Asociación Argentina de Astronomía
Third La Plata International School on Astronomy and Geophysics:
Chaos, diffusion and non–integrability in Hamiltonian Systems
Applications to Astronomy , 2012
P. M. Cincotta, C. M. Giordano & C. Efthymiopoulos, eds.

ORAL COMMUNICATION

Simple Instability in a 3D Autonomous Hamiltonian system of galactic type

M. Katsanikas[1,2]

(1) Research Center for Astronomy, Academy of Athens
Soranou Efessiou 4, GR-11527 Athens, Greece
mkatsan@academyofathens.gr
(2) Section of Astrophysics, Astronomy and Mechanics
Department of Physics, University of Athens, Greece

Abstract. In this paper we study the orbital behavior in the neighborhood of simple unstable periodic orbits in a 3D rotating galactic potential. We use the method of color and rotation to visualize the 4D spaces of section. We found four types of structures in the 4D space of section that correspond to four types of orbits. The first three types are sticky chaotic orbits and in the last one the orbit visits all available phase space.

1. Introduction

In this paper we give a short review of recent results from studies about the structure of the phase space in the neighborhood of simple unstable periodic orbits in a 3D autonomous hamiltonian system of galactic type (Katsanikas *et al.* 2011b). In 3D autonomous hamiltonian systems the phase space is six dimensional and the space of section is 4D. For this reason we use the method of color and rotation to visualize this space (Patsis & Zachilas 1994). By using this method we first consider 3D projections and rotate the 3D figures on a computer screen to observe the figure from all its sides. Every point is colored according its value in the 4th dimension. The smooth color variation corresponds to ordered behavior and the mixing of colors to chaotic behavior in the 4th dimensional space.

For the study of stability of periodic orbits we use the method of Broucke (1969) and Hadjidemetriou (1975). According to this method we compute firstly the elements of the monodromy matrix of our 4D Poincaré map. Using the elements of the monodromy matrix, we find four types of periodic orbits according to their stability: (1) stable (2) simple unstable (3) double unstable and (4) complex unstable (Contopoulos $ Magnenat 1985, Contopoulos 2002 p.286). We study here the case of simple instability. In this case the periodic orbits have two complex eigenvalues of the monodromy matrix of our 4D Poincaré map that are on the unit circle and two real eigenvalues that are off the unit circle.

The system that we used for our applications is the one used by Katsanikas $ Patsis (2011) with $\Omega_b = 0.045$ in our units. Our potential in its axisymmetric form can be considered as an approximation of the potential for the Milky Way (Miyamoto $ Nagai 1975).

2. 4D surfaces of section

We examine first the different types of orbital behavior in the neighborhood of simple unstable periodic orbits. In previous works Magnenat (1982) found orbits that are represented with double loops in the 2D projections of the 4D space of section in the neighborhood of simple unstable periodic orbits. Here we study the morphology of these double loops in the 4D space. Besides this type of orbits we found totally four types of orbital behavior in the neighborhood of simple unstable periodic orbits. The first three types are associated with the phenomenon of stickiness (Contopoulos $ Harsoula 2008). In these three types the chaotic orbits are guided by the manifolds close to the regions that are occupied by the rotational tori (Vrahatis *et al.* 1997, Katsanikas and Patsis 2011) associated with the stable periodic orbits existing in the region. All of them are associated with motion close to a transition of a mother family from stability to simple instability or vice versa. The fourth type of orbital behavior (last type) is associated with orbits that visit very fast all available phase space.

The four types are represented in the surface of section from four different types of structures in the 4D space of section:

1. The first type is represented by a double loop with smooth color variation (as in the example of Fig. 1). The smooth color variation means that the 4th dimension supports this 3D topological surface (double loop in the 3D subspace of our 4D space of section) in the 4D space. This means that the double loop is a 4D object. At the intersection of the double loop we can see the meeting of different colors (red with violet). This indicates that this intersection does not exist in the 4D space. The previous structure is encountered for a specific number of intersections. For a larger number of intersections we observe that the consequents form a Θ-like structure with mixing of colors (like the Fig. 2). For even larger number of intersections the consequents leave this structure and form clouds of points that correspond to motion in a chaotic sea.

2. The second type is represented only by a Θ-like structure with mixing of colors (like the Fig. 2) for a number of intersections. For larger number of intersections we have again as in the first type of orbits the formation of clouds of points.

3. The third type of orbits (Fig. 3) is represented by a double loop with smooth color variation as the double loop of the first type of orbits. However, in this case we have a difference. The region of the intersection of the double loop has now the same color (green - Fig. 3). This means that this intersection is a real 4D intersection. This difference reflects properties of the family of periodic orbits under consideration (e.g. lack of a symmetric family with respect to the equatorial plane).

4. The fourth type of orbits correspond to motion in a chaotic sea. The chaotic sea is represented in the 4D surface of section with clouds of points with mixing of colors. The mixing of colors means that the scattered points are not only in the 3D subspaces of the space of section but are scattered in the 4D space. This means that these clouds are 4D.

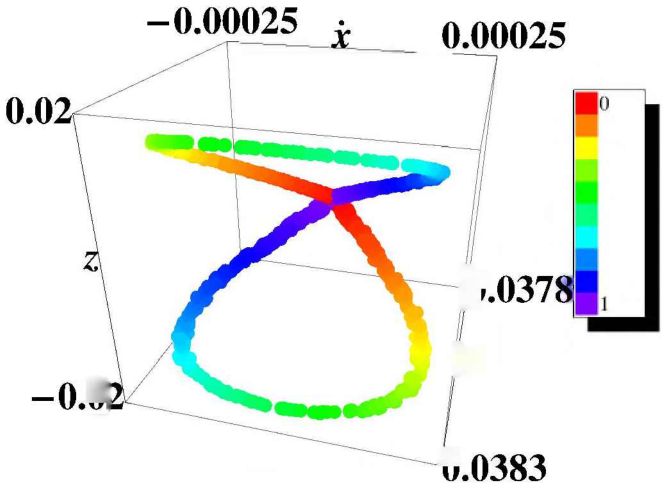

Figure 1. An orbit that is represented by a double loop with no real intersection for 2580 consequents in the 4D space of section. This orbit is in the neighborhood of a simple unstable periodic orbit of x1v2 for E_j=-4.66 (Katsanikas *et al.* 2011b). The consequents are given in the $(x, \dot{x}, z)$ projection, while every point is colored according to its $\dot{z}$ value. We observe that a double loop ribbon is formed with a smooth color succession on it (cf. with the color bar on the right). Our point of view in spherical coordinates is $(\theta, \varphi) = (30^o, 108^o)$.

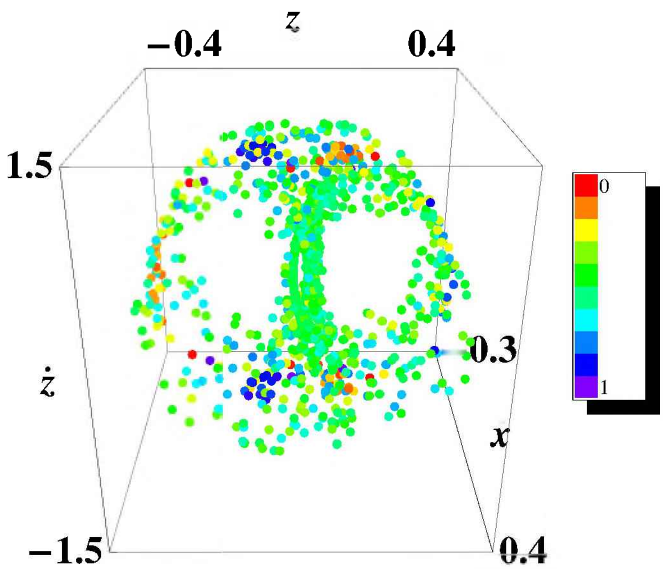

Figure 2. Mixing of colors for the first 1000 consequents of the orbit in the Fig. 1 (beyond the first 2580 consequents) that deviate from the 4D ribbon surface. The 3580 points totally are depicted in the 3D subspace $(\dot{x}, z, \dot{z})$ and the color represents the 4th dimension x of the points. Our point of view in spherical coordinates is $(\theta, \varphi) = (120^o, 0^o)$.

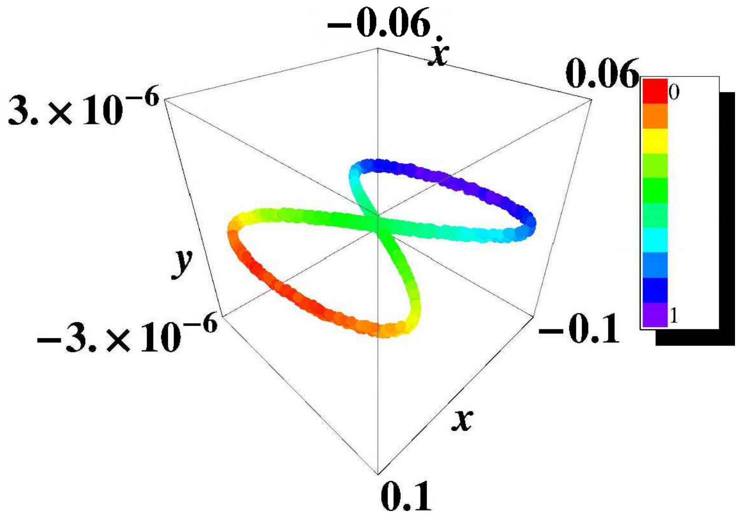

Figure 3. An orbit that is represented by a double loop with real intersection in the 4D space of section. This orbit is in the neighborhood of a simple unstable periodic orbit of z-axis family for a slow pattern model of our system $\Omega_b = 5 \times 10^{-6}$ for E_j=-6.6342 (Katsanikas *et al.* 2011b). Thus, we consider now the $z = 0$ surface as surface of section with $\dot{z} > 0$. The consequents are given in the $(x, \dot{x}, y)$ projection, while every point is colored according to its $\dot{y}$ value. We observe that a double loop ribbon is formed with a smooth color succession on it. Our point of view in spherical coordinates is $(\theta, \varphi) = (60^o, 60^o)$.

The results about the 4D structure of phase space close to a periodic orbit have been presented in a recent series of papers (Katsanikas & Patsis 2011, Katsanikas *et al.* 2011a,b, Katsanikas *et al.* 2011c, Katsanikas 2011).

References

Broucke R. 1969, NASA Tech. Rep. 32-1360, 1

Contopoulos G. 2002, *Order and Chaos in Dynamical Astronomy*, Springer-Verlag, New York Berlin Heidelberg

Contopoulos G., & Magnenat P. 1985, Celest. Mech. & Dynam. Astron., 37, 387

Contopoulos G., & Harsoula M. 2008, Int. J. Bif. Chaos, 18, 2929

Hadjidemetriou J. 1975, Celest. Mech. & Dynam. Astron., 12, 255

Katsanikas M., & Patsis P.A. 2011, Int. J. Bif. Chaos, 21, 467

Katsanikas M., Patsis P.A., & Contopoulos G. 2011a, Int. J. Bif. Chaos, 21, 2321

Katsanikas M., Patsis P.A., & Contopoulos G. 2011b, Int. J. Bif. Chaos (is submitted).

Katsanikas M., Patsis P.A., & Pinotsis A.D. 2011c, Int. J. Bif. Chaos, 21, 2331

Katsanikas M. 2011, Ph.D thesis, University of Athens (in greek - to be submitted).

Magnenat P. 1982, Celest. Mech. & Dynam. Astron., 28, 319

Miyamoto M., & Nagai R. 1975, Publ. Astron. Soc. Japan, 27, 533

Patsis P.A., & Zachilas L. 1994, Int. J. Bif. Chaos, 4, 1399

Vrahatis M.N., Isliker H., & Bountis T.C. 1997, Int. J. Bif. Chaos, 7, 2707