets are made putting bonds between neighboring spins of the same domain with probability $P(T) = 1 - \exp(2J/kT)$ [19].

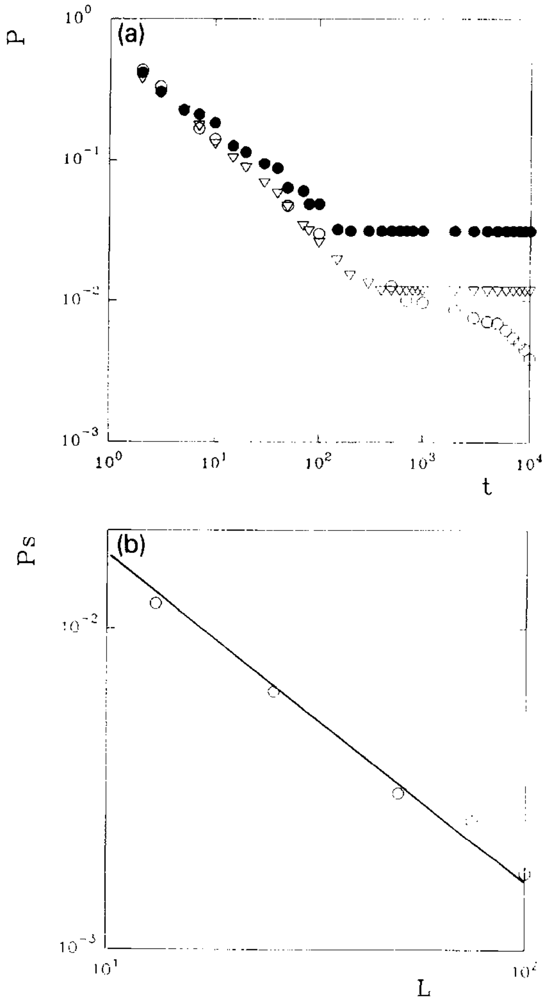

Fig. 4. (a). Log-log plots of $P(t)$ versus t for lattices of different sizes. (●) $L = 25$, (▽) $L = 75$, (□) $L = 100$. (b) Log-log plot of P_s versus L. The straight line has slope $x = 0.93$.

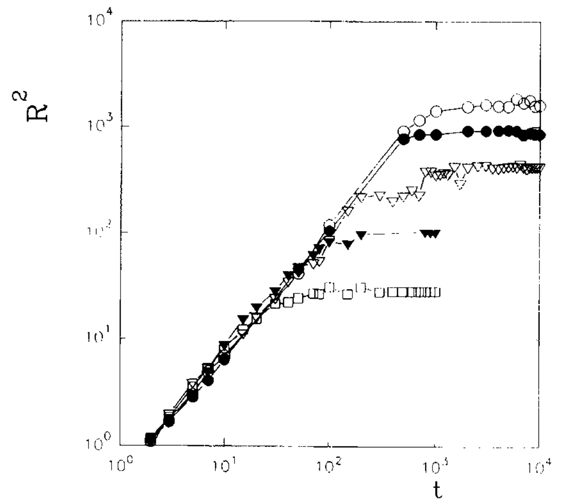

Fig. 5. Log-log plots of R^2 versus t for lattices of different sizes (□) $L = 12$, (▼) $L = 25$, (▽) $L = 50$, (●) $L = 75$, (○) $L = 100$.

The numerical determination of the fractal dimension of the damaged cloud in the 2D Ising ferromagnet at T_c has given a value $d_f \simeq 1.87 \pm 0.02$ [3], in good agreement with the present result.

To conclude, the dynamics of damage spreading has been studied in the two-dimensional Ising magnet at criticality. The number of damaged sites, the survival probability of the damage and the average square distance over which the damage spreads obey a simple power law behavior with dynamic critical exponents $\eta \simeq 1.11 \pm 0.03$, $\delta \simeq 0.58 \pm 0.03$ and $z^* \simeq 1.19 \pm 0.03$, respectively. By means of the scaling relation $d_f = 2\eta/z^*$ the fractal dimension of the Ising droplets given by $d_f = d - \beta/\nu$ is obtained, within error bars. These results lead us to conjecturing the following relationship,

$$d(1 - \eta/z^*) = \beta/\nu, \quad d = 2,$$

which relates the static critical exponents β and ν with the dynamic critical exponents η and z^* characteristic of the damage spreading process.

A microscopic damage due to a single flipped spin could induce a complete inversion of the damaged lattice with respect to the reference one.

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