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On the Breaking of the $U(1)$ Peccei–Quinn Symmetry and Its Implications for Neutrino and Dark Matter Physics

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Abstract: The Standard Model of electroweak interactions is based on the fundamental $SU(2)_{weak} \times U(1)_{elect}$ representation. It assumes massless neutrinos and purely left-handed massive $W^{\pm}$ and Z^0 bosons to which one should add the massless photon. The existence, verified experimentally, of neutrino oscillations poses a challenge to this scheme, since the oscillations take place between at least three massive neutrinos belonging to a mass hierarchy still to be determined. One should also take into account the possible existence of sterile neutrino species. In a somehow different context, the fundamental nature of the strong interaction component of the forces in nature is described by the, until now, extremely successful representation based on the $SU(3)_{strong}$ group which, together with the confining rule, give a description of massive hadrons in terms of quarks and gluons. To this is added the minimal $U(1)$ Higgs group to give mass to the otherwise massless generators. This representation may also be challenged by the existence of both dark matter and dark energy, of still unknown composition. In this note, we shall discuss a possible connection between these questions, namely the need to extend the $SU(3)_{strong} \times SU(2)_{weak} \times U(1)_{elect}$ to account for massive neutrinos and dark matter. The main point of it is related to the role of axions, as postulated by Roberto Peccei and Helen Quinn. The existence of neutral pseudo-scalar bosons, that is, the axions, has been proposed long ago by Peccei and Quinn to explain the suppression of the electric dipole moment of the neutron. The associated $U(1)_{PQ}$ symmetry breaks at very high energy, and it guarantees that the interaction of other particles with axions is very weak. We shall review the axion properties in connection with the apparently different contexts of neutrino and dark matter physics.

Keywords: dark matter; $U(1)$ symmetry; axions; neutrino mass

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1. Introduction

The current knowledge about the fundamental forces in nature is based on the description of particles and their interactions [1]. It consists of a classification of the elementary forces by means of symmetries and their breaking by couplings among the constituent particles and mediators of the forces. The standard picture consists of the group chain $SU(3)_{strong} \times SU(2)_{weak} \times U(1)_{elect}$ supplemented by an extra $U(1)$ group, with a generator, that is, the minimal Higgs boson, which gives mass to all other particles except to the neutrinos [1].

The electroweak sector is still confined to an $SU(2)_{weak}$ left-handed representation. This representation would prove incomplete if we considered the existence of massive neutrinos, as indicated by neutrino oscillation experiments showing evidence of oscillations between different flavor states, or if we included right-handed electroweak currents within the framework of electroweak theory.

On the other hand, with reference to the astrophysical and cosmological sector of the theory, the existence, nature and/or composition of dark matter is challenging the basis of physics, both at small and large scales.

Here, we shall explore the case of neutrinos by allowing interactions with axions as a possible mass mechanism, then proceed to discuss our results for neutrino–axion interactions, at one loop level. We write the Lagrangian density of the coupling between fermions and bosons, namely, the neutrinos and the axions. Then, by breaking the $U(1)_{PQ}$ symmetry, and assigning a non-zero vacuum expectation value to the corresponding bosonic sector, we calculate the neutrino mass. To support this picture, we have assumed that axions are the main components of dark matter.

The aim of this paper is to show the close connection existing between the neutrino mass problem, the need to extend the minimal version of the Standard Model of electroweak interactions in order to include massive neutrinos and right-handed currents, that is, by adding an extra $SU(2)_{right}$ group, and the breaking of the $U(1)_{PQ}$ symmetry group represented by massive axions. In parallel, we shall discuss the potential existence of a boson condensate of massive axions and make some estimations of the properties attached to the condensate.

Although the Peccei and Quinn hypothesis [2,3] has been argued upon and, moreover, extended to describe axion–two-pion vertices mediated by quark–antiquark pairs and axion–two-photon vertices mediated electron–positron pairs, we shall put the emphasis on the axion–neutrino couplings and on the possible manifestation of an axion-related phase at cosmological scale.

The paper is organized as follows: The basic elements of the theory are given in Section 2. We shall then proceed in the following sequence:

(i) Review of the properties of the axions, in the Peccei–Quinn model and its extensions by Weinberg and Wilczek [4,5]. The basics notions about the symmetry breaking and associated couplings are discussed in connection with a possible mass mechanism for the neutrino sector of weak decay. The mechanism, based on the breaking of the extra $U(1)$ group symmetry introduced by Peccei and Quinn, is discussed together with the introduction of triangular vertices as done by Weinberg and Wilczek [4,5]. In addition, we shall define the renormalized mass propagator for neutrinos, which we are considering as a possible mass mechanism generated by the coupling between axions and neutrinos;

(ii) Review of the notions related to electroweak transitions which are dependent on the neutrino mass. Particularly, we shall discuss the case of the neutrinoless double beta decay and compare the values of the neutrino mass extracted from the limits of the non-observation of the decay with those obtained from the coupling between neutrinos and axions. We shall discuss the consequences of the inclusion of massive neutrinos in the formalism of the minimal Standard Model of electroweak processes, that is, the $SU(2)_{left} \times U(1)_{elect}$, which could become an extended group of the $SU(2)_{left} \times SU(2)_{right} \times U(1)_{elect}$;

(iii) Review of the cosmological aspects related to massive axions as a component of dark matter. The fact that massive neutrinos may be the main component of dark matter poses the question about the thermodynamic properties of the associated phase. In this respect, massive non-interacting axions may be delocalized in space if the conditions for a Bose–Einstein condensate evolve.

The following are a few words about the overall scope of the present work:

(a) Each of the topics, like the extensions of the Standard Model of electroweak interactions to include right-handed currents [6], the consequences upon it resulting from the observation (or non-observation) of the neutrinoless double beta decay [7], the appearance of the axion model to deal with the CP violation at the level of the neutron electric dipole moment [2–5], the role of axion-like particles (e.g., majorons) [8] as mediators of lepton-number violating processes and, finally, the question about the composition of dark matter [9], have been treated separately in detail in the previously quoted works. Here, we shall explore the notion of the existence of an intersection between all of these subjects;

(b) In the present work, these aspects will be discussed while keeping in mind the existence of such common features;

(c) Since the literature is very rich, and it would be impossible to mention the results of all published papers, we have chosen a few of them where the topics listed in (a) are discussed.

The results of the calculations that we have performed are presented and discussed in Section 3 and, finally, our conclusions are presented in Section 5. The material presented here is partially based on our previous work [7,10].

2. Formalism

In this section, we shall present the essentials of the formalism upon which we have based our discussion. It consists of a short presentation of the Peccei and Quinn formalism in Section 2.1, where the mass mechanism for axions is analyzed. In Section 2.2, we shall relate the findings about neutrino mass resulting from coupling to axions with the limits extracted from the non-observation of electroweak decays, which are forbidden by the minimal Standard Model, in order to compare both types of results. Finally, in Section 2.3, we shall present a view of the dark matter problem based on massive axions.

2.1. Massive Axions in the Peccei and Quinn Picture and Neutrino–Axion Couplings

The gauge field tensor of the QCD Lagrangian is written [1]

$$L = -(1/2) g_{\alpha\beta} F^{\alpha}_{\mu\nu} F^{\beta,\mu\nu} \,, \tag{1}$$

where $F^{\alpha}_{\mu\nu}$ is the gauge field tensor, α and β are structure constant indexes, μ and ν are Lorentz indexes, and $g_{\alpha\beta}$ is a constant matrix. If the CP and T invariances are not assumed, one may add the term

$$L' = -(1/2)\Theta_{\alpha\beta}\epsilon^{\mu\nu\rho\sigma} F^{\alpha}_{\mu\nu} F^{\beta}_{\rho\sigma} \,, \tag{2}$$

where $\Theta_{\alpha\beta}$ are the elements of another constant matrix, and $\epsilon^{\mu\nu\rho\sigma}$ is the Levi-Civita tensor. The term L' induces a neutron electric dipole moment, given by the expression $d_n \approx abs(\Theta) e \frac{m_\pi^2}{m_N^2}$, where m_π and m_N are the masses of the pion and of the neutron, respectively, and e is the absolute value of the electron charge.

Then, if the neutron electric dipole moment is of the order of (or smaller than) $d_n \leq 10^{-26}$e cm [11], it implies the constraint $\Theta \leq 10^{-9} \to 10^{-11}$.

The following are a few words about the CP properties of the electroweak Hamiltonian. The total Hamiltonian may be decomposed into two parts, one even (H_{even}) and one odd (H_{odd}), under CP transformations, like $H = H_{even} + H_{odd}$, where $H_{even/odd} = (1/2)(H \pm CPHP^{\dagger}C^{\dagger})$ [12]. The present evidences indicate that the odd component under CP is much weaker than the even one. Then, as explained in [12], when the ratio between these components of the electroweak Hamiltonian is fixed at the super weak scale ($\frac{H_{odd}}{H_{even}} \approx 10^{-10}$), the neutron electric dipole moment should then be, as we have mentioned before, of the order of (or smaller than) 10^{-26}e cm.

In order to solve this problem, R. Peccei and H. Quinn (1977) [2,3] have proposed the inclusion of a pseudo-scalar field $a(x,t)$, the axion, such that

$$\Theta \to \Theta + \frac{a(x,t)}{f} \,, \tag{3}$$

f being a strength constant. The non-vanishing vacuum expectation value of $a(x,t)$, with the subsequent breaking of the $U(1)$ symmetry associated with it, i.e., $\langle a(x,t) \rangle \neq 0$, causes Θ to vanish, $\Theta \to 0$.

This assumption was later extended, separately, by S. Weinberg and F. Wilczeck in 1978 [4,5], who wrote the Lagrangian

$$\begin{aligned} L &= -(1/2)\partial_\mu\phi\partial^\mu\phi \\ &+ \frac{1}{64\pi^2}(\Theta + \frac{\phi}{f})\epsilon^{\mu\nu\rho\sigma}F^\alpha_{\mu\nu}F^\beta_{\rho\sigma} \\ &- i\frac{f_u}{f}\partial_\mu\phi\bar{u}\gamma_5\gamma^\mu u \\ &- i\frac{f_d}{f}\partial_\mu\phi\bar{d}\gamma_5\gamma^\mu d\,, \end{aligned} \tag{4}$$

where $\phi(x,t)$ is a neutral scalar boson field, e.g., the axion, and the last two terms are the interactions of up (u) and down (d) quarks with axions.

The departure from the Peccei–Quinn axion [2,3] is just the transformation of this Lagrangian to a pion–axion basis. From it, the axion mass naturally arises from triangular vertices of the axion–two-pions type mediated by quark and antiquark loops. This is possible because of the quark–antiquark pair structure of the pions.

The effective Lagrangian which describes this process is written

$$\begin{aligned} L_{\pi-\phi} &= -(1/2)\partial_\mu\pi^0\partial^\mu\pi^0 \\ &- (1/2)\partial_\mu\phi\partial^\mu\phi \\ &- (1/2)\rho^T M_0^2\rho\,, \end{aligned} \tag{5}$$

where M_0 is a 2×2 mass-matrix and $\rho^T = (\pi^0, \phi)$. The eigenvalues of M_0^2 are m_π^2 and m_ϕ^2. The axion mass then becomes [1]

$$m_\phi \simeq 6\times 10^{-6}\ \mathrm{eV}\left[\frac{10^{12}\ \mathrm{GeV}}{f}\right]. \tag{6}$$

From the point of view of a fundamental symmetry, the axion of Peccei and Quinn is the Goldstone boson of the $U(1)$ symmetry which breaks at the scale $f \geq 4\times 10^8$ GeV, and in the Weinberg–Wilczeck [4,5] formulation, it results from the condition $\left\langle \Theta + \frac{\phi}{f} \right\rangle = 0$.

With values of the mass of the axion smaller than fractions of meV, its lifetime is of the order of 10^{24} s, which is more than enough to travel cosmological distances without decaying.

The scale factor f has lower limits which vary from 10^8 GeV, based on symmetry arguments, to 10^{11} GeV, which is the typical value associated with the Peccei–Quinn scale [1,2]. Both the Weinberg [4] and Wilczeck [5] approaches determine the relevant role of the scale factor f, and the corresponding limits may be extracted from the measurement of the decay of axions into two pions. In a similar manner, a term describing the decay of axions into two X-ray photons, that is, a triangular vertex mediated by electron–positron pairs, may be added to the effective Lagrangian of Equation (4). The coupling of neutrinos and axions may also be a way to determine that scale, as we shall discuss next.

Most of the matter in the Universe is dark [13,14], its existence is manifest from astronomical evidence [14]. Basically, it is non-baryonic and collision-less, but its composition is unknown.

Axions may be a dominant part of the cold dark matter, as postulated by Sikivie and Yang in their original paper [9] and by other authors [15–20].

Among the experiments devoted to the direct detection of dark matter particles, we should mention ADMeX (Axion Dark Matter electron-X), which aims to detect axions by the measurement of X-rays produced by the interaction of axions with electrons, mediated by electron–positron pairs. It is the equivalent of the production of two pions by the interaction between axions and quarks, mediated by quark–antiquark pairs, which is allowed as a second-order process by the effective Lagrangian of Weinberg and Wilczek [4,5].

If one assumes that axions are indeed the main component of dark matter, and that they have a non-localized distribution in space, it could be possible to couple them with neutrinos by means of a derivative term in the axion sector coupled to the neutrino current. Then, by taking the non-zero vacuum expectation value of the axion field, it results in a mass term for the neutrinos, as it is shown next.

By adding to the Lagrangian of axions an axion–neutrino coupling term, it could be possible to give mass to the neutrinos, as we shall explain next. From From a more fundamental point of view, the extension of the Lagrangian to include the interactions between neutrinos and axions follows from the work of Weinberg [4] and Wilczek [5]. It is expressed like Equation (4), by taking the four-potential as the covariant derivative of the axion field ($\partial_\mu \phi$) and the four-current $\bar{\nu}\gamma^\mu\gamma^5\nu$ of the neutrino sector. Since the time derivative of the axion field in the proper frame is just the mass of the axion, we define the coupling constant $g_{a\nu}$ scaled by the mass of the axion. We start from the Lagrangian

$$\mathcal{L}_{int} = i\frac{g_{a\nu}}{m_a}\bar{\nu}\gamma^\mu\gamma^5\nu\partial_\mu\phi\,, \tag{7}$$

which describes the derivative coupling between neutrinos (ν) and axions (ϕ), with $g_{a\nu}$ being the strength of their coupling divided by the mass of the axion (remember that we have taken $c = 1$). The time derivative of the axion field then cancels out that mass dependence (see below) since, in the proper frame, $\phi \approx \langle\phi\rangle_0 e^{-im_a t}$. As explained later, the mass scale resulting from the time derivative is given in terms of the coupling $g_{a\nu}$ and of the expectation value of the axion field $\langle\phi\rangle_0$ (see Equation (9)).

The breaking of the $U(1)$ symmetry of the axion sector is represented by the Higgs-like potential

$$V(\phi) = -\frac{\mu^2}{2}(|\phi|^2 - \frac{1}{f^2}|\phi|^4)\,. \tag{8}$$

The variation of this potential leads to the extremes

$$\langle\phi\rangle_0 = 0 \;\;(\text{unstable point})\,, \tag{9}$$

and

$$\langle\phi\rangle_0 = \frac{f}{\sqrt{2}}\,. \tag{10}$$

As a consequence of it, from the structure of the Lagrangian $\mathcal{L}_{int}$, the time derivative and the spatial derivatives can be written separately, leading to the expression

$$\mathcal{L}_{int} = i\frac{g_{a\nu}}{m_a}\nu^\dagger\vec{\sigma}\nu.\vec{\nabla}\phi + i\frac{g_{a\nu}}{m_a}\nu^\dagger\gamma^5\nu\partial_0\phi\,. \tag{11}$$

The second term of Equation (11) can be interpreted as a mass term, after performing the time derivative of the axion field in the proper frame and giving to the axion the non-zero extreme value of Equation (10). This mechanism is analogous to the conventional Higgs mechanism. The breaking of this extra symmetry, represented by the non-zero value of the expectation value of the axion field, as determined by the potential of Equation (8), gives mass to the neutrinos. Going beyond this level of approximation requires the exchange of axions at one loop level. The corrections to the zeroth-order mass of the neutrinos

$$m_\nu^0 = g_{a\nu}\langle\phi\rangle_0 = g_{a\nu}\frac{f}{\sqrt{2}}\,, \tag{12}$$

are represented by the mass propagator $\Sigma(p)$, which depends on the momentum exchanged between the axion and the neutrino. In the next paragraphs, we shall show the final results of the calculation of the mass propagator. The calculation is based on the use of conventional methods of quantum field theory and the results can be expressed in terms of scalar (Σ_m) and vector (Σ_p) components of the propagator.

The physical mass of the neutrino can be computed as

$$m_\nu \quad = \quad m + \Sigma(p)_{p^2=m^2} \; . \tag{13}$$

After evaluating $\Sigma(p)$ on shell, that is, by taking $p^2 = m^2$, the one-loop correction to the neutrino mass due to the interaction with axions is contained in the kernel $\Sigma(p)$ [21]

$$\Sigma(p) \quad = \quad \frac{g_{a\nu}^2}{8\pi^2}(\not{p}\Sigma_p + m\Sigma_m) \; , \tag{14}$$

where its vector and scalar components are Σ_p and Σ_m, respectively.

The one-loop neutrino propagator is then written [10,21]

$$\begin{aligned} \delta S \quad &= \quad \frac{1}{\not{p} - m - \Sigma(p)} \\ &= \quad \frac{1}{\not{p} - m - \Sigma(p)\Big|_{p^2=m^2}} \left(1 - \frac{\partial \Sigma(p)}{\partial \not{p}}\Big|_{p^2=m^2}\right)^{-1} . \end{aligned} \tag{15}$$

The derivation of the previous equations involves the ordering of higher-order corrections to the propagator, as well as fixing the value of the coupling $g_{a\nu}$ for each mass scale of the axion.

The following are some final words about neutrino–axion coupling:

(i) The breaking of the $U(1)$ symmetry proposed by Peccei and Quinn, $U(1)_{PQ}$, at the level of the Lagrangian which describes the interaction between the axion and the neutrino, at the zeroth order, gives mass to the neutrino (Equation (12)). That mass is dependent upon the coupling constant of the Lagrangian ($g_{a\nu}$) and of the constant (f), which determines the value of the mass of the axion;

(ii) The one-loop corrections to the zeroth-order neutrino mass are also dependent upon these constants, but they are non-divergent (Equations (13)–(15));

(iii) In order to complete the scheme, one has to take into account the squared mass differences between the three light-mass eigenstates Δm_{ij}^2 (both in the normal and inverse ordering) and the amplitudes U_{ij} relating the mass and flavor states in the light-mass sector, as well as the amplitudes V_{ij} of the heavy-mass sector.

To date, the calculations have been restricted to neutrino light-mass eigenstates, but to be consistent with the claim that the representation should have a right-handed channel, one may also have to include heavy-mass neutrinos in the picture, in order to express left- and right-handed lepton doublets [7].

2.2. Extensions of the Minimal Standard Model of Electroweak Interactions

The minimal version of the Standard Model of electroweak interactions [1] is based on purely left-handed currents, both for fermions and bosons, massless neutrinos, left-handed lepton doublets for negatively charged e^-, μ^-, and τ^-, with the corresponding antineutrinos $\overline{\nu}_e$, $\overline{\nu}_\mu$, and $\overline{\nu}_\tau$, and singlets of e^+, μ^+, and τ^+ leptons.

The conservation rules of the minimal representation imposed lepton number and lepton flavor conservation. The existence of neutrino flavor oscillations were confirmed experimentally by the SNO and Kamiokande collaborations, the results for which A. McDonald and T. Kajita received the Nobel Prize for Physics in 2015. The existence of neutrino mass oscillations implies the existence of massive neutrinos.

The coefficients of the linear combination between three light-mass neutrino mass eigenstates ϕ_{m_j} are defined in the expression

$$\psi_{\nu_e} = \sum_j U_{(e,j)} \lambda_j^{CP} \phi_{m_j} \, , \tag{16}$$

for the electron neutrino.

The electron neutrino sector depends on the amplitudes and CP phases of the linear combination of neutrino mass eigenstates $U_{(e,j)}$ and λ_j^{CP}, respectively; similar expressions are written for the other two flavors. Table 1 gives the values of the mixing angles and squared mass differences between neutrino mass eigenstates, for normal (NH) and inverse (IH) mass hierarchies.

Table 1. Neutrino oscillation parameters, for the normal (NH) and inverted (IH) mass hierarchies. Solar (δ^2_{solar}) and atmospheric (δ^2_{atm}) squared mass differences and mixing angles (θ_{ij}) are listed in the table. The values are taken from the review written by M.C. Gonzalez-Garcia (YITP, Stony Brook; ICREA, Barcelona; ICC, U. of Barcelona) and M. Yokoyama (Tokyo U.; Kavli IPMU (WPI), U. Tokyo). Particle Data Group (https://pdg.lbl.gov/2019/reviews/rpp2019-rev-neutrino-mixing.pdf (accessed on 1 September 2023)).

$sin^2(\theta_{12}) = 0.297$
$sin^2(\theta_{13}) = 0.0215$
$sin^2(\theta_{23}) = 0.425$ (NH)
$sin^2(\theta_{23}) = 0.589$ (IH)
$\delta^2_{atm} = m_3^2 - m_1^2 = 2.56 \times 10^{-3}$ eV^2
$\delta^2_{solar} = m_2^2 - m_1^2 = 7.37 \times 10^{-5}$ eV^2

Starting from these expressions, the amplitudes and probabilities associated with the flavor conversion from flavor α at $t = 0$ to flavor β at time t ($\alpha, \beta = e, \mu, \tau$) are written:

$$A_{\alpha\beta}(t) = e^{i\delta_1^2(t)} \sum_k U_{\alpha k} U^*_{\beta k} e^{i(\delta_k^2(t) - \delta_1^2(t))} . \tag{17}$$

and

$$\begin{aligned} P_{\alpha\beta}(t) &= |A_{\alpha\beta}(t)|^2 = (\text{Re}\, A_{\alpha\beta}(t))^2 + (\text{Im}\, A_{\alpha\beta}(t))^2 \\ &= \sum_{kk'} U_{\alpha k} U^*_{\beta k} U^*_{\alpha k'} U_{\beta k'} \cos(\delta_k^2(t) - \delta_{k'}^2(t)) , \end{aligned} \tag{18}$$

respectively, where the symbol δ_k^2 is the squared mass differences between the mass eigenstates k and k'.

The pattern of neutrino oscillations in a vacuum may differ from the pattern of neutrino oscillations in the presence of interactions between neutrinos and, for instance, dark matter particles. The interaction mediated by the exchange of virtual massive particles is depicted in the diagram given in Figure 1 and the results are shown in the panels of Figure 2. The details of the calculations are given in [22].

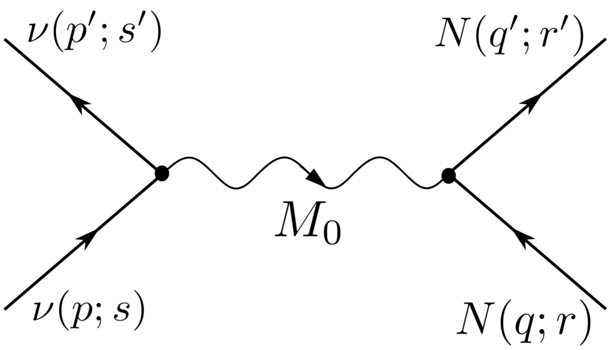

Figure 1. Interaction between neutrinos and particles belonging to the environment.

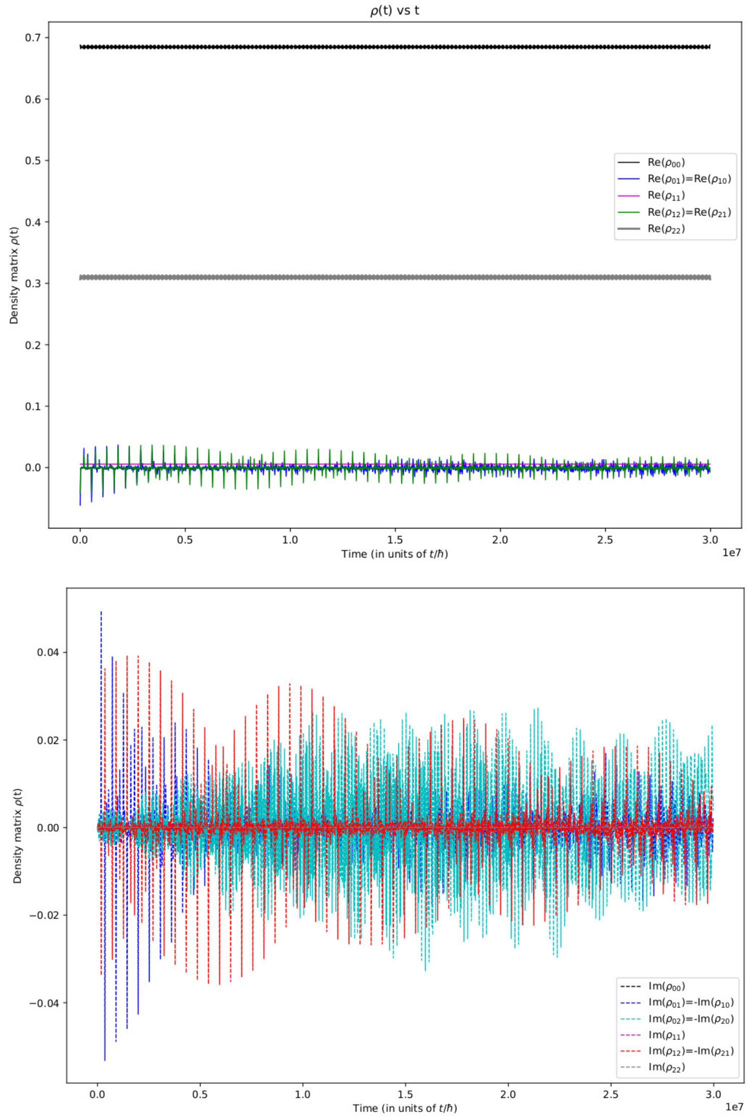

Figure 2. Real (upper plot) and imaginary (lower plot) matrix elements of the neutrino density matrix as a function of time for the three-flavor scheme. We used the following parameters: $\sigma = 20$ and $\lambda_{coup} = 1.0$ for the Gaussian function and the coupling to the environment. The relative time scale (t divided by $\hbar$) is expressed in units of 10^7.

Table 2 shows the results for the survival and conversion probabilities between the three neutrino flavors, where we have included the results obtained by diagonalizing neutrino–neutrino interactions.

Table 2. Probabilities for lepton flavor transformations between the three neutrino flavors in vacuum and as a function of interactions with diagonal and non-diagonal terms in the neutrino–neutrino interactions.

Interaction	$\nu_e \to \nu_e$	$\nu_e \to \nu_\mu$	$\nu_e \to \nu_\tau$
vacuum	0.56	0.27	0.16
diag	0.56	0.27	0.16
diag	0.56	0.27	0.16
diag	0.56	0.27	0.16
non-diag	0.60	0.24	0.15
non-diag	0.65	0.19	0.14
non-diag	0.55	0.28	0.16

In addition to these oscillations between neutrinos of different flavors, which do not require any interaction between the neutrinos or between neutrinos and hadrons, there exist other processes which, if observed, would demonstrate the need to extend the minimal version of the Standard Model. They can be possible if neutrinos are massive. They are the neutrinoless double beta decay, which violates lepton number conservation, and the neutrino flavor violation. Both processes require the interaction of nucleons and pair of leptons, mediated by charged and/or neutral bosons. Among these exotic processes, the neutrinoless double beta decay ($0\nu\beta\beta$) has been, and still is, the object of the attention of theoreticians and experimentalists for several decades [7]. In the following paragraphs, we shall introduce the basic elements of the theory related to the double beta decay processes. This process consists of the decay of a mother nucleus with N neutrons and Z protons into a daughter nucleus with N − 2 neutrons and Z + 2 protons, accompanied by the emission of two electrons with an energy equal to the Q-value of the decay. The decay schemes are shown in Figure 3.

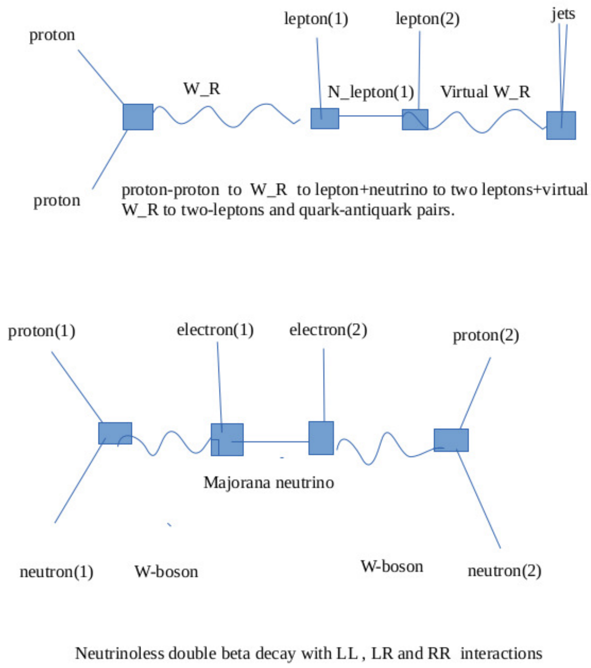

Figure 3. Diagrams showing the double beta decay transitions, for the case of zero neutrino double beta decay channels, and an equivalent process mediated by W bosons. The neutrinoless double beta decay process is not allowed by the Standard Model, since it implies that the neutrino is a Majorana particle.

The experimental signal would be the detection of the two electrons flying in opposite directions. The decay implies the exchange of a massive neutrino between the two vertices. It can also take place if right-handed currents are included in the electroweak Lagrangian, also implying the existence of left–right couplings. The observation of this process indicates the need to extend the Standard Model, because it is forbidden, in the current formalism, to accommodate massive neutrinos and/or right-handed currents with the corresponding triplet of right-handed bosons.

Values of the electron neutrino mass can be extracted by comparing the theoretical rates and the experimental lower limits for the non-observation of the neutrinoless double beta decay ($0\nu\beta\beta$) half-life, which are of the order of 10^{25} years or larger [7].

By using average values of the nuclear matrix elements, evaluated in the context of different nuclear structure models, the resulting average neutrino mass is of the order of 0.5 eV, but this value is very much dependent on the nuclear structure models used to evaluate the nuclear matrix elements of the participant electroweak operators.

The expression of the inverse half-life of the $0\nu\beta\beta$ decay is separable into three factors, which represent the contributions coming from the neutrino sector, the nuclear structure sector, and phase space factors.

The non-standard electroweak Hamiltonian, which includes left- and right-handed currents and mediators, is written

$$H_{LR} = \frac{G}{\sqrt{2}} \cos\theta_{\rm CKM} \left(j_{\rm L} J_{\rm L}^{\dagger} + \eta j_{\rm R} J_{\rm L}^{\dagger} + \lambda j_{\rm R} J_{\rm R}^{\dagger} \right) + {\rm h.c.} \tag{19}$$

The simpler mechanism to explain lepton number violation consists of the second-order treatment of the first term of the Hamiltonian of Equation (19) acting on two neutrons belonging to the initial nucleus, leading to the transformation of them into two protons, followed by the emission of two electrons [7], as is shown by the diagram of Figure 3. The matrix elements of the Hamiltonian of Equation (19) on nuclear states, for operators of the Fermi (F), Gamow–Teller (GT), and Tensorial (T) types are written

$$\left(\frac{g_{\rm A}}{g_{\rm A}^{\rm b}} \right)^2 \left[M_{\rm GT}^{(0\nu)} - \left(\frac{g_{\rm V}}{g_{\rm A}} \right)^2 M_{\rm F}^{(0\nu)} + M_{\rm T}^{(0\nu)} \right]. \tag{20}$$

The calculation of these matrix elements implies the definition of a neutrino potential

$$h_K(r_{mn}, E_k) = \frac{2}{\pi} R_{\rm A} \int dq \frac{q h_K(q^2)}{q + E_k - (E_{\rm i} + E_{\rm f})/2} j_0(q r_{mn}), \tag{21}$$

with matrix elements between nuclear states of the form

$$M_K^{(0\nu)} = \sum_{J^\pi, k_1, k_2, J'} \sum_{pp'nn'} (-1)^{j_n + j_{p'} + J + J'} \sqrt{2J'+1} \begin{Bmatrix} j_p & j_n & J \\ j_{n'} & j_{p'} & J' \end{Bmatrix} (pp' : J' || \mathcal{O}_K || nn' : J'), \tag{22}$$

with two-particle transition amplitudes given by the expressions

$$(0_f^+ || \left[c_{p'}^{\dagger} \tilde{c}_{n'} \right]_J || J_{k_1}^{\pi}) \langle J_{k_1}^{\pi} | J_{k_2}^{\pi} \rangle (J_{k_2}^{\pi} || \left[c_p^{\dagger} \tilde{c}_n \right]_J || 0_i^+). \tag{23}$$

In these equations, the index K stands for Fermi, Gamow–Teller, and Tensorial operators.

With them, one can write transition densities depending on nuclear wave functions. In the framework of the Quasiparticle Random Phase Approximation (QRPA) [7], as an example, they acquire the form

$$|J_k^{\pi} M\rangle = \sum_{pn} \left(X_{pn}^{J^\pi k} \left[a_p^{\dagger} a_n^{\dagger} \right]_{JM} - Y_{pn}^{J^\pi k} \left[a_p^{\dagger} a_n^{\dagger} \right]_{JM}^{\dagger} \right) |{\rm QRPA}\rangle \tag{24}$$

$$(0_f^+ || [c_{p'}^{\dagger} \tilde{c}_{n'}]_J || J_{k_1}^{\pi}) = \sqrt{2J+1} \left[\bar{v}_{p'} \bar{u}_{n'} \bar{X}_{p'n'}^{J^\pi k_1} + \bar{u}_{p'} \bar{v}_{n'} \bar{Y}_{p'n'}^{J^\pi k_1} \right] \tag{25}$$

$$(J_{k_2}^{\pi} || [c_p^{\dagger} \tilde{c}_n]_J || 0_i^+) = \sqrt{2J+1} \left[u_p v_n X_{pn}^{J^\pi k_2} + v_p u_n Y_{pn}^{J^\pi k_2} \right], \tag{26}$$

for all possible terms resulting from the multipole expansion of the neutrino potential.

The resulting expression for the half-life, restricted to the mass sector, is given by Equation (27):

$$(T_{1/2}^{(0\nu)})^{-1} = C_{mm}^{(0\nu)} \frac{\langle m_\nu \rangle^2}{m_e{}^2}. \tag{27}$$

The nuclear structure sector $C_{mm}^{(0\nu)}$ is written in terms of the matrix elements of the multipole operators which participate in the transitions between nuclear states. The so-called mass term is expressed in terms of nuclear matrix elements of the Gamow–Teller operator $\sigma_{1\mu}\tau^{\pm}$:

$$C_{mm}^{(0\nu)} = G_1^{(0\nu)} (M_{\rm GT}^{(0\nu)} (1 - \chi_{\rm F}))^2 \tag{28}$$

The form factors $G_1^{(0\nu)}$ are written in terms of radial integrals of electron wave functions [6]. Unlike the double beta decay with neutrinos ($2\nu\beta\beta$), which is observed with half-lives some orders of magnitude faster, and which is not dependent on neutrino properties [7], the values of the nuclear matrix elements which participate in the $0\nu\beta\beta$ decay are much more stable along the nuclear systems, with values which are of the order of 3–5, much larger than the values needed to reproduce the $2\nu\beta\beta$ channel, which are strongly suppressed.

The theory and experiments related to both the $2\nu\beta\beta$ and $0\nu\beta\beta$ decays have been intensively developed during the last three decades .

Some results of the calculated neutrino mass, extracted from the systematics on calculated half-lives for $0\nu\beta\beta$ decay processes, are shown in Section 3. The cases are those of the decays of the nuclei ^{76}Ge and ^{136}Xe, which have been, and still are, the object of leading experimental efforts [23,24]. For the case of $2\nu\beta\beta$ decay, we show, as examples, the results corresponding to the allowed decay of the mother nuclei ^{128,130}Te.

2.3. Axions in Cosmology

In addition to the effects associated with the assumed existence of axions, with reference to neutrino properties, axions may play a crucial role in the composition of dark matter. The literature is rich in the exploration of this possibility [13,14], which is very crucial to our understanding of the Universe. As postulated by Sikivie et al. long ago [9], axions may indeed be the main components of dark matter and eventually reach thermal equilibrium, induced by gravity, to form a Bose–Einstein (BEC) condensate [17–19]. This is to be contrasted with the opinion of other authors who claim that the value of the galaxy phase-space density excludes the formation of a Bose–Einstein condensate [20] or that the rate of entropy creation apparently is not consistent with the gravitational thermalization rate [18]. Moreover, the condensate of axions may be unstable and it could collapse as result of the competition between attractive and repulsive interactions, as pointed out by Guth et al. [15]. The conventional picture of a BEC state implies long-range space correlations. However, if the collapse takes place, it could be that some sort of local BEC-like configurations might be favored instead. The example of such a formation are bosonic droplets. This is still an open question, which deserves much attention, particularly when making assumptions in cosmology. Nevertheless, although the existence of the BEC may be disputed, none of the arguments against it rule out the possibility that axions may be the main components of dark matter. In our opinion, the very interesting question is related to the origin of the neutrino mass, and we strongly support the notion that the coupling of axions and neutrinos can, indeed, be the proper mechanism responsible for non-zero neutrino masses. In this respect the combination of limits extracted from nuclear and astrophysical observations of processes beyond the Standard Model could bring the answer to this question.

3. Results and Discussions

In this section, we shall present some of the results which we have obtained by applying the formalism presented in the previous section. Far from being a complete presentation of results, we shall focus on the relation between the axion and the neutrino mass problem, its consequences upon the Standard Model of electroweak interactions, and, more briefly, on the effects of it in cosmology.

3.1. The Axion–Neutrino Couplings

Some results about the neutrino mass obtained from the coupling to axions are shown in Tables 3 and 4. In these tables, we estimate the dependence of the neutrino mass, for values of the axion mass within the range allowed by Equation (12). The values for the neutrino mass are upper limits, which are consistent with the ones extracted from other processes mediated by neutrinos, like the neutrinoless double beta decay, as we shall explain below. The values shown in these two tables can be interpreted from different views, namely, (a) either the mass of the axion is determined by fixing the coupling f_a and, independently, the neutrino mass is related to it by the coupling $g_{a\nu}$, as done to obtain the results shown in Table 3, or (b) by considering both masses simultaneously as done when calculating the results shown in Table 4.

Table 3. Calculated values of the neutrino mass m$_\nu$ as a function of the axion mass m$_a$ and of the coupling constant $g_{a\nu}$.

$g_{a\nu}$	$\log_{10}(m_a/\text{eV})$	m_ν (eV)
10^{-24}	-7.0	0.05
	-6.0	0.02
10^{-22}	-5.0	0.04
	-4.0	0.01
10^{-20}	-3.0	0.03
	-2.0	0.01

Table 4. Values of the axion mass m_a and of the coupling constants $g_{a\nu}$ and f_a for fixed values of the neutrino mass m_ν.

$(m_a/6)$ (eV)	f_a (GeV)	$g_{a\nu}/\sqrt{2}$ (m_ν = 0.1 (eV))	$g_a/\sqrt{2}$ (m_ν = 0.01 (eV))
10^{-10}	10^{16}	10^{-26}	10^{-27}
10^{-9}	10^{15}	10^{-25}	10^{-26}
10^{-8}	10^{14}	10^{-24}	10^{-25}
10^{-7}	10^{13}	10^{-23}	10^{-24}
10^{-6}	10^{12}	10^{-22}	10^{-23}
10^{-5}	10^{11}	10^{-21}	10^{-22}
10^{-4}	10^{10}	10^{-20}	10^{-21}
10^{-3}	10^{9}	10^{-19}	10^{-20}
10^{-2}	10^{8}	10^{-18}	10^{-19}
10^{-1}	10^{7}	10^{-17}	10^{-27}

3.2. Neutrinoless Double Beta Decay and the Neutrino Mass

In other words, we advocate the structure $SU(2)_{left} \times SU(2)_{right} \times U(1)_{elect}$, with massive neutrinos, both left- and right-handed ones, with masses, for the three flavors, which are linear combinations of three light and three heavy neutrino mass eigenstates. Naturally, it also implies the inclusion of three right-handed bosons, $(W^+, W^-, Z_0)_{right}$, with masses of the order of 1–3 TeV or larger. As said before, the neutrino mass could then come from the coupling with axions in the presence of the spontaneous symmetry breaking of the $U(1)_{PQ}$ group [1,2].

The results for values of the neutrino mass induced by the coupling with axions do provide evidence supporting the need to consider the neutrino as a massive particle and the subsequent need to extend the Standard Model of electroweak interactions, meaning that to the $SU(2)_{left}$ sector, resulting from the addition of $SU(2)$ of gauge bosons to the $U(1)$ Higgs associated with the maximal symmetry breaking of the left-handed currents, one should add an $SU(2)_{right}$ sector. The same statement is valid for neutrinos, because to the light-mass eigenstates, one should add a heavy-mass triplet.

Table 5 shows two of the cases where the experimental limits on the non-observation of the neutrinoless double beta decay are determined. From these limits, and using average nuclear matrix elements, the values of the electron neutrino mass, extracted from

Equation (27), are compatible with the values which we had obtained by implementing the scheme of the coupling between axions and neutrinos. Here, we shall give, as examples, the cases of the decay of ^{76}Ge [23] and ^{136}Xe [24].

Table 5. Experimental half-life lower limits and extracted neutrino mass, for the cases of Ge and Xe nuclei. The values of the electron neutrino mass have been extracted from the experimental lower limit of the neutrinoless double beta decay, using average nuclear matrix elements and the corresponding form factors.

Nucleus	Half-Life Lower Limit (years)	Extracted Neutrino Mass (eV)
^{76}Ge	1.9×10^{25}	0.1
^{136}Xe	1.1×10^{25}	0.5

To compare with the somehow large values of the nuclear matrix elements associated with the neutrinoless double beta decay, we shall show some results corresponding to the allowed double beta decay accompanied by the emission of two neutrinos. The decay chains are those of the mother nuclei ^{128,130}Te. The suppression mechanism controlling the behavior of the $2\nu\beta\beta$ has been the subject of continuous studies. Table 6 shows the comparison between measured and calculated nuclear matrix elements.

Table 6. Comparison between experimental and theoretical results for the half-life of the allowed two-neutrino double beta decay of ^{128,130}Te.

Nuclear Mass	Experimental Half Life	Extracted Matrix Elements	Calculated Matrix Elements
128	$2.5 \pm 0.3\ 10^{24}$ years	0.025 ± 0.005	0.016
130	$0.9 \pm 0.1\ 10^{21}$ years	0.001 ± 0.005	0.012

We have set limits on the coupling constants of the right-right and right-left handed interactions of the Lagrangian, by taking the lower limits of the half-life of neutrinoless double beta decay transitions Table 7 shows the values of the extracted coupling constants of the left-right and right-right sectors of the Hamiltonian of Equation (19), $\langle\eta\rangle$ and $\langle\lambda\rangle$, respectively, which are compatible with the lower limits of the half-life of the neutrinoless double beta decay of ^{76}Ge and ^{136}Xe. The values, extracted from the minimization procedure, corresponds to average neutrino masses of the order of 0.25 eV.

Table 7. Values of the coupling of right-handed currents $\langle\lambda\rangle$, and the coupling of right-left currents $\langle\eta\rangle$, which are compatible with the present limits on the half-life for the non-observation of the neutrinoless double beta decay of ^{76}Ge and ^{136}Xe. The values are give in units of eV.

Nucleus	$\langle\eta\rangle$	$\langle\lambda\rangle$
^{76}Ge	-3.69×10^{-9}	1.47×10^{-7}
^{136}Xe	-1.17×10^{-9}	6.71×10^{-8}

4. Some Consequences of the Proposed Axion–Neutrino Couplings

We have discussed the following two aspects of the physics related to axions and neutrinos:

- The value of the neutrino mass resulting from the coupling with axions;
- The compatibility of the neutrino mass obtained from the coupling to axions with those extracted from the non-observation of the neutrinoless double beta decay.

In the calculations that we have performed, which are consistent with other authors calculations (see [25]), the values of the neutrino mass, both in the normal and inverse mass hierarchy, do not differ much. For values of the neutrino mass larger than 0.1 eV, the axion–neutrino coupling yields a degenerate scheme, which is broken for values of the ratio between the axion–neutrino coupling and the axion mass of the order of $\approx 10^{-18}$. The results show a definite scaling depending on this ratio. On the other hand, the calculated values of the neutrino mass, at one loop level of the coupling, are fully consistent with the limits imposed by the non-observation of the neutrinoless double beta decay. Both facts seems to support the notion of the validity of the neutrino mass generation mechanism induced by the coupling with axions. The interesting thing here is the potential link between neutrino and dark matter physics, with the constraints determined by measurements of lepton number violation processes, like the neutrinoless double beta decay.

5. Conclusions

Axions may be a dominant component of non-baryonic dark matter of the Universe. The axions, neutral scalar bosons, in addition to their role in solving the strong CP problem, exhibit interesting properties in connection with cosmology and with extensions of the Standard Model of electroweak interactions. The coupling between neutrinos and axions could be the dominant mechanism which gives non-zero mass to the neutrinos. In this picture, the neutrinos become Majorana particles and nuclear decay processes like the double beta decay without neutrinos, which are forbidden by the Standard Model in the minimal $SU(2)_{left}$ formalism, may indeed be allowed and eventually observed. The role of axions in cosmology is also central and some crucial aspects of it are related to the space distribution of axions. As we have discussed, the onset of a Bose–Einstein condensation of axions, a phase which does not requires the pre-existence of interactions among the axions, could result in a complete space delocalization of the axions. Our findings can be summarized as follows:

1. The Peccei and Quinn proposal, complemented by the extensions separately written by S. Weinberg and F. Wilczek about the inclusion of an extra $U(1)$ group, the breaking of which gives rise to non-zero values of the mass of the axions and the coupling of axions with a pair of mesons and a pair of X-ray photons, may also be extended to include the coupling with neutrinos;
2. The coupling of axions and neutrinos induces non-zero values of the mass for the neutrinos;
3. The values of the mass of the neutrinos resulting from this coupling are indeed comparable with limits extracted from nuclear decays, which are otherwise forbidden by the minimal Standard Model of electroweak interactions, like the neutrinoless double beta decay transitions;
4. The role of massive axions in cosmology is also crucial, since they may be the dominant component of dark matter and also determine the onset of a BEC phase.

In this note, we tried to convey the notion that a new interphase between elementary particle physics, nuclear structure physics, and cosmology is developing and it may indeed be the subject of intensive research activities. Figure 4 illustrate the interphase existing in this very challenging and exciting area of physics.

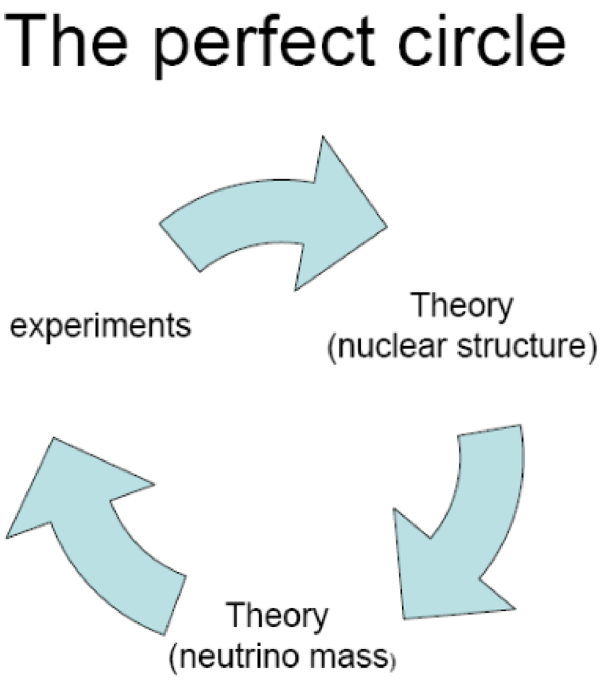

Figure 4. View of the close connection existing between experimental, nuclear, and particle physics focusing on the determination of neutrino properties, as well as on the non-trivial extensions of the minimal Standard Model.

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