INVERSION OF POROELASTIC PARAMETERS AND SONIC WAVE VELOCITY MODELING IN VACA MUERTA FORMATION

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RESUMEN

La Formación Vaca Muerta, ubicada en la Cuenca Neuquina, Argentina, está entre los mayores reservorios no convencionales de tipo shale (lutitas orgánicas) a nivel mundial, resultando un objetivo de alto interés en exploración y caracterización geofísica. Debido a su génesis y a su composición multiminerálica dichas lutitas presentan una gran heterogeneidad espacial, especialmente en la dirección vertical, pudiendo alcanzar espesores hasta 350 m. Dadas las características de estas rocas, la construcción de modelos elásticos que además de la existencia de poros y fluidos, incorporen su descripción litológica detallada así como parámetros físicos realistas para cada mineral y materia orgánica constituyen un desafío. Con estas ideas, utilizaremos la teoría poroelástica desarrollada por Brown y Korringa (1975), extendiendo la de Gassmann (1951) a medios porosos de matriz no homogénea, para modelar y ajustar velocidades sónicas P y S medidas en un pozo que atraviesa gran parte de la mencionada formación. En este modelo la compresibilidad monominerálica es reemplazada por otras dos: una asociada al volumen poral y otra a la matriz multifásica, cuyos valores pueden determinarse en laboratorio pero no son simples de calcular analíticamente. La falta de conocimiento de los parámetros de este modelo ha limitado fuertemente su aplicación práctica. Esto constituye una de las motivaciones de este trabajo, en el cual proponemos su determinación mediante técnicas de inversión numérica, lo cual no había sido hecho hasta el momento. La implementación de este procedimiento implica la modelar las propiedades elásticas de la matriz, habiéndose elegido el modelo de porosidad crítica, ampliamente aceptado en rocas sedimentarias clásticas. Como resultado de este estudio se obtiene una representación poroelástica macroscópica por intervalos de profundidad, se comprueba el grado de ajuste entre las velocidades medidas y calculadas, se analiza estadísticamente la distribución de los parámetros invertidos y se analiza la heterogeneidad elástica .

Palabras Clave: física de rocas, poroelasticidad, inversión, modelado, Vaca Muerta

INTRODUCTION

As is well known, the Vaca Muerta (VM) formation is the main source rock within the Neuquén basin (Argentina), which during the last decade has gained great interest as unconventional reservoir. From a lithological point of view it is characterized by organic rich shales, with variable organic content and mineralogical composition, mainly given by clay minerals, carbonates, quartz, feldspars and pyrite. The elastic modeling in this type of rocks is a complex problem due to the ambiguity in the determination and definition of its porosity, being a parameter of great influence on its mechanical behavior. On the other hand, the properties of the matrix are difficult to model due to the effects of rock-fluid interaction, particularly associated to the high volume fraction of clay minerals, with very variable elastic properties (Dvorkin et al., 2007). On the other hand, the physical properties of the kerogen are difficult to measure and the information about them is scarce. They vary according to its type, composition and maturation state, with few measurements reported in the literature (Vernik, 2016). Understanding the relation between the bulk elastic parameters and wave velocities in these rocks is a very important task in unconventional rock physics.

In their classical paper Brown and Korringa (1975) developed a theory to describe the elastic behavior of porous saturated rocks with microheterogenous frames, valid for isotropic and anisotropic media. However, it has been almost unused for practical applications in geophysics due to the difficulties in the determination of the parameters involved. With these ideas, in this work we propose and test a simple modeling workflow for composite porous rocks which does not require detailed petrophysical information about mineral volume fractions and their corresponding elastic properties. This is particularly convenient when dealing with multiphase rocks, such as organic rich shales, which are formed by many different minerals and organic matter (besides of pore fluids). The physical parameters of such amount of constituents may introduce errors and uncertainty in the computations. To overcome this problem we show that it is possible to calibrate the model coefficients using a numerical inversion procedure without using a detailed description by means of Brown and Korringa and Gassmann (1951) formulations. The application of the procedure is illustrated using real data of a well across the Vaca Muerta shale formation. We analyze appropriate search ranges for the different model coefficients, with special attention on the feasibility of the inversion of the unjacketed pore modulus from velocity data. The overall goodness of fit between real and synthetic velocities is quantified, the statistical distribution of the inverted coefficients, as well as the determination of the critical porosity and the degree of elastic heterogeneity in this shale, are also discussed.

THEORETICAL BACKGROUND

The modeling and analysis of the elastic behavior of porous saturated rocks under variable pressures has long been studied by many authors from different fields. To take into account the role that pore fluid pressure plays in the deformation of porous media under different conditions (drained, undrained, jacketed, unjacketed), different compressibilities can be defined. This was analyzed by different authors, being the pioneer works of Gassmann (1951), Brown and Korringa (1975), Zimmerman et al. (1986) and Zimmerman (1991) for homogeneous rocks and single-phase fluids of particular importance in this subject.

Let us consider a composite fluid saturated rock of volume V_b composed by a solid rock matrix volume V_m formed by a mineral aggregate (which may also include organic mater, non-connected pores and unmovable fluids), and a pore volume V_p formed by connected pores. From now on we assume elastic isotropic behavior and we denote ϕ the connected or effective porosity of the medium. In what follows we review some fundamental concepts and definitions.

Undrained compression for homogeneous rock matrix: *Gassmann*'s equation

The compression of porous saturated rocks under *undrained conditions* (i.e. when fluids are not allowed to escape from the sample) is relevant in the deformation of low permeability rocks and wave propagation in porous media saturated with viscous fluids. In this situation the external confining pressure P_c and the pore pressure P_p are coupled to each other. For this case the associated bulk compressibility and modulus are defined as

$$C_{\text{sat}} = \frac{-1}{\bar{V}_b}\left(\frac{\partial V_b}{\partial P_c}\right)_{m_f} = \frac{1}{K_{\text{sat}}}, \tag{1}$$

for constant fluid mass (i.e. no flow). For the case of an homogeneous monomineralic rock matrix, we introduce an intrinsic mineral compressibility $C_0 = 1/K_0$ being $K_0$the corresponding mineral bulk modulus. This compressibility can be obtained from an *unjacketed test*, in which the rock matrix is compressed uniformly from inside and outside at the same time, so that the involved elastic modulus is that of the solid grains forming the matrix (Carcione, 2007, Zimmerman, 1991). Also, we denote K_{dry} the bulk modulus of the dry rock and K_f the pore fluid bulk modulus. The coefficient K_{sat} is known as the *Gassmann's* bulk modulus, and can be written in the form (Berryman and Milton 1991, Saxena et al. 2018)

$$K_{\text{sat}} = \frac{1}{C_{\text{sat}}} = K_{\text{dry}} + \frac{\alpha^2}{\frac{\alpha}{K_0}+\phi\left(\frac{1}{K_f}-\frac{1}{K_0}\right)}, \quad \alpha = 1 - \frac{K_{\text{dry}}}{K_0} \tag{2}$$

The coefficent α is the well known Biot and Willis coefficient for the poroelastic effective pressure law (Biot and Willis, 1957).

Undrained compression for heterogeneous matrix: *Brown and Korringa* equation

Taking into account the mixed mineralogy of most rocks Brown and Korringa (1975) extended Gassmann's equation to allow for arbitrarily mixed mineralogy. This extension was accomplished by adding one additional compressibility and replacing the mineral bulk modulus with a more general coefficient, somewhat less intuitive (Mavko and Mukerji, 2013): the *unjacketed bulk compressibility* C_M and its inverse K_M

$$C_M = \frac{-1}{\bar{V}_b}\left(\frac{\partial V_b}{\partial P_p}\right)_{P_d} = \frac{1}{K_M}, \tag{3}$$

which corresponds to an unjacketed compression applied on the rock at constant differential pressure. Since the rock matrix is compressed from inside and outside, with the same incremental pressure, the change in volume depends on the deformation of the mineral grains. Thus C_M represents the change in bulk volume of the heterogeneous solid aggregate. In the particular case of homogeneous (monomineralic) matrix clearly

$$C_M = C_0 \text{ and } K_M = K_0. \tag{4}$$

Since most rocks are multimineralic it is important to distinguish C_M from C_0. The other additional compressibility, and the least intuitive of this theory, is the *unjacketed pore compressibility,* and its associated *pore bulk modulus,* defined as

$$C_\phi = \frac{-1}{\bar{V}_p}\left(\frac{\partial V_p}{\partial P_p}\right)_{P_d} = \frac{1}{K_\phi}, \tag{5}$$

which quantify the variations in the pore volume under the same incremental pore and confining pressures. For the particular homogeneous case, the following equalities hold (Brown and Korringa,1975, Saxena et al. 2018)

$$C_{\phi}=C_{M}=C_0 \text{ and } K_{\phi}=K_{M}=K_0. \quad (6)$$

The undrained compressibility and bulk modulus in BK theory can be written in the form (Berryman and Milton, 1991)

$$K_{\text{sat}}=\frac{1}{C_{\text{sat}}}=K_{\text{dry}}+\frac{\beta^2}{\frac{\beta}{K_M}+\phi\left(\frac{1}{K_f}-\frac{1}{K_{\phi}}\right)}, \quad \beta=1-\frac{K_{\text{dry}}}{K_M} \quad (7)$$

It must be remarked the similarity of this expression with Gassmann's equation (2), being clearly a particular case for monomineurallic solids. Different authors have proposed experimental procedures for the determination of these compressibilities (such as Duranti 2018, Müller and Sahay 2012 and others), while others analyzed their analytical computation using effective medium theories for simplified geometries. The values of C_M,C_{ϕ} were discussed in several papers such as Berge and Berryman (1995), Berge (1998), Mavko and Mukerji (2013), Wollner and Mavko (2017) and Duranti (2018). As explained in the following section, in this paper the coefficients $K_M K_{phi}$ and K_0 (and others) will be determined using a *numerical inversion procedure*.

Shear modulus and wave velocities

To complete the elastic description of the rock, it is necessary to recall the rigidity modulus of the composite saturated rock μ, which in agreement with Gassmann's theory, is taken equal to that of the solid frame (dry rock) μ_{dry}

$$\mu=\mu_{\text{dry}}. \quad (8)$$

This also holds for multimineral matrix, taking into account that in a static pure shear experiment on the porous saturated medium, the pore fluid does not support shear stresses and consequently does not change the rigidity of the rock. Finally, using (7) and (8) and denoting ρ_b the bulk density of the porous saturated rock, we can compute the elastic compressional and shear wave velocities for this model in the form

$$V_P^m=\sqrt{\frac{K_{\text{sat}}+\frac{4}{3}\mu}{\rho_b}},\ V_S^m=\sqrt{\frac{\mu}{\rho_b}}. \quad (9)$$

Elasticity of the rock frame: the *critical porosity* model

For modeling purposes to estimate the elastic properties of the dry rock we choose the *critical porosity* model proposed by Nur et al. (1998) based on the observation that for most porous materials there is a limiting value for porosity, denoted ϕ_c, from which the mechanical behavior of the aggregate is that of a suspension (Mavko et al. 2009). Beyond this critical value, the rock loses its rigidty and the bulk and shear moduli of the dry rock can be estimated using the simple Reuss average. For porosities lower than ϕ_c the mineral grains are load-bearing and the moduli decrease linearly from the mineral values at zero porosity K_0,μ_0 to the suspension values at the critical porosity. Then in the isotropic homogeneous case, the moduli are given by the following linear functions

$$K_{\text{dry}}=\text{K}_0\left(1-\frac{\phi}{\phi_c}\right), \quad \mu_{\text{dry}}=\mu_0\left(1-\frac{\phi}{\phi_c}\right). \tag{10}$$

For the multimineral matrix case, according to BK theory, the modulus K_0 is *replaced* by the unjacketed bulk modulus K_M and, for consistency, we also introduce a generalized shear modulus μ_M that replaces μ_0. It is important to remark that for the practical application the organic matter, will be considered as a part of the rock frame and treated as an additional mineral within K_M,μ_M. As pointed out by Mavko et al. (2009) the relationship between these elastic parameters, the elastic moduli of the individual mineral constituents and their volume fractions is not clear. That encouraged us to determine these coefficients numerically using an inversion technique as explained in the next section.

APPLICATION USING DATA FROM VACA MUERTA SHALE

Description of data

For the practical application of the elastic workflow described we use information of a vertical well located in the transition between the black oil to live oil zones within Neuquén basin, whose name and location are kept confidential.

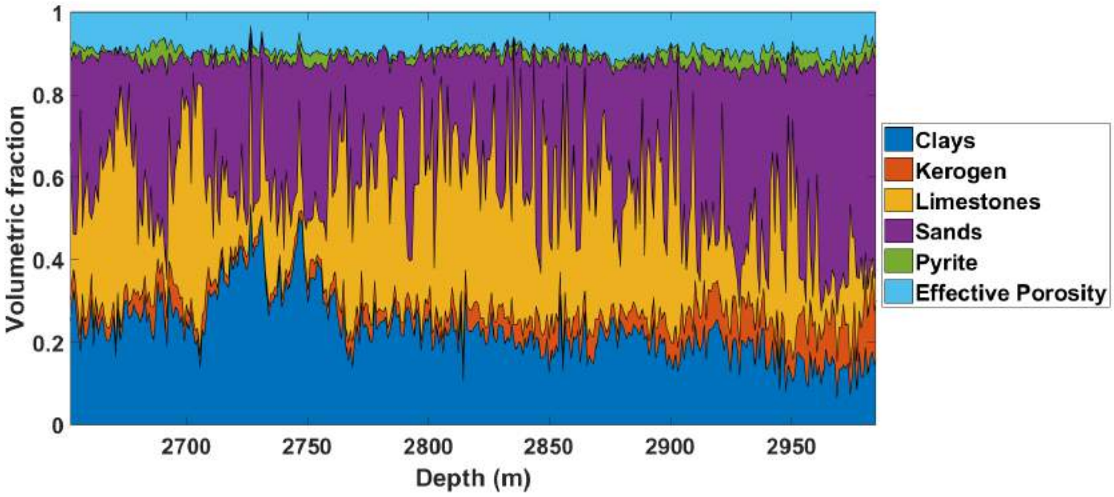

Figure 1. Lithological description of VM shale including kerogen fraction and effective porosity.

We selected a depth interval within Vaca Muerta shale formation between 2650 to 2985 m, (335 meters thick), in which we have sonic compressional and shear wave velocities (2196 data spaced every 0.15 m), ranging from 2700 to 5030 m/s for P waves and from 1500 to 2400 m/s for S waves. Bulk density, effective porosity, lithological description (mineral volume fractions), kerogen fraction and water saturation logs are also available. The lithological description profiles are given in terms of mineral groups, mainly: clay, carbonate, pyrite and sands in variable proportions. However, for the implementation of the workflow we don't use these detailed volume fractions. This information is shown in Figure 1 to illustrate the vertical heterogeneity in the well at different scales. This same data set was previously used by Ravazzoli et al. (2017) and Blanco et al. (2018), who presented highly detailed elastic workflows.

The characterization of the hydrocarbon fluids was done in laboratory using PVT analysis, obtaining that the API gravity of oil is 41.2 and the specific gravity of hydrocarbon gas is 0.732. Moreover, petrophysical measurements were done on a set of core samples in which the saturation of water, gas and oil were determined from which the oil/gas fraction will be assumed constant along the well being in average 75% oil, 25% gas. The physical properties of the fluids (density and bulk modulus) were estimated using the semi-empirical equations of Batzle and Wang (1992). For the computations, the effective bulk modulus of the mixture of pore fluids were obtained using the saturations and the classical isostress Reuss's average. The effective fluid density was computed as weighted average of the individual values. The effective porosity ranges from 0.5 to 15%, with an average of 9%.

Implementation of the modeling and inversion workflow

For the implementation of the workflow we subdivided the logs in a number of non overlapping windows containing N data points (velocities, bulk density, effective porosity, fluid saturations), with depth spacing of 0.15 m. Our goal is to obtain an elastic macroscopic representation valid within each window of the profiles. In this example we take 366 windows including N=6 data points, so that we obtain a macromodel for a window scale of about 0.9 m. However the window size can be selected according to the vertical resolution desired.

The calibration of the model requires the determination of the poroelastic parameters: $K_{\phi}, K_M, \mu_M, \phi_c$ within each window, using the measured log densities and porosities. To determine the unknown parameters we define for each window, an L_2 norm scalar cost function $Q(K_{\phi}, K_M, \mu_M, \phi_c)$, measuring the departure between real and synthetic velocity data computed using eqns. (9) combined with (7), (8) and (10). This velocity model is hereafter denoted as **BKCP**.

To obtain the optimum parameters, those for which Q results minimum, we use the *pattern search* algorithm. To obtain significant results we need to define appropriate search ranges for each parameter. For K_M, μ_M, the search range was taken sufficiently large (between 2 and 100 GPa) to include the elastic moduli for most minerals, using standard values taken from the literature (Mavko et al. 2009). Regarding the selection of a suitable numerical range for K_{ϕ}, it should be noted that this parameter is the least intuitive of all. Berge and Berryman (1995) and Berge (1998) found that $K_{\phi}$and K_M are independent parameters and also that the pore compressibility of certain composite materials can be negative when the bulk moduli of the components differ by at least a factor of 5, a situation possible in a mixture of sands and clays. Many years later Mavko and Mukerji (2013) found positive values lower than 30 GPa and Wollner and Mavko (2017) found values in the range 20-60 GPa and recently Duranti (2018) reported experimental values for sandstones between about 0 and 15 GPa. The optimum critical porosities were searched in the range 0.2 to 0.75 in agreement with Bachrach et al. (2013).

For further analysis, we also implemented a similar inversion procedure based on the classical Gassmann's model for *homogeneous frames*, using eqns. (2) combined with (8), (9) and (10). In this case the parameters inverted are K_0,$\mu_{0,}\phi_c$ and this model will be referred to as **GCP**. It is worthwhile to remark that for the computations we use the measured bulk density log. Once we determined the model parameters we compute the synthetic velocity on each data point and evaluate the goodness of fit using the root mean square error (RMS). We remark that for each combination of parameters within the model space, the positive definiteness of the elastic strain energy was verified.

RESULTS AND DISCUSSION

Using the workflow described in the previous section, next we show its application, which involves the inversion of the set of poroelastic parameters, to fit the sonic velocities in a thick interval of the Vaca Muerta formation. In what follows we focus on the following topics:

- fitting of the measured sonic P and S wave velocities using BKCP model for mixed mineralogy, evaluating the overall goodness of fit;
- comparison of fitting results using the simpler GCP approach for an *equivalent homogeneous* frame;
- analyze the feasibility of inverting the unjacketed pore modulus K_ϕ for each window and an appropriate search range;
- inversion and analysis of the unjacketed moduli K_M,μ_M within each window;
- comparison with the moduli obtained using GPC, i.e K_0,μ_0 and
- the determination of critical porosities $\phi_c$along the well.

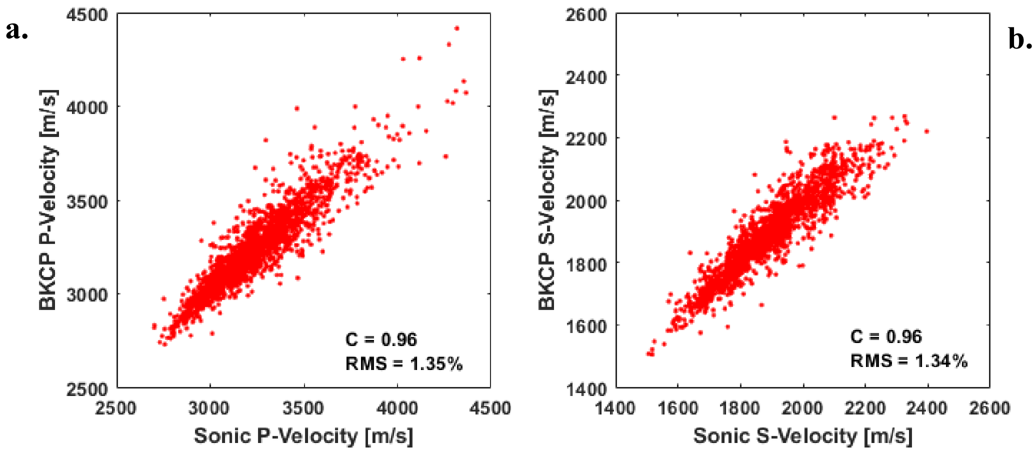

Figure 2. Synthetic vs. real sonic velocities using BKCP model **a.** for P-waves, **b.** for S-waves.

Figure 2 shows cross plots between observed and calculated velocities using BKCP model, in which, despite of the apparent dispersion, we obtained very good quality of fitting. This is quantified by means of the low RMS errors and high correlation coefficient C obtained, being on the same order for both models.

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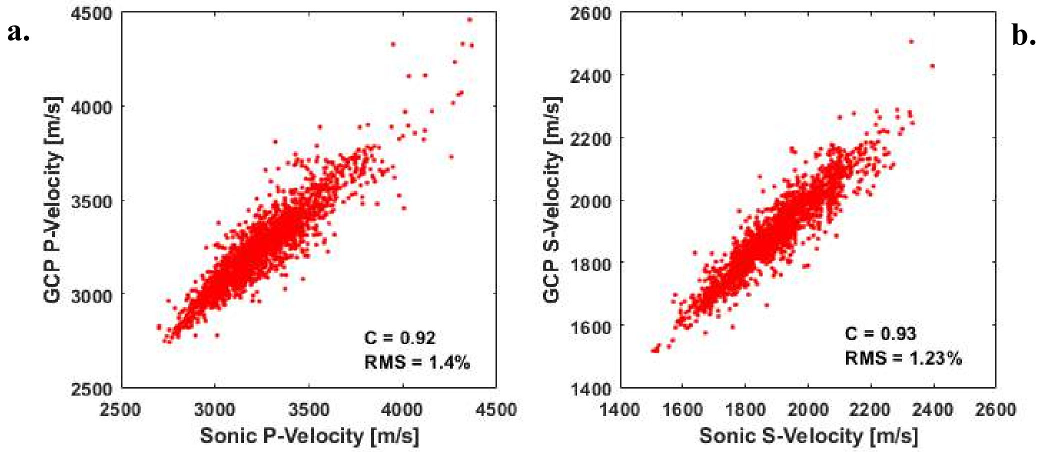

Figure 3. Synthetic vs. real sonic velocities using GCP model, **a.** for P-waves, **b.** for S-waves.

In Figure 3 we show the analogous results using the homogeneous approach given by the GCP model, obtaining similar quality of fit.

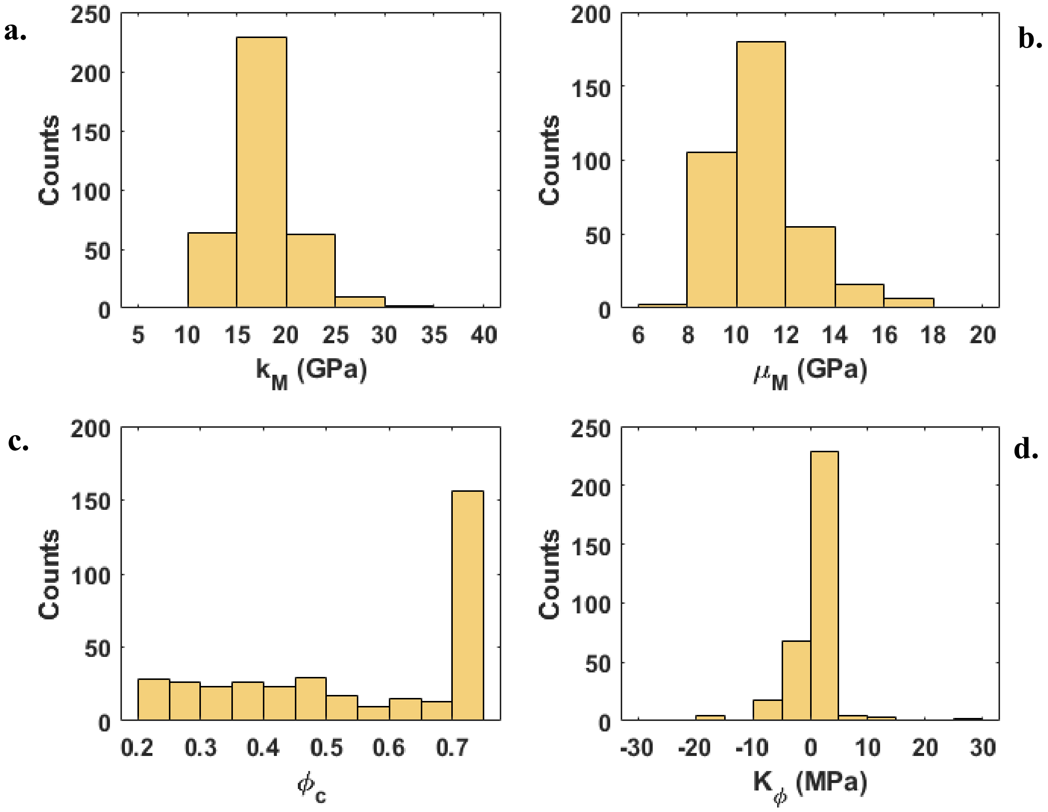

Figure 4. Histograms of parameters inverted using BKCP. **a.** K_M, **b,** μ_M, **c.** ϕ_c, **d.** K_ϕ.

The parameters inverted for the BKCP model are shown in Figure 4 using histograms to analyze their statistical significance along the well. We observe that the most significant values of modulus K_M are in the range 10 – 30 GPa and for μ_M between 6 – 18 GPa. The critical porosity parameter shows values almost equally distributed in the whole range, except near the maximum acceptable value, showing a large amount of points. The significance of these estimates should be compared with other data sets and other inversion strategies, given that there are no values reported for organic shales.

The estimation of the unjacketed pore modulus K_ϕ deserves some more analysis. First we made some tests using strictly positive values within the same range used for K_M,μ_M (i.e., 2 – 100 GPa), however we did not find stable solutions. In that range, the large numerical difference between $1/K_\phi$and $1/K_f$ in equation (7) makes K_{sat} almost insensitive to K_ϕ, unless a proper search range is defined for the inversion. This lack of sensitivity of K_{sat} to K_ϕ (and consequently in the computed velocities), was also pointed out by Zimmerman (1991). This observation and the analysis made by Berge and Berryman (1995) encouraged us to choose a smaller range, from -5 GPa to 5 GPa, allowing for negative and positive values. In this way, we obtained significant results which are plotted in the histogram shown in Figure 4(d), where we restricted the image to the significant interval between about -20 MPa to 30 MPa. The practical validity of these results needs further analysis and, if possible, an experimental verification, which implies careful laboratory procedures (Duranti, 2018).

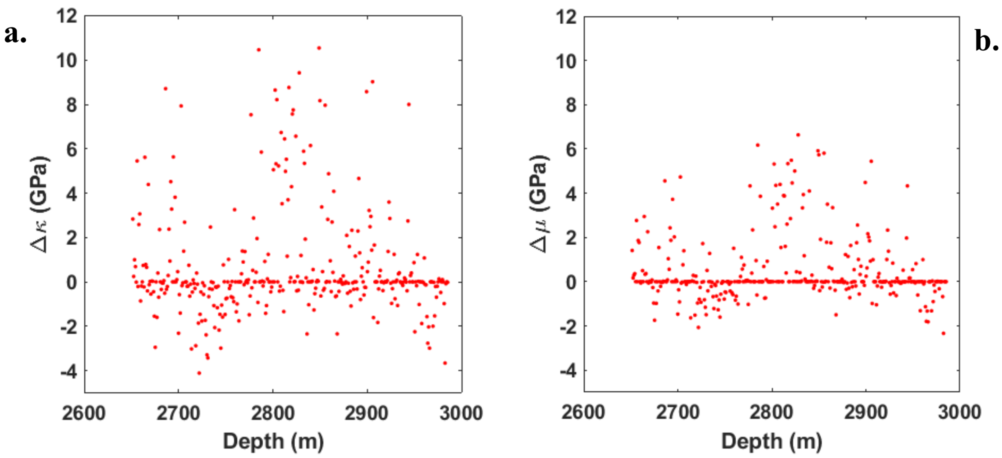

Figure 5. Departures between elastic parameters of GCP and BKCP, **a.** unjacketed bulk modulus, **b.** generalized shear modulus.

Finally, in Figure 5 we analyze the differences between the elastic moduli obtained with the two models described, denoted as $\Delta K=(K_0 - K_M)$ and $\Delta\mu=(\mu_0 - \mu_M)$. As can be seen, although there are a large number of depths in which the differences are near zero, the departures indicate that statistically, the elastic behavior of the rock cannot be completely represented using the equivalent homogeneous approach based on Gassmann's theory (GCP). Taking into account equation (4), those departures can be interpreted as a measure of elastic heterogeneity of the medium at scales smaller than the window size. A similar reasoning, but considering the discrepancies with respect to the classical Voigt-Reuss-Hill averages, was proposed by Duranti (2018), who found differences on the same order of magnitude. We remark that no statistical correlation was found between ΔK,$\Delta\mu$ and any of the volume fractions of the shale (shown in Fig. 1), from which we conclude that those discrepancies must be related to other petrophysical characteristics of the rocks.

CONCLUSIONS

In this work we presented and applied an original elastic workflow for modeling and inversion of velocities and poroelastic parameters based on the classic Brown and Korringa (1975) theory combined with the critical porosity model. In this way we fitted sonic log velocities corresponding to a thick interval of the Vaca Muerta organic shale formation, Argentina. Although this procedure does not require detailed information about the volume fractions and physical properties of the different rock constituents, it gives very good fitting results, with overall RMS errors lower than 1.5% along the well. This workflow involves the solution of an inverse problem to determine the set of poroelastic parameters in the formulation: the unjacketed pore and bulk moduli, a generalized shear modulus and the critical porosity of the rocks. The procedure was implemented using non overlapping windows, which define a length scale for the computations and the results.

The inverted unjacketed bulk and shear moduli are analyzed statistically, giving values very reasonable taking into account the mineralogy of the rocks under study, without the need of using effective medium theories with their underlying assumptions. Regarding the inversion of the unjacketed pore modulus from velocity data, we remark that its determination is conditioned by the sensitivity of the compressional wave velocity to that coefficient. The search range for this parameter requires a careful selection, having found convenient to consider negative and positive values as well. The reliability of these results must still be checked by independent methods and with data sets for different kinds of rocks.

By comparing the quality of velocity fitting between BKCP procedure with a simpler workflow based on Gassmann (1951) formulation for homogeneous rocks (GCP), we conclude that the parameter K_ϕ is not crucial for velocity modeling and wave propagation problems. We also compared the unjacketed bulk and shear elastic moduli resulting from both procedures, which allowed us to analyze statistically the elastic heterogeneity of the medium at scales smaller than the window size. From this point of view we conclude that the elastic behavior of the rock cannot be accurately represented using the equivalent homogeneous moduli obtained from Gassmann's theory at the window scale.

The critical porosity values were also determined along the well, obtaining results which are in good agreement with those expected for shales. These estimations are relevant taking into account the scarcity of published information on this parameter for organic shales.

It is worthwhile to remark that the methods described are not limited to the example analyzed in this study and can be applied to any other porous rock type. They can be useful for applications such as fluid substitution, elastic and velocity upscaling and for pore fluids sensitivity analysis of geomechanical parameters, seismic velocities and related attributes.

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