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PRICE DISCRIMINATION WITH DIVISIBLE GOODS

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Abstract

This paper studies second-degree price discrimination (SPD) in cases involving products with divisible quantities and sold in many presentations (carbonated soft drink, beer, packaged bread, ready-to-eat cereals, laundry products, etc.). Differently from the standard case, consumers may have the option to choose more than one package when comparing different product presentations; therefore, standard SPD may not cover all self-selection constraints.

We solve an extended SPD problem (ESPD) and characterize the solution for two types of preferences. As in SPD, the seller provides the efficient quantity to high-WTP consumers and introduces inefficiencies in packages designed for low-WTP consumers. But the distortion is less than that suggested by SPD, provided that the seller attends both consumer types. Closing the market for low-WTP consumers (when SPD suggests to keep it open) is also a possibility if the two distortions introduced in the ESPD –the standard trade off between inefficiency and consumer surplus, and the n-arbitration constraint for high-WTP consumers– are too costly to the seller. Given the possibility for high-WTP consumers to consume more than one unit of small packages, the seller offers deeper quantity discounts, provided that he finds profitable to sell both consumer types.

Resumen

Este paper estudia el problema de discriminación de precios de segundo grado (DPS) aplicado a productos con cantidades divisibles y múltiples presentaciones (bebidas gaseosas, cerveza, pan lactal, cereales para desayuno, jabones en polvo, etc.). A diferencia del caso estándar, los consumidores pueden tener la opción de elegir más de un paquete al comparar diferentes presentaciones. En estos casos, la solución estándar de DPS puede no cubrir todas las restricciones relevantes de auto-selección.

En la solución al problema de DPS ampliado (DPSA) para el caso de información privada del consumidor particionada en dos *types*, el vendedor provee la cantidad eficiente para consumidores de alta valoración e introduce ineficiencias en el paquete diseñado para consumidores de baja valoración. Pero la distorsión es menor que la sugerida en DPS, si el vendedor atiende ambos tipos de consumidores. Cerrar el mercado para consumidores de baja valoración (cuando DPS sugiere atenderlos) es óptimo si las dos distorsiones que entran en juego –el *trade off* entre eficiencia y extracción de excedente del consumidor y la restricción de arbitrar con n paquetes por parte de los consumidores de alta valoración– son demasiado costosas para el vendedor. Dado que los consumidores de alta valoración tienen la opción de consumir más de un paquete pequeño, el vendedor debe ofrecer mayores descuentos por cantidad cuando decide mantener abierto el mercado para consumidores de baja valoración.

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Price discrimination with divisible goods

1 Introduction

The solution to second-degree price discrimination (SPD) already introduced by Pigou (1920) has been fully developed by, among others, Goldman, Leland and Sibley (1980), Oi (1971), Roberts (1979) and Spence (1977), and summarized neatly by Maskin and Riley (1984). Similar problems that share the same framework have been developed for quality discrimination (Mussa and Rosen, 1979), regulation and procurement (Baron and Myerson, 1982, Laffont and Tirole, 1986) and many other economic applications (taxation, financial system, etc.). Nowadays, this topic is a must in all Industrial Organization books.[1]

The solution to the standard SPD problem is as follows. Assume asymmetric information between several groups of buyers (with private information about their type –say, willingness to pay, or WTP–) and a seller with monopoly power. The optimal solution involves menu pricing with quantity discounts, trading off efficiency in allocations (quantities) for all but the highest type against consumer surplus (information rent) for all but the lowest type. This solution may involve bunching subsets of types.

However, the successful implementation of packages that maximize profit to the seller depends strongly on the premise that consumers may arbitrate among alternatives by choosing only <u>one</u> of them. This premise may be obvious in cases of quality discrimination (usually with unit demand), e.g., air tickets, because it is reasonable to assume that a passenger chooses one seat among first, business and tourist classes, which involve different perceived quality to her. In other words, there is no economic meaning in analyzing the case of a passenger comparing one business-class seat and two tourist-class seats. But the

[1] See, for example, Tirole (1988), recent books by Pepall, Richards and Norman (2006) and Belleflame and Peitz (2010), and also the chapters in the Handbook of Industrial Organization (Varian, 1988, Stole, 2007).

premise may be wrong if buyers are able to choose more than one unit of a product, as it is the case in many applications that involve quantity discounts: quantity is divisible by design, and the consumer cannot be restrained from choosing two or more units of a certain package. Examples abound: they relate to almost every product sold in many presentations (carbonated soft drink, beer, packaged bread, ready-to-eat cereals, laundry products, etc.)

This paper develops a model with two groups of buyers (high and low WTP) and shows that the standard solution to SPD does not apply when the high-WTP consumers find attractive to consume more than one small package (this is developed in Section 2). Section 3 develops the solution to the extended SPD problem (ESPD). The already known non-distortion-at-the-top result still holds. Under particular cases, the solution to the SPD problem is the solution to the ESPD problem: they involve low incremental value (i.e., differential valuation of the good) and low proportion of high-WTP consumers. Otherwise, the seller introduces less distortions than those suggested by the standard SPD case, provided that he attends both markets. But the seller may close the market for the low-WTP consumers (under conditions such that SPD suggests to keep it open) when the two distortions introduced in ESPD –the standard trade off between inefficiency and consumer surplus, and the *n-arbitrage constraint*, which induces high-WTP consumers not to choose extra packages designed for low-WTP consumers– are too costly to the seller. Finally, deeper quantity discounts are part of the solution when the incremental value of high-WTP is high, provided that the seller attends both consumer types.

2 Model and the standard case

2.1 Setup and second-degree price discrimination (SPD)

A seller produces and sells a product with total cost $c.x$, where x is quantity. Consumer preferences are summarized as follow:

$$U(x, T; \theta) = u(x, \theta) - T$$

where $u(x, \theta)$ is gross consumer surplus, T is the amount spent, and therefore $U(x, T; \theta)$ is (net) consumer surplus. There are two consumer types, θ_1 and θ_2, with $c < \theta_1 < \theta_2 < \infty$. The seller does not observe θ_i but knows the distribution of types,with $\lambda = Pr(\theta = \theta_1)$.

Throughout the paper we use a specific functional form

$$u(x, \theta) = \theta x - \frac{x^2}{2} \tag{1}$$

with associated (inverse) demand function $p(x, \theta) = \theta - x$. We define $\Delta\theta_1 = \theta_1 - c$ as the "value" of type-θ_1 consumers to the seller, and $\Delta\theta_2 = \theta_2 - \theta_1$ as the "incremental value" of type-θ_2 consumers to the seller. This way, we obtain closed form solutions that depend on fundamental main parameters of preferences $\Delta\theta_1$, $\Delta\theta_2$ and population λ.[2]

If the seller discriminates pefectly, for each pair seller - type-θ_i consumer, he offers an allocation x_i that maximizes total surplus ($u(.) - cx$) and the payment/transfer (from consumers to producer) that fully extracts consumer surplus. Using the functional form (1) quantities-transfers are

$$x_i^{D1}(\theta_i) = \theta_i - c \qquad T_i^{D1} = \frac{\theta_i^2 - c^2}{2} \tag{2}$$

[2] This specific functional form for gross consumer surplus satisfies the standard assumptions of monotonicity ($u(0, \theta) = 0$, $u_x > 0, u_{xx} < 0, u_\theta > 0$), single-crossing property ($u_{x\theta} > 0$), and no income effect. Inverse demand is $p(x, \theta)$ and demand is $x(p, \theta)$, such that $p(0, \theta) > c, \forall\theta$.

When the seller does not observe consumer types, he chooses pairs $P_i = (x_i, T_i)$ for $i = 1, 2$ to maximize profit

$$\max_{x_1, x_2, T_1, T_2} \pi = \lambda \left[T_1 - c x_1 \right] + (1 - \lambda) \left[T_2 - c x_2 \right] \tag{3}$$

subject to consumers' participation and self-selection constraints

$$\begin{aligned}
&u(x_1, \theta_1) - T_1 \geq 0 && (PC_1) \\
&u(x_2, \theta_2) - T_2 \geq 0 && (PC_2) \\
&u(x_1, \theta_1) - T_1 \geq u(x_2, \theta_1) - T_2 && (IC_1) \\
&u(x_2, \theta_2) - T_2 \geq u(x_1, \theta_2) - T_1 && (IC_2)
\end{aligned}$$

The pair $P_i = (x_i, T_i)$ is known in the literature as *bundle* or *package* to implement second-degree price discrimination (SPD).

From standard proofs, PC_1 and IC_2 constraints are binding while PC_2 and IC_1 constraints are slack (provided that $x_2 \geq x_1$). Payments from consumers to producer are set equal to

$$\begin{aligned}
T_1 &= u(x_1, \theta_1) \\
T_2 &= u(x_2, \theta_2) - \left[u(x_1, \theta_2) - u(x_1, \theta_1) \right]
\end{aligned}$$

which means that the seller fully extracts consumer surplus from type-θ_1 consumers and leaves type-θ_2 consumers indifferent between packages P_2 and P_1.

The SPD problem simplifies to

$$\max_{x_1, x_2} \pi = \lambda \left[u(x_1, \theta_1) - c x_1 \right] + (1 - \lambda) \left[u(x_2, \theta_2) - (u(x_1, \theta_2) - u(x_1, \theta_1)) - c x_2 \right]$$

From first-order conditions x_2^{D2} is such that $u_x(x_2, \theta_2) = c$ and x_1^{D2} is such that

$$\lambda \left(u_x(x_1, \theta_1) - c \right) = (1 - \lambda) \left[u_x(x_1, \theta_2) - u_x(x_1, \theta_1) \right]$$

When preferences are represented as (1), quantities are

$$\begin{aligned}
x_2^{D2} &= \theta_2 - c && (4) \\
x_1^{D2} &= \Delta\theta_1 - \frac{1 - \lambda}{\lambda} \Delta\theta_2 && (5)
\end{aligned}$$

and transfers are set such that type-θ_1 consumers get no surplus while type-θ_2 consumer surplus is $(x_1, \theta_2) - u(x_1, \theta_1) = \Delta\theta_2 - x_1$. The solution to the SPD problem is $x_2^{D2} = x_2^{D1}$, i.e., non distortion at the top,[3] and $x_1^{D2} < x_1^{D1}$, i.e., the seller distorts x_1 trading off inefficiency in sales to type-θ_1 consumers against consumer surplus (or information rent) in sales to type-θ_2 consumers. Moreover, $x_1^{D2} < x_2^{D2}$ guarantees that IC_1 is indeed slack. Finally, the solution is interior ($x_1^{D2} > 0$) for $\lambda > \underline{\lambda}$, which is

$$\underline{\lambda} = \frac{\theta_2 - \theta_1}{\theta_2 - c} = \frac{\Delta\theta_2}{\Delta\theta_2 + \Delta\theta_1} \tag{6}$$

in the special case. Otherwise, the solution to SPD is the same as the solution to FPD for θ_2 combined with the exclusion of θ_1 consumers. Intuitively, if the ratio of high to low consumer preferences $(1 - \lambda)$ or the ratio incremental value of type-θ_2 consumers to value of type-θ_1 consumers (here, $\Delta\theta_2/\Delta\theta_1$) are sufficiently high, it is optimal for the seller to close the market for type-θ_1 consumers and sell exclusively to type-θ_2 consumers.

2.2 SPD does not consider n-arbitrage constraints

The SPD problem in Section 2.1 explicitly restricts itself to the case that type-θ_2 consumers arbitrate surplus choosing between one package P_2 and one package P_1. As discussed in the introduction, this is a reasonable assumption in quality discrimination (consumers arbitrate between one first-class seat and one tourist seat) and other applications, but it may not apply to products with divisible allocations (for example, quantity). For example, if the product is carbonated soft drinks and the allocation x is the content (fl oz) of different packages (a can, a bottle, etc.), consumers do have the option of arbitrating between one package P_2 (e.g., one 1-liter bottle of soda) and as many packages P_1 as they want (e.g., one, two or three 12-ounce cans of soda). The principle underlying this example extends to all products that involve many presentations with different quantities.

Next we show that the solution (4)-(5) to SPD fails in satisfying the constraint that type-θ_2 does not arbitrate choosing n packages P_1 to obtain a surplus of $u(nx_1, \theta_2) - nT_1$

[3] This result is known from the optimal taxation literature (see Mirrlees, 1971, and Seade, 1977).

(named the "n-arbitration constraint"). With preferences (1) this surplus ascends to

$$u(nx_1, \theta_2) - nu(x_1, \theta_1) = n(\theta_2 - \theta_1)x_1 - \frac{n(n-1)x_1^2}{2} \tag{7}$$

Proposition 1 *Suppose the seller implements the solution to the SPD problem (3) with packages $P_1^{D2} = (x_1^{D2}, T_1^{D2})$ and $P_2^{D2} = (x_2^{D2}, T_2^{D2})$. Type-$\theta_2$ consumers arbitrate surplus by choosing 2 or more packages P_1^{D2} for low values of λ. Under preferences (1) this happens when $\lambda < \Delta\theta_2/\Delta\theta_1$.*

Proof: In the particular case of preferences (1), type-θ_2 consumer surplus with $2x_1^{D2}$ (i.e., equation (7) for $n = 2$) is higher than her surplus with x_1^{D2} (i.e., equation (7) for $n = 1$) if $\lambda < \Delta\theta_2/\Delta\theta_1$.

When λ is low the distortion in x_1^{D2} is high (equation 5) and type-θ_2 consumers may even choose more than 2 packages P_1^{D2}.

The proof does not depend on the functional form (1). For general utility functions, we only have to show that there are values of λ low enough (but higher than the corresponding $\underline{\lambda}$) that the allocation x_1 chosen by the seller allows type-θ_2 consumers for this possibility. Figure 1 helps to show this point. The example in the left-panel shows a distortion in x_1^{P2} relatively small, compared to x_1^{P1}. Type-θ_2 consumers derive more surplus with x_1^{P2} $(u(x_1, \theta_2))$ that with $2x_1^{P2}$ $(u(2x_1, \theta_2))$. But the example in the right-panel shows that if the distortion in x_1^{P2} is large relative to x_1^{P1}. Type-θ_2 consumers derive more surplus with $2x_1^{P2}$ or even $3x_1^{P2}$ ($u(2x_1, \theta_2)$ and $u(3x_1, \theta_2)$, respectively) that with x_1^{P2} $(u(x_1, \theta_2))$. *Q.E.D.*

Proposition 1 states that when the distortion in x_1^{D2} opens the option for high-demand consumers to arbitrate, then (4)-(5) do not take into account incentive compatibility constraints correctly. Even more,

Corollary 1 *With preferences (1), if $\Delta\theta_2 > \Delta\theta_1$ packages $P_1^{D2} = (x_1^{D2}, T_1^{D2})$ and $P_2^{D2} = (x_2^{D2}, T_2^{D2})$ are not the solution to second-degree price discrimination.*

Example 1: Assume $\theta_1 = 50$, $\theta_2 = 80$, $\lambda = 0.6$ and $c = 10$. The solution to SPD is a pair $P_1^{D2} = (20, 800)$ and $P_2^{D2} = (70, 2550)$. However, type-θ_2 consumers will find attractive 2 packages P_1^{D2}. At the proposed solution, their surplus is $u(70, \theta_2 = 80) - 2550 = 600$ which equals their surplus if they choose package P_1^{D2} (i.e., $u(20, \theta_2 = 80) - 800 = 600$). But their surplus from consuming 2 packages P_1^{D2} is $u(2 \times 20, \theta_2 = 80) - 2 \times 800 = 800$, which is higher than 600.

Example 2: Assume $\theta_1 = 50$, $\theta_2 = 150$, $\lambda = 0.85$ and $c = 10$. The solution to SPD is a pair $P_1^{D2} = (22.40, 867.82)$ and $P_2^{D2} = (140, 8964.71)$. However, type-$\theta_2$ consumers will find attractive 5 packages P_1^{D2}. At the proposed solution, their surplus is $u(140, \theta_2 = 150) - 8964.71 = 2235.29$ which equals their surplus if they choose package P_1^{D2} (i.e., $u(22.40, \theta_2 = 150) - 867.82 = 2235.29$). But their surplus from consuming 5 packages P_1^{D2} is $u(5 \times 22.40, \theta_2 = 150) - 5 \times 867.82 = 5916.96$, which is higher than 2235.29.

Next section formulates the extended SPD problem that takes all relevant incentive constraints into consideration to ensure self-selection. The use of preferences (1) is helpful to show neat results.

3 Extended second-degree price discrimination

The extended-SPD (ESPD) problem consists of finding pairs $P_1 = (x_1, T_1)$ and $P_2 = (x_2, T_2)$ that maximize profit

$$\max_{x_1, x_2, T_1, T_2} \pi = \lambda [T_1 - cx_1] + (1 - \lambda) [T_2 - cx_2] \tag{8}$$

subject to consumers' participation and (overall) self-selection constraints:

$$
\begin{aligned}
&u(x_1,\theta_1) - T_1 \geq 0 && (PC_1)\\
&u(x_2,\theta_2) - T_2 \geq 0 && (PC_2)\\
&u(x_1,\theta_1) - T_1 \geq u(x_2,\theta_1) - T_2 && (IC_1)\\
&u(x_2,\theta_2) - T_2 \geq u(x_1,\theta_2) - T_1 && (IC_2 \times 1)\\
&u(x_2,\theta_2) - T_2 \geq u(2x_1,\theta_2) - 2T_1 && (IC_2 \times 2)\\
&\ldots\\
&u(x_2,\theta_2) - T_2 \geq u(nx_1,\theta_2) - nT_1 && (IC_2 \times n)\\
&\ldots\\
&u(\theta_2,x_2) - T_2 \geq u(Nx_1,\theta_2) - NT_1 && (IC_2 \times N)
\end{aligned}
$$

Lemma 1 *Generically, (PC_1) constraint and only one $(IC_2{\times}n)$ constraint will be binding in the solution to problem (8).*

Proof: The result that condition (PC_1) is binding and condition (PC_2) is slack follows from the solution to the standard SPD problem. That is, the seller leaves type-θ_1 consumers with no surplus, while he leaves type-θ_2 consumers with a surplus related to their arbitrage possibilities.

Condition $IC_2 \times 1$ may not bind if type-θ_2 consumers derive more surplus choosing n packages P_1, in which chase condition $IC_2 \times n$ binds.[4] *Q.E.D.*

Given Lemma 1, we can restrict ourselves to solve problem (8) subject to the binding conditions PC_1 and $IC_2 \times n$. For a pair of allocations (x_1, x_2), payments / transfers from consumers to the seller are

$$T_1 = u(x_1,\theta_1)$$

[4] Type-θ_2 consumers may be indifferent between two options ax_1 and bx_1. Given that in this case both $IC_2 \times a$ and $IC_2 \times b$ hold in equality, and that the role of these constraints is to define the surplus that the seller must leave to type-θ_2 consumers, we choose the constraint with lower quantity as part of the solution to the ESPD problem.

$$T_2 \quad = \quad u(x_2, \theta_2) - [u(nx_1, \theta_2) - nu(x_1, \theta_1)]$$

Problem (8) simplifies to

$$\max_{x_1, x_2} \pi = \lambda \left[u(x_1, \theta_1) - cx_1 \right] + (1 - \lambda) \left[u(x_2, \theta_2) - (u(nx_1, \theta_2) - nu(x_1, \theta_1)) - cx_2 \right]$$

It is useful to define the interior solution of this problem

$$\lambda \left[u_x(x_1, \theta_1) - c \right] = (1 - \lambda) n \left[u_x(nx_1, \theta_2) - u_x(x_1, \theta_1) \right] \quad \text{and} \quad u_x(x_2, \theta_2) - c$$

When preferences are represented as (1), quantities are

$$x_2^{D2} \quad = \quad \theta_2 - c \tag{9}$$

$$x_1(n) \quad = \quad \frac{\lambda \Delta\theta_1 - n(1 - \lambda)\Delta\theta_2}{\lambda - n(n-1)(1 - \lambda)} \tag{10}$$

Next proposition states the main result of the paper.

Proposition 2 *In the solution to problem (8) the seller does not distort* x_2^{D2} *(equation (9)) corresponding to package* P_2^{D2}*. In order to define* $x_1^{D2}(n)$ *corresponding to package* P_1^{D2}*, define* $\hat{\lambda}$*,* $x_1(n, \hat{\lambda}(n))$ *and* $\hat{\hat{\lambda}}$ *as follows*

$$\hat{\lambda}(n) \quad = \quad \frac{n\Delta\theta_2}{(n-1)\Delta\theta_2 + n\Delta\theta_1} \tag{11}$$

$$x_1(n, \hat{\lambda}(n)) \quad = \quad x_1(n)|_{\lambda = \hat{\lambda}(n)} = \frac{\Delta\theta_2^2 - (n-1)\Delta\theta_1\Delta\theta_2}{n\Delta\theta_2 - n(n-1)\Delta\theta_1} \tag{12}$$

$$\hat{\hat{\lambda}}(n) \quad = \quad \frac{n(n+1)\Delta\theta_2^2 - n(n-1)(n+1)\Delta\theta_1\Delta\theta_2}{[n(n+1) - 1]\Delta\theta_2^2 - 2n(n-1)\Delta\theta_1^2 - [1 + n(n-2)(n+2)]\,\Delta\theta_1\Delta\theta_2} \tag{13}$$

Set $\Delta\theta_1$*. For* $\Delta\theta_2 \in ((n-1)\Delta\theta_1, n\Delta\theta_1]$

$$x_1^{D2}(n) = \begin{cases} x_1(n) \text{ from (10)} & \text{if } \hat{\lambda}(n) \leq \lambda \\ x_1(n, \hat{\lambda}(n)) \text{ from (12)} & \text{if } \hat{\hat{\lambda}}(n) \leq \lambda < \hat{\lambda}(n) \\ 0 & \text{if } \lambda < \hat{\hat{\lambda}}(n) \end{cases} \tag{14}$$

Transfers are set to leave consumer surplus $U(\theta_1) = 0$ *and*

$$U(\theta_2) = u(nx_1^{D2}(n), \theta_2) - nu(x_1^{D2}(n), \theta_1) = n\Delta\theta_2 x_1^{D2}(n) - \frac{n(n-1)\left(x_1^{D2}(n)\right)^2}{2} \tag{15}$$

Proof: See Appendix.

This proposition covers several results. First, we show the conditiones under which the solution to the SPD is the solution to the ESPD.

Corollary 2 *The solution to SPD (problem (3)) is the solution to ESPD (problem (8)) if* $\Delta\theta_2 < \Delta\theta_1$ *and* $\lambda \geq \hat{\lambda}(1) = \Delta\theta_2/\Delta\theta_1$.

Corollary 2 shows the limitations of the SPD solution: both the incremental value $(\theta_2 - \theta_1)$ and the share of type-θ_2 consumers must be low. Otherwise, the seller leaves open extra options for these consumers to arbitrate.

Second, continuing with the case of $\Delta\theta_2 < \Delta\theta_1$ we show the distortions in allocations imposed in the solution to the ESPD problem.

Corollary 3 *Assume* $\Delta\theta_2 < \Delta\theta_1$ *and* $\lambda < \hat{\lambda}(1)$. *The solution to ESPD problem involves under-distortions for* $\hat{\hat{\lambda}}(1) \leq \lambda < \hat{\lambda}(1)$ *and over-distortions for* $\underline{\lambda} < \lambda < \hat{\hat{\lambda}}(1)$.

Corollary 3 shows that the distortion in x_1 is less than the predicted in SPD for intermediate values of λ because if the seller distorted x_1 more, type-θ_2 consumers would choose 2 packages P_1^{D2}. In such instances, the seller is dealing with a <u>double distortion</u>: the standard trade-off between inefficiency in allocations and extraction of consumer surplus, plus the n-arbitration constraint (on additional arbitrage opportunities for type-θ_2 consumers). When the combination of preferences and share of consumers implies both distortions become too costly to the seller, it is optimal for him not to sell packages P_1^{D2}. This way, the seller over-distorts the allocation x_1 (compared to the solution to SPD).

Notice also that in the *under*-distortion scenario the quantity discount implicit in package $P_2^{D2} = (x_2^{D2}, T_2^{D2})$ is lower than in the standard SPD case.

Figure 2 shows the results in Corollaries 2 and 3 for the secial case of $\Delta\theta_1 = 40$ and $\Delta\theta_2 = 30$ (i.e., $\Delta\theta_2 \leq \Delta\theta_1$). The upper panel shows the relationship between λ and x_1^{P2} while the lower panel does the same for λ and profits.

Third, when preferences are represented as (1) the combination of $\Delta\theta_1$ and $\Delta\theta_2$ define the number n of packages that type-θ_2 consumers would choose if they decided to arbitrate.

Corollary 4 *Assume $\Delta\theta_2 \in ((n-1)\Delta\theta_1, n\Delta\theta_1)$. If the seller chooses a positive x_1 he has to set T_2 in order to leave enough surplus to type-θ_2 consumers for them not to choose n packages P_1^{D2}. Quantity discounts must be more aggressive than in standard SPD.*

Corollary 4 shows that if $\Delta\theta_2 > \Delta\theta_1$ quantity discounts must be aggressive. The possibility of under-distortion in x_1 may temper partially the deep discounts. Corollary 5 is more specific about the general distortions of x_1.

Corollary 5 *Assume $\Delta\theta_2 \in ((n-1)\Delta\theta_1, n\Delta\theta_1)$.*

(i) If $\hat{\lambda}(n) \leq \lambda$, $x_1^{D2}(n) > x_1^{D2}$.

(ii) If $\hat{\hat{\lambda}}(n) \leq \lambda < \hat{\lambda}(n)$, $x_1^{D2}(n) > x_1^{D2}$.

(iii) If $\underline{\lambda} \leq \lambda < \hat{\hat{\lambda}}(n)$, $x_1^{D2}(n) < x_1^{D2}$.

Corollary 5 provides a neat result on distortions (already anticipaded in Corollary 3 for a special case). In general, ESPD involves less (or *under*-)distortions in x_1 than SPD does, unless the seller does not provide packages P_1^{D2}, in which case he *over*-distorts x_1.

Corollary 6 *Given $\Delta\theta_1$ (linked to the relationship between $\Delta\theta_1$ and $\Delta\theta_2$), $\underline{\lambda} < \hat{\hat{\lambda}}(n) < \hat{\lambda}(n)$ for all n. Moreover, all $\underline{\lambda}$, $\hat{\hat{\lambda}}$ and $\hat{\lambda}$ increase in $\Delta\theta_2$.*

Lastly, Corolary 6 ensures that all cases –interior, constrained and corner values of x_1– are possible solutions for a given relationship between $\Delta\theta_1$ and $\Delta\theta_2$. But the three treshold values converge (to 1) as $\Delta\theta_2$ grows large (of course, the fact that $\underline{\lambda} \rightarrow 1$ in this case is included in the solution to SPD problem).

The results in Collaries 4 to 6 can be seen in Figure 3, in the particular example of $\Delta\theta_1 = 40$ and $\Delta\theta_2 = 100$ (i.e., $2\Delta\theta_1 < \Delta\theta_2 \leq 3\Delta\theta_1$). The upper panel shows the relationship between λ and x_1^{P2} while the lower panel does the same for λ and profits.

4 Concluding Remarks

This paper develops a second-degree price discrimination model with two types of buyers (high and low demand) to show that the standard solution to the SPD problems may noy apply when consumers arbitrate among alternative packages by choosing more than one unit of any of them. This is a clear possibility when products are divisible and sold in many presentations (carbonated soft drink, beer, packaged bread, ready-to-eat cereals, laundry products, etc.). We provide a solution to the extensive SPD (ESDP) problem. The seller provides the efficient quantity to high-WTP consumers and introduces inefficiencies in the quantity of packages designed for low-WTP consumers. However, distortions are less than those suggested by standard SPD, provided that the seller attends both markets (except in the case of low incremental value and low proportion of high-WTP consumers). Closing the market for low-WTP consumers (when standard SPD suggests to keep it open) is also a possibility when the two existing distortions –the standard trade off and the extra constraint for high-WTP consumers to consume an extra package designed for low-WTP consumers– are too costly to the seller. Given the possibility to consume more than one small packages, the seller will have to offer deeper quantity discounts to high-WTP consumers.

The reader may suspect that the assumption of asymmetric information partitioned in two types is too strong. The extension to three or more (but not continuous) types will generate different combinations of incremental values and shares of type-θ_i consumers (even in partial-pooling cases) under which one of the possible solutions shown here will hold.

Although we do not pursuit it in this paper, the results do not depend specific functional for consumer surplus (1). Every model applied to SPD with general preferences $U(x, T; \theta)$ and discrete types includes a combination of parameters that generates low x_1. As stated by Maskin and Riley (1984), "As the ratio of high to low demanders increases, the offer $< q_1^{**}, R_1^{**} >$ moves further to the left until eventually $< q_1^{**}, R_1^{**} >=< 0, 0 >$"

(see p.175). At some point, a distortion in x_1 opens the possibility for high-WTP consumers to consume more than one small-package option, and the proposed solution to SPD will not satisfy wide incentive compatibility conditions.

Appendix

Proof of Proposition 2.

Fix $\Delta\theta_1$ and partition the possible values of $\Delta\theta_2$. We divide the proof in two parts, the special case $\Delta\theta_2 \leq \Delta\theta_1$ and the general case $\Delta\theta_2 \in ((n-1)\Delta\theta_1, n\Delta\theta_1]$. In all cases we start with high values of λ ($\rightarrow 1$) and then reduce λ in order to distort x_1 downwards and reassess the solution to ESPD.

- $\Delta\theta_1 \geq \Delta\theta_2$. Assume a high value λ so $x_1^{D2}(1) \approx x_1^{D1}$ from (2). Type-θ_2 consumers arbitrate at most with one package P_1^{D2}. As λ decreases, so does the distortion of $x_1^{D2}(1)$. If λ becomes lower than a certain treshold, which is defined by $\hat{\lambda}(1)$, type-θ_2 consumers find attractive to consume 2 packages P_1^{D2}. Let $x_1(1, \hat{\lambda}(1))$ be the allocation of P_1^{D2} at $\lambda = \hat{\lambda}(1)$. If the seller distorts x_1 below $x_1(1, \hat{\lambda}(1)$ he must reduce T_2 in order to give an extra surplus to type-θ_2 consumers beyond the standard trade-off.

Figure 4 is helpful to understand this result: let $\hat{x}_1 = x_1(1, \hat{\lambda}(1))$. At this quantity the is trading off the distortion of x_1 against the type-θ_2 surplus (area $\theta_1\theta_2 ab$, more specifically the height ab). If $\lambda < \hat{\lambda}(1)$ the seller would set $x_1' < \hat{x}_1$ (trading off more distortion to θ_1 consumers against less surplus to type-θ_2 consumers, i.e., area $\theta_1\theta_2 d'b'$). But type-θ_2 consumers find attractive to consume 2 packages P_1^{D2} because it gives them extra surplus $a'd'f'e'$. Given the corresponding λ, the marginal distortion in allocation at x_1' is equal to the marginal increase in T_2, the amount $a'd'f'd'$ (wich becomes positive once type-θ_2 consumers find attractive 2 packages P_1^{D2}) is a sacrifice that the seller must support through a reduction in T_2.

The seller would rather not distort x_1 more than $x_1(1, \hat{\lambda}(1))$ and keep type-θ_2 consumers from arbitrating with 2 packages P_1^{D2}. His profit is

$$\begin{aligned}
\pi &= \lambda \left[\theta_1 x_1(1, \hat{\lambda}(1)) - \frac{\left(x_1(1, \hat{\lambda}(1))\right)^2}{2} - c(x_1(1, \hat{\lambda}(1)) \right] \\
&\quad + (1-\lambda) \left[\theta_2 x_2^{DP2} - \frac{\left(x_2^{DP2}\right)^2}{2} - \Delta\theta_2 (x_1(1, \hat{\lambda}(1)) - c x_2^{DP2} \right] \\
&= (1-\lambda)\frac{(\theta_2 - c)^2}{2} + \left[\lambda\Delta\theta_1 - \left(1 - \frac{\lambda}{2}\right)\Delta\theta_2 \right] \Delta\theta_2
\end{aligned}$$

But in this case, the seller is introducing a new distortion to the standard trade-off between efficiency and information rent: he is controlling that type-θ_2 consumers do not select 2 packages P_1^{D2}. At some point these two distortions become too costly to the seller so that he may close the x_1 market and provide only θ_2 consumers, in which case he gets a profit equal to $\lambda(\theta_2 - c)^2/2$. Therefore, the sign of

$$\lambda\Delta\theta_1 - \left(1 - \frac{\lambda}{2}\right)\Delta\theta_2$$

defines whether $x_1^{D2}(1) = x_1(1, \hat{\lambda}(1))$ (for $\lambda \geq \hat{\hat{\lambda}}(1)$) or $x_1^{D2}(1) = 0$ (for $\lambda < \hat{\hat{\lambda}}(1)$. Finally, it is easy to check that $\underline{\lambda} < \hat{\hat{\lambda}}(1) < \hat{\lambda}$.

- General case: $\Delta\theta_2 \in ((n-1)\Delta\theta_1, n\Delta\theta_1]$. Assume a high value of λ so that $x_1^{D2}(n) \approx x_1^{D1}$ from (2). Type-θ_2 consumers arbitrate with n packages P_1^{D2}. As λ decreases, so does the distortion to $x_1^{D2}(n)$. If λ is lower than the treshold $\hat{\lambda}(n)$, type-θ_2 consumers find attractive to consume $n+1$ packages P_1^{D2}. Let $x_1(n, \hat{\lambda}(n))$ be the allocation of P_1^{D2} at $\lambda = \hat{\lambda}(n)$. If the seller distorts x_1 below $x_1(n, \hat{\lambda}(n))$ he must reduce T_2 in order to give an extra surplus to type-θ_2 consumers beyond the standard trade-off efficiency-information rent because high-demand consumers may switch from n to $n+1$ packages P_1^{D2}. The trade-off is the same as in the case with $n = 1$. The treshold $\hat{\lambda}(n)$ is obtained from

$$\begin{aligned}
u\left((n+1)x_1^{D2}, \theta_2\right) - (n+1)u\left(x_1^{D2}, \theta_1\right) &= u\left(nx_1^{D2}, \theta_2\right) - nu\left(x_1^{D2}, \theta_1\right) \\
\Delta\theta_2 x_1^{D2} &= \left[\frac{(n+1)n}{2} - \frac{n(n-1))}{2}\right](x_1^{D2})^2 \\
\Delta\theta_2 &= nx_1^{D2} \\
\Delta\theta_2 &= n\frac{\lambda\Delta\theta_1 - n(1-\lambda)\Delta\theta_2}{\lambda - n(n-1)(1-\lambda)} \\
&\dots \\
\hat{\lambda}(n) &= \frac{n\Delta\theta_2}{n\Delta\theta_1 + (n-1)\Delta\theta_2}
\end{aligned}$$

This is equation (11). Next, we replace $\hat{\lambda}(n)$ into $x_1(n)$ to get $x_1(n, \hat{\lambda}(n))$.

$$\begin{aligned}
x_1(n, \hat{\lambda}(n)) &= \frac{\lambda\Delta\theta_1 - n(1-\lambda)\Delta\theta_2}{\lambda - n(n-1)(1-\lambda)}|_{\lambda=\hat{\lambda}(n)} \\
&\dots \\
&= \frac{\Delta\theta_2^2 - (n-1)\Delta\theta_1\Delta\theta_2}{n\Delta\theta_2 - n(n-1)\Delta\theta_1}
\end{aligned}$$

This is equation (12). If the seller sets $x_1(n, \hat{\lambda}(n))$ and lets type-θ_2 consumers arbitrate with n packages P_1^{D2} instead of setting $x_1 < x_1(n, \hat{\lambda}(n))$ and letting type-θ_2 consumers arbitrate with $n+1$ packages P_1^{D2}, his profit is

$$\begin{aligned} \pi &= \lambda \left[\theta_1 x_1 - \frac{x_1^2}{2} - c x_1 \right] + (1-\lambda) \left[\left(\theta_2 x_2 - \frac{x_2^2}{2} \right) - \left(n \Delta\theta_2 x_1 - \frac{n(n-1)}{2} x_1^2 \right) - c x_2 \right] \\ &= (1-\lambda) \frac{(\theta_2 - c)^2}{2} + \beta(n, \lambda) \end{aligned}$$

where

$$\beta(n, \lambda) = \left[\lambda \Delta\theta_1 - (1-\lambda) n \Delta\theta_2 \right] x_1(n, \hat{\lambda}(n)) - \left[\lambda - (1-\lambda) n(n-1) \right] \frac{\left(x_1(n, \hat{\lambda}(n)) \right)^2}{2}$$

As stated before, the seller is adding a new distortion to the the efficiency-information rent trade-off (in this case, with n packages P_1^{D2}), i.e., he discourages type-θ_2 consumers from arbitrating with an extra package P_1^{D2}. At some point these two distortions become too costly to the seller so that he may close the x_1 market and provide only θ_2 consumers, in which case he gets a profit equal to $\lambda(\theta_2 - c)^2/2$. The sign of $\beta(n, \lambda)$ defines whether $x_1^{D2}(n) = x_1(n, \hat{\lambda}(n))$ (if $\lambda \geq \hat{\hat{\lambda}}(n)$) or $x_1^{D2}(n) = 0$ (if $\lambda < \hat{\hat{\lambda}}(n)$. Substituting $x_1(n, \hat{\lambda}(n))$ into $\beta(n, \lambda)$ and equating the latter to zero

$$\begin{aligned} 0 &= 2 \left[\lambda \Delta\theta_1 - (1-\lambda) n \Delta\theta_2 \right] - \left[\lambda - (1-\lambda) n(n-1) \right] x_1(n, \hat{\lambda}(n)) \\ & \dots \\ \hat{\hat{\lambda}}(n) &= \frac{n(n+1)\Delta\theta_2^2 - n(n-1)(n+1)\Delta\theta_1 \Delta\theta_2}{[n(n+1)-1]\Delta\theta_2^2 - 2n(n-1)\Delta\theta_1^2 - [1 + n(n-2)(n+2)] \Delta\theta_1 \Delta\theta_2} \end{aligned}$$

Finally, comparing equations (6), (11) and (13) it is easy to check that $\underline{\lambda} < \hat{\hat{\lambda}}(n) < \hat{\lambda}(n)$. *Q.E.D.*

Figure 1: Arbitrage possibilities open to high-demand consumers

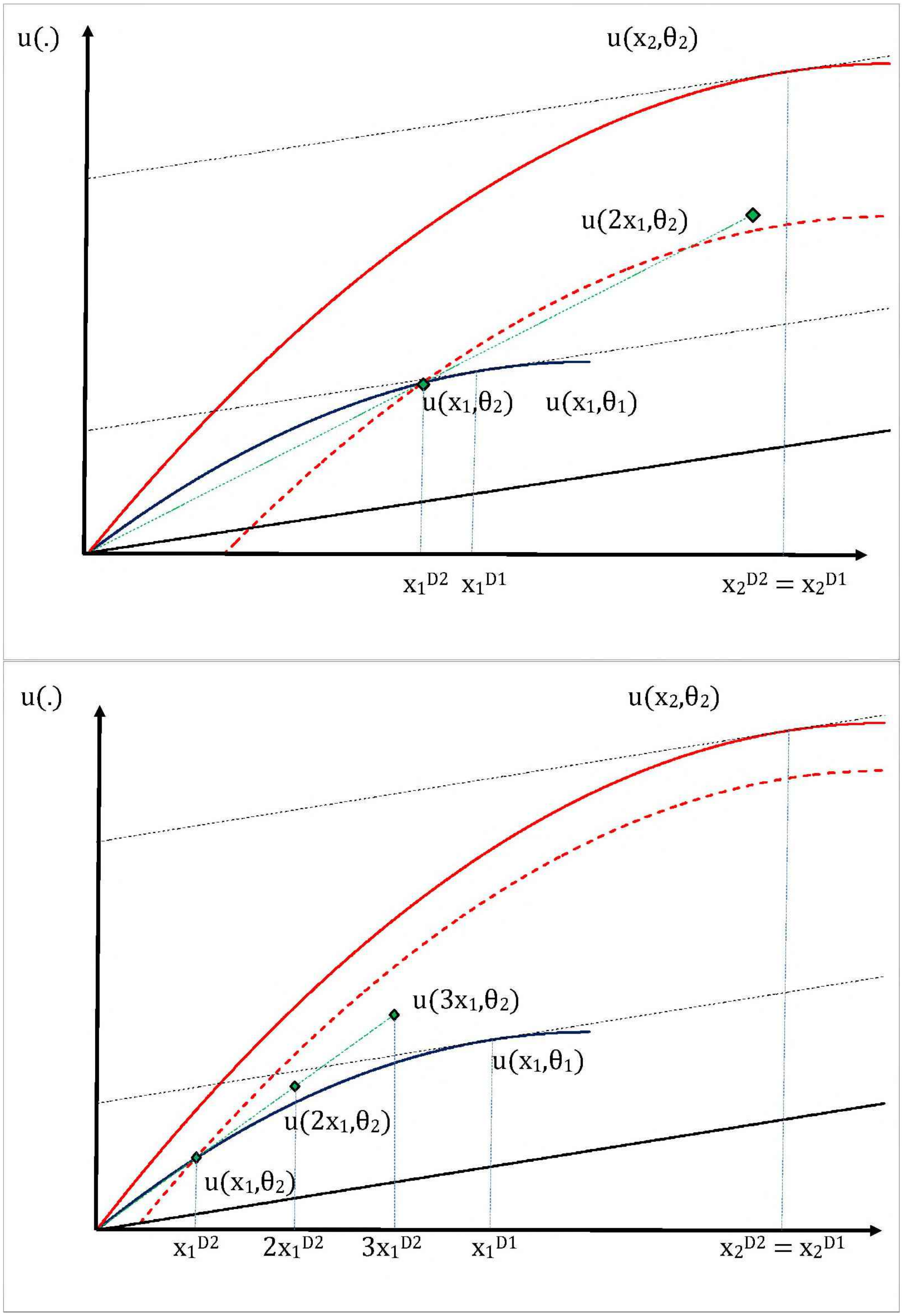


Figure 2: Solution to SPD - Case 1: $\Delta\theta_1$= 40 ; $\Delta\theta_2$= 30

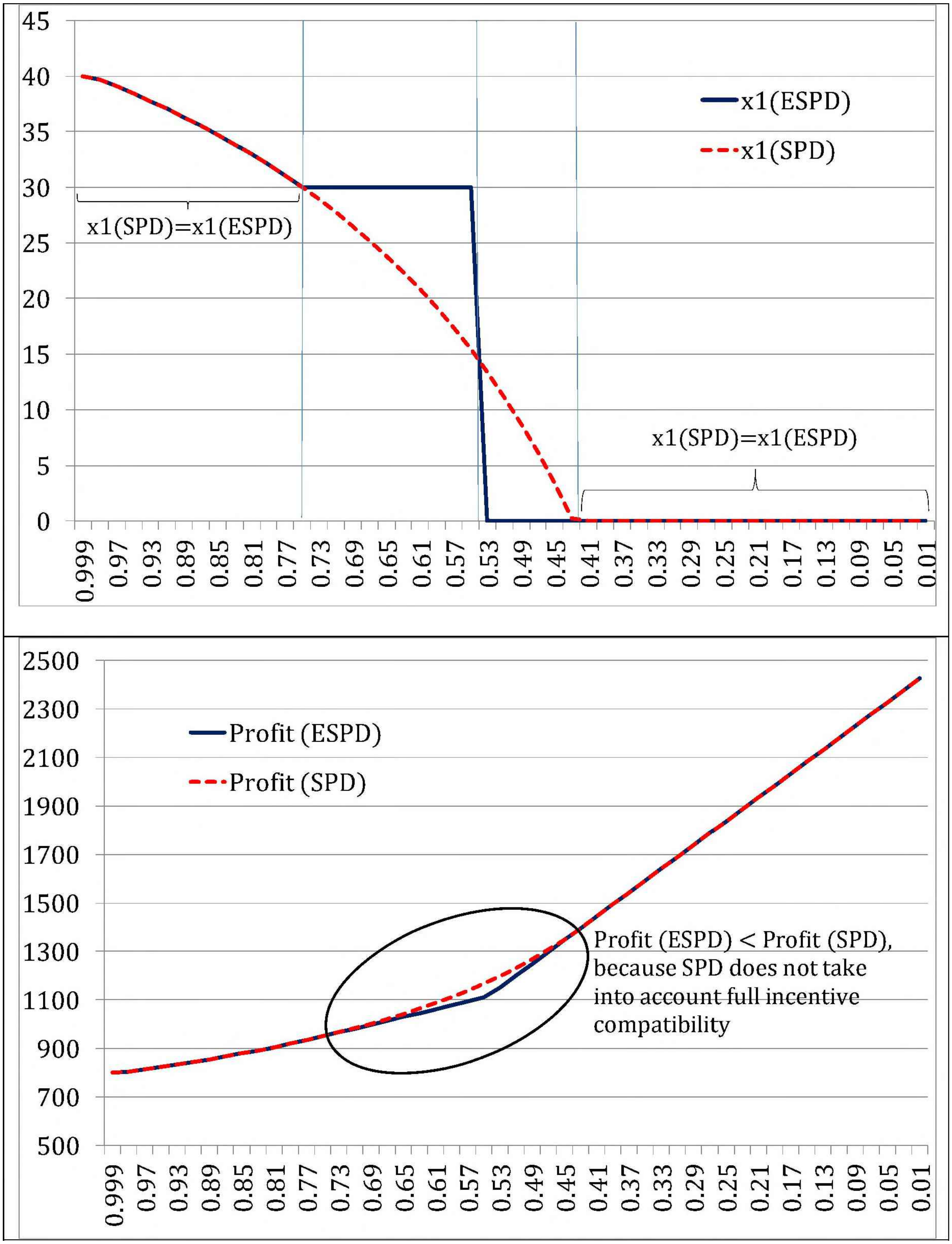


Figure 3: Solution to SPD - Case 2: $\Delta\theta_1$= 40 ; $\Delta\theta_2$= 100

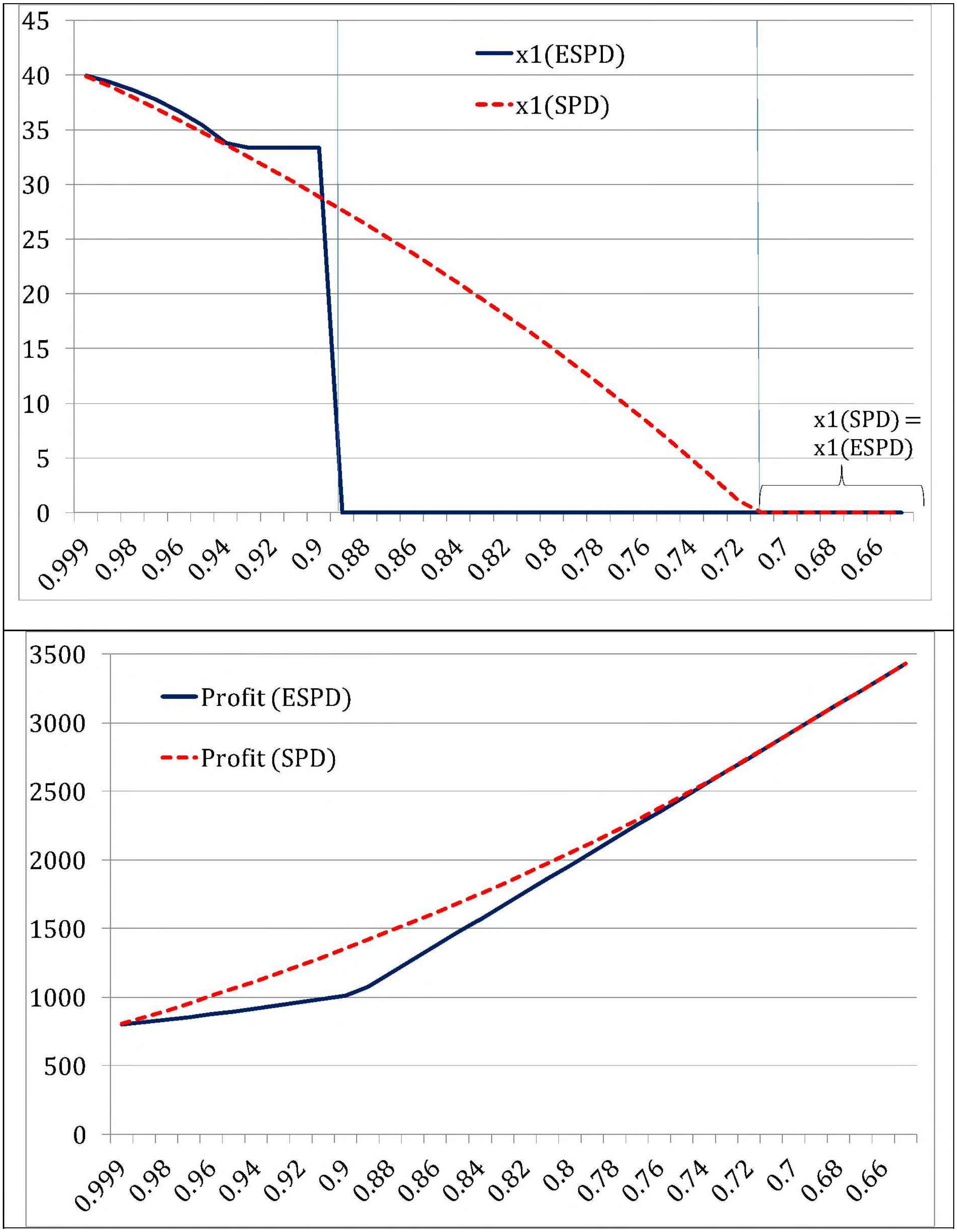
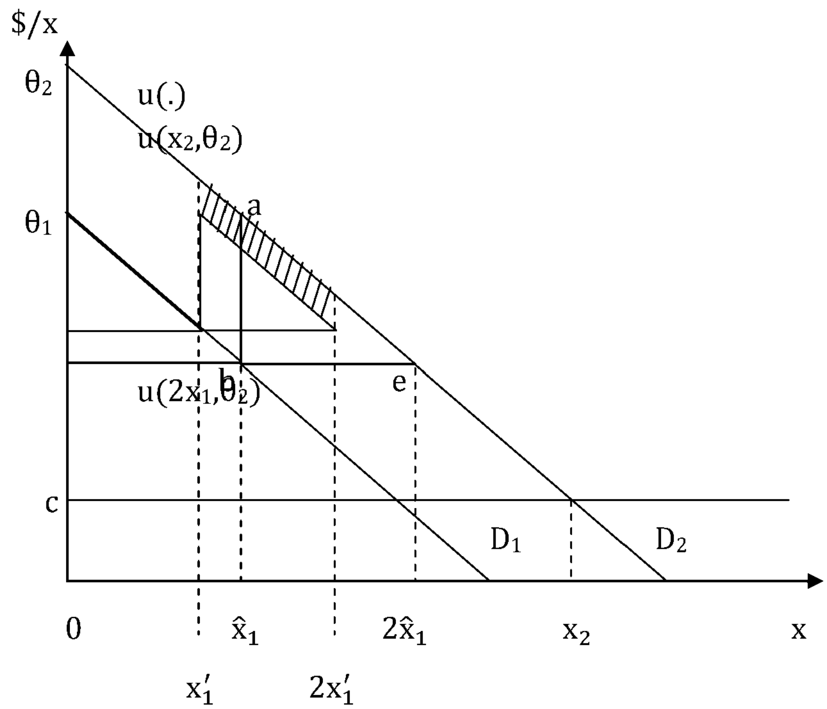


Figure 4: Type-θ_2 consumer alternatives



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