Unrestricted multivariate medians for adaptive filtering of color images

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Abstract. Reduction of impulse noise in color images is a fundamental task in the image processing field. A number of approaches have been proposed to solve this problem in literature, and many of them rely on some multivariate median computed on a relevant image window. However, little attention has been paid to the comparative assessment of the distinct medians that can be used for this purpose. In this paper we carry out such a study, and its conclusions lead us to design a new image denoising procedure. Quantitative and qualitative results are shown, which demonstrate the advantages of our method in terms of noise reduction, detail preservation and stability with respect to a selection of well-known proposals.

Keywords: Adaptive filtering, multivariate median, impulse noise, edge detection.

1. Introduction

Filtering of color images is aimed at reducing noise while at the same time chromaticity, edges and details are preserved [1]. This can be done by either component-wise or vector methods [2]. The main difference between these families of methods is that component-wise methods can introduce new color artifacts in the resulting image, because they process each pixel color channel independently without considering the correlation between channels. On the other hand, the vector methods process the color channels of each pixel as a vector, thus avoiding inconvenient chromaticity changes in the resulting image [2], [3]. For this reason, the vector methods are more effective for noise reduction and preservation of color image chromaticity. Among classical non-linear vector based filters, we have the Vector Median Filter (VMF) [4]; the Basic Vector Directional Filter (BVDF) [5]; and the Directional Distance Filter (DDF) [6]. These filters are uniformly applied on the image; therefore they tend to modify both noisy pixels and edge pixels. Consequently, effective noise removal is achieved at the expense of blurred and distorted features. In order to better preserve the image structure, the vector median and directional weighted adaptive filters have been proposed [7], [8]. The vector filters based on an adaptive switching scheme [3], [9], [10], [11] consider an impulse detector to determine which pixels should be filtered and which pixels should be preserved. These filters are simple and effective in preserving image details.

We propose a new filter for noise reduction in color image, based on the filter unrestricted multivariate medians, in an adaptive switching scheme. Median based filters such as the VMF are commonplace in literature; there has been little interest in studying the comparative advantages of the different multivariate medians that could be used for image denoising. Here we do such a study, and its result leads us to propose a new denoising filter for color images corrupted by impulse noise. The results of experiments and simulations have shown that the proponed filter is better than many other existing adaptive filters, in terms of capacity for noise reduction, preservation of edges, thin detail and image color.

The paper is organized as follows. Section 2 describes the different multivariate medians that can be used for color image denoising, and then carries out a study of their relative strengths. In light of the results of this study, we propose in Section 3 a new adaptive filter. Experimental results and comparisons between the proposed filter and several well-known nonlinear adaptive filters are presented in Section 4. Finally, the conclusions are set out in Section 5.

2. Median Filters

In this section we examine the use of median filters for impulse noise removal. Impulse noise is classified into two types [11], [3]:

1. Fixed-valued impulse noise, also known as salt and pepper noise, pollutes pixels with random values that can be either 0 or 255.
2. Uniform impulse noise, pollutes the pixels following a uniform random distribution over the full range [0, 255]. We have considered this filter model for the experiments.

Under impulse noise, the pixels of the *k-th* channel of a color image (*k=1,2,3*) are distorted according to the following equation:

$$y_k(x_1, x_2) = \begin{cases} \hat{y}_k(x_1, x_2) & \text{with probability } 1-\rho \\ n_k(x_1, x_2) & \text{with probability } \rho \end{cases} \quad \textbf{(1)}$$

where $\hat{y}_k(x_1, x_2)$ and $y_k(x_1, x_2)$ are the pixel values of channel k at position (x_1, x_2) of the original image and noisy image, respectively; $n_k(x_1, x_2)$ is an impulse noise value which is independently chosen in the three channels; and ρ denotes the noise ratio. In order to simplify the discussion, for the rest of this section we will assume that $n_k(x_1, x_2)$ comes from a uniform distribution (uniform impulse noise), as this is the impulse noise type which is the most difficult to remove.

Perhaps the most basic procedure for noise removal is low pass filtering. This amounts to replacing the old value by a weighted mean of the pixel color values $\mathbf{y}_j$ in a small window W around the pixel of interest. If we assume equal weights for the sake of simplicity we arrive at the sample mean:

$$smn(W) = \frac{1}{|W|}\sum_{\mathbf{y}_j \in W} \mathbf{y}_j = \arg\min_{\mathbf{z}} \sum_{\mathbf{y}_j \in W} \left\| \mathbf{z} - \mathbf{y}_j \right\|^2 \quad \textbf{(2)}$$

where $|W|$ stands for the number of pixels of the windows W. However, the mean is known to be very sensitive to outliers [12]. As seen in (2), this is because the mean minimizes the sum of squared distances. Consequently, this approximation can be dominated by outliers. In this way, when the noise is made of impulses, the corrupt data dominate the computation of the weighted average, leading to poor results.

A classical approach to solve this problem is to use the component-wise median:

$$cmedian(W) = \left(\arg\min_{z_k} \sum_{\mathbf{y}_j \in W} \left\| z_k - y_{jk} \right\| \right)_{k \in \{1,2,3\}} \quad \textbf{(3)}$$

The decision to use *cmedian* is very common when applying a denoising method designed for grayscale data to color images, since it is equivalent to filtering the three color channels separately by means of the univariate median. Despite being a robust statistic, *cmedian* does not make use of the tridimensional structure of the color space, which leads to chromatic shifting problems [3]. Moreover, it is not invariant to similarity transformations.

Robust statistics have been developed, which are fully adapted to the characteristics of multivariate data [13]. In particular, the multivariate median (also called L_1-median) has an efficient learning algorithm [14] and has been proven to experience little degradation when outliers are present [15], [16]. It has a key advantage over *cmedian*, namely its invariance with respect to all similarity transformations. The rationale behind the *Lmedian* statistic is to drop the square in the minimization (2) to arrive at the L_1-median of the set (we use equal weights like before to simplify matters):

$$Lmedian(W) = \arg\min_{\mathbf{z}} \sum_{\mathbf{y}_j \in W} \left\| \mathbf{z} - \mathbf{y}_j \right\| \quad \textbf{(4)}$$

so that the outliers have a much lesser impact on the minimization.

A typical simplification of *Lmedian* for color image denoising is to restrict the possible outcomes to be among the input data [10], [3], [4]. In this way we arrive to the *restricted* L_1-median:

$$rmedian(W) = \arg\min_{\mathbf{y}_i \in W} \sum_{\mathbf{y}_j \in W} \left\| \mathbf{y}_i - \mathbf{y}_j \right\| \quad \textbf{(5)}$$

The four discussed strategies (*smn, cmedian, Lmedian* and *rmedian*) are estimators of the true value $\hat{\mathbf{y}}$ of the central pixel, where the input samples for the estimation come from the window W. Perhaps the most standard way to compare the performance of two estimators $\tilde{\mathbf{y}}$ and $\tilde{\mathbf{y}}'$ is to compute their *relative efficiency* (*REF*); see [17], [18]. For a tridimensional estimated parameter $\hat{\mathbf{y}}$, which is our case, it reads as follows [19]:

$$REF\left(\tilde{\mathbf{y}}, \tilde{\mathbf{y}}'\right) = \left(\frac{\det\left(\operatorname{cov}\left(\tilde{\mathbf{y}}'\right)\right)}{\det\left(\operatorname{cov}\left(\tilde{\mathbf{y}}\right)\right)} \right)^{\frac{1}{3}} \tag{6}$$

where $\operatorname{cov}\left(\tilde{\mathbf{y}}\right)$ and $\operatorname{cov}\left(\tilde{\mathbf{y}}'\right)$ are the covariance matrices of the estimators $\tilde{\mathbf{y}}$ and $\tilde{\mathbf{y}}'$ respectively. The estimator $\tilde{\mathbf{y}}$ is judged to be better than $\tilde{\mathbf{y}}'$ if and only if $REF\left(\tilde{\mathbf{y}}, \tilde{\mathbf{y}}'\right) > 1$, while $\tilde{\mathbf{y}}'$ is better than if and only if $REF\left(\tilde{\mathbf{y}}, \tilde{\mathbf{y}}'\right) < 1$. Since we have four estimators to compare, and not only two, it is more convenient to compute the following *estimator efficiency*:

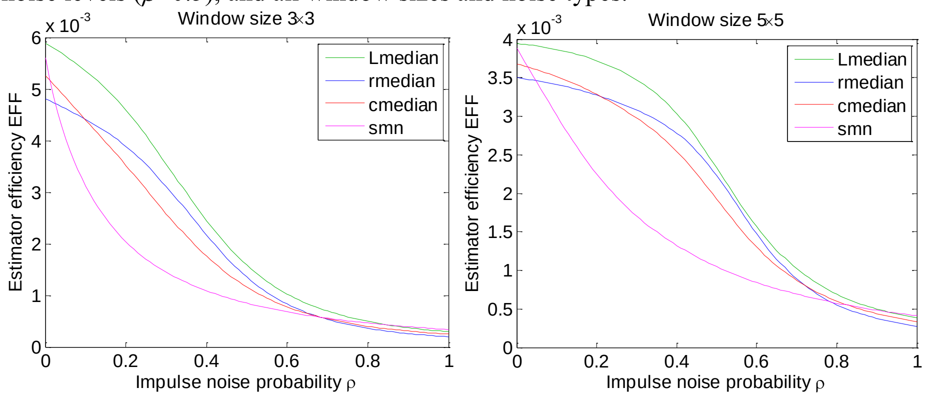
$$EFF\left(\tilde{\mathbf{y}}\right) = \left(\frac{1}{\det\left(\operatorname{cov}\left(\tilde{\mathbf{y}}\right)\right)} \right)^{\frac{1}{3}} \tag{7}$$

Higher values of $EFF\left(\tilde{\mathbf{y}}\right)$ mean that the estimator $\tilde{\mathbf{y}}$ is better, since from (6) and (7) we have:

$$REF\left(\tilde{\mathbf{y}}, \tilde{\mathbf{y}}'\right) > 1 \Leftrightarrow EFF\left(\tilde{\mathbf{y}}\right) > EFF\left(\tilde{\mathbf{y}}'\right) \tag{8}$$

We have tested the four considered approaches without any impulse detection method, in order to compare their intrinsic performance against uniform noise (Fig. 1); the well known Baboon image has been used for this purpose [20]. The four most used window sizes have been considered, i.e. 3 ×3, 5×5, 7×7 and 9×9. The impulse noise probability ρ has been varied from 0 to 1 in 0.01 increments.

As seen, the best performing filter is *Lmedian* for the most commonly considered noise levels (ρ<0.5), and all window sizes and noise types.



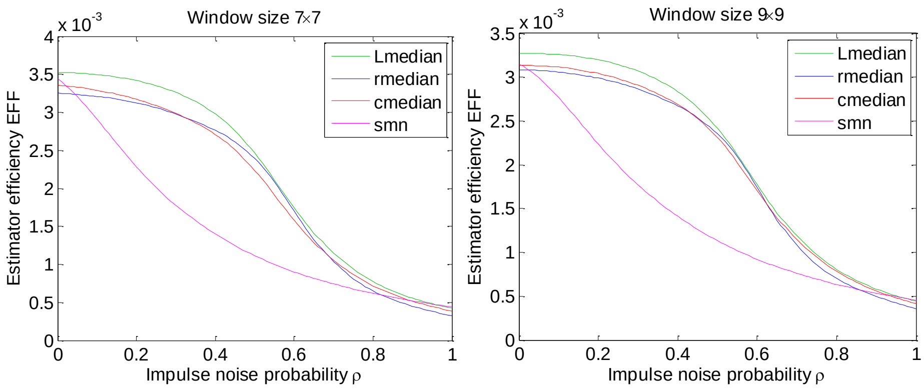


Fig. 1. Efficiency of the estimators under uniform impulsive noise for the Baboon image.

The best window size is the smallest (3×3) for low noise (ρ<0.2). This is because the increased smoothing of the larger sizes does not offer any advantage at so low noise levels. However, when the noise is higher, larger sizes are better. The 5×5 size is the best performing for moderate noise levels, so it is a reasonable tradeoff, as seen before. From the preceding, it can be concluded that we can get some advantages in impulse noise removal by considering the *Lmedian* strategy, in particular with a 5×5 window size. Next we design a nonlinear image filtering scheme which is based on this observation.

3. Adaptive Filtering

Before we can apply the unrestricted multivariate median *Lmedian* to image denoising, we need a procedure to detect impulse corrupted pixels reliably. Let $\mathbf{x} = (x_1, x_2)$ be the position of a pixel in an image of size $NumRows \times NumCols$, and let be $\mathbf{y} : \{1, \ldots, NumRows\} \times \{1, \ldots, NumCols\} \to \mathbf{R}^3$, where

$$\mathbf{y}(\mathbf{x}) = (y_1(\mathbf{x}), y_2(\mathbf{x}), y_3(\mathbf{x})) \tag{9}$$

is the function which gives the three-dimensional color value at position $\mathbf{x}$. If the pixel $\mathbf{x}$ were not an impulse, it would imply that $\mathbf{y}$ is differentiable at $\mathbf{x}$. Hence we could express each of its components as a Taylor series,

$$y_k(x_1 + \varepsilon, x_2 + \delta) = y_k(x_1, x_2) + \varepsilon \frac{\partial y_k}{\partial x_1}(x_1, x_2) + \delta \frac{\partial y_k}{\partial x_2}(x_1, x_2) + \ldots \tag{10}$$

Then we get from (10) that the directional derivative in the direction (ε, δ) is zero to first order approximation:

$$\frac{y_k(x_1+\varepsilon, x_2+\delta) - y_k(x_1, x_2)}{\varepsilon^2+\delta^2} \approx 0 \qquad \textbf{(11)}$$

That is, there should be a small constant $\lambda > 0$ such that

$$\frac{\left|y_k(x_1+\varepsilon, x_2+\delta) - y_k(x_1, x_2)\right|}{\varepsilon^2+\delta^2} < \lambda \qquad \textbf{(12)}$$

Please note that the vector (ε, δ) points in the direction of the level curve that crosses the point (x_1, x_2). One could check whether (x_1, x_2) is not an impulse by looking for a vector (ε, δ) which fulfils (12). We restrict our search to those which correspond with easily realizable gradient estimators (edge detection filters):

$$(\varepsilon, \delta) \in \{(0,1), (1,0), (1,1), (-1,1)\} \qquad \textbf{(13)}$$

The corresponding masks for the 5×5 window size are as follows:

$$\mathbf{M}(0,1) = \begin{pmatrix} 0 & 0 & -1 & 0 & 0 \\ 0 & 0 & -1 & 0 & 0 \\ 0 & 0 & 4 & 0 & 0 \\ 0 & 0 & -1 & 0 & 0 \\ 0 & 0 & -1 & 0 & 0 \end{pmatrix} \quad \mathbf{M}(1,1) = \begin{pmatrix} 0 & 0 & 0 & 0 & -1 \\ 0 & 0 & 0 & -1 & 0 \\ 0 & 0 & 4 & 0 & 0 \\ 0 & -1 & 0 & 0 & 0 \\ -1 & 0 & 0 & 0 & 0 \end{pmatrix} \qquad \textbf{(14)}$$

$$\mathbf{M}(1,0) = (\mathbf{M}(0,1))^T \quad \mathbf{M}(-1,1) = (\mathbf{M}(1,1))^T \qquad \textbf{(15)}$$

Please note that equation (12) should hold for all three color components if the pixel were not an impulse. Consequently, we declare that pixel (x_1, x_2) is an impulse if and only if there exists a color component such that no vector (ε, δ) which satisfies (13) can be found to verify condition (12).

If the pixel at position $\mathbf{x}$ is not an impulse, it is not changed in the restored output $\tilde{\mathbf{y}}(\mathbf{x})$. Otherwise, we substitute it by the unrestricted multivariate median of its 5×5 window $W_{\mathbf{x}}$, as justified in Section 2:

$$\tilde{\mathbf{y}}(\mathbf{x}) = Lmedian(W_{\mathbf{x}}) \qquad \textbf{(16)}$$

Now that we have defined our proposal, we are ready to assess its performance, which is done in the following section.

4. Experimental Results

In this section we compare the performance of the proposal we have just presented with that of several well known impulse noise removal filters. We have considered various benchmark images of 512 × 512 pixels, 24-bit RGB. These images were obtained from the Southern California University images database [20]. We have obtained quantitative and qualitative results similar to the test images. We only present the experimental results obtained with Baboon (Fig. 2). We considered a filter window 5×5 to experiment with our filter, as previously explained. We chose a threshold value $\lambda = 5$, which proved to yield robust results across the tested benchmark images. The set of alternative multi-channel filters which has been considered for the comparative evaluations is shown in Table 1.

Fig. 2. Original Image (Baboon).

Table 1. Filters considered for comparison with the proposed filter. Please note that the first four filters operate using a filter window 3 × 3 and the rest of the filters operate using a filter window 5 × 5.

Notation	Filter	Parameter	Ref.
VMF	Vector Median Filter		[4]
BVDF	Basic Vector Directional Filter		[5]
DDF	Directional Distance Filter	$p = 0.25$	[6]
HVF	Hybrid Vector Filter	$Tol \geq 10$	[11]
SVMF	Switching Vector Median Filter	$Tol \geq 3$	[10]
RASVMF	Rank Adaptive Sigma Vector Median Filter	$\lambda = 5$	[9]

4.1. Quantitative results

Here we compare the methods in terms of quantitative noise reduction, faithful color reproduction, detail preservation and stability. To this end, we have selected three performance measures for the filter evaluations: Peak signal-to-noise ratio (PSNR), higher is better; mean absolute error (MAE), lower is better; and normalized color difference (NCD), lower is better. PSNR reflects noise suppression level [21]. MAE reflects the capability to preserve image details. NCD reflects the capability to preserve the image chromaticity [22]. The stability of the methods has been assessed by computing the mean value and standard deviation of these three measures over 10 simulation runs with different pseudorandom seeds for random noise generation.

In order to evaluate the stability of the filters' performance, we added impulsive noise to the test images; we processed ten times the test images with each filter, for every impulsive noise ratio considered in this paper.

Table 2 present the results obtained with the mean and standard deviation for each performance measure (PSNR, MAE and NCD), with different impulsive noise ratios.

Our method shows the best performance in chromaticity preservation (NCD) in all cases, while it also preserves details satisfactorily (MAE). This validates the theoretical results of Section 2, where we aimed to preserve the information which is associated to the three dimensional structure of the color data. On the other hand, it also attains good PSNR results, although RASVMF outperforms it. As we will see in the next subsection, RASVMF yields rather poor qualitative results in spite of having a high PSNR. This is because of its problems with color faithfulness, which we can be seen on table 2: RASVMF is the worst method with respect to NCD in several situations. This table shows that the performance of the proposed filter is more stable than that of most other filters.

Table 2. Quantitative results (standard deviations in parentheses) on Baboon image corrupted by uniform impulsive noise.

	10%			20%		
Filter	PSNR	MAE	NCD	PSNR	MAE	NCD
VMF	30.(0.01)	7.2 (0.02)	0.10 (0.00)	29 (0.01)	8.5 (0.01)	0.11 (0.00)
BVDF	29.(0.01)	9.9 (0.03)	0.12 (0.00)	29 (0.01)	11.1 (0.05)	0.13 (0.00)
DDF	31.(0.01)	6.1 (0.01)	0.09 (0.00)	30 (0.01)	6.8 (0.01)	0.09 (0.00)
SVMF	33.(0.02)	4.6 (0.02)	0.07 (0.00)	30 (0.01)	7.4 (0.02)	0.11 (0.00)
HVF	31.(0.01)	5.6 (0.02)	0.08 (0.00)	30 (0.01)	7.2 (0.02)	0.10 (0.00)
RASVMF	**38.(0.02)**	**2.0 (0.01)**	0.07 (0.00)	**35 (0.02)**	**4.7 (0.19)**	0.10 (0.02)
Proposed	34.(0.02)	3.1 (0.01)	**0.05 (0.00)**	32 (0.01)	4.9 (0.01)	**0.07 (0.00)**

4.2. Qualitative results

The proposed filter is qualitatively more adequate than the other filters, in the three above mentioned criteria. For example, in Fig. 3, it can be observed that the thin white hairs of the Baboon face, are very well preserved, and noise, successfully reduced.

It must be highlighted that bad color reproduction leads to visually deficient denoised images, even if the pixel values are numerically accurate. This can be seen in RASVMF results, where a very good PSNR does not lead to perceptually pleasant output images. The reason for this is that a relatively small number of very badly colored pixels can spoil the restored image as seen by a human.

5. Conclusions

The usage of multivariate medians for removal of impulse noise in color images has been examined. The different medians suitable for this purpose have been defined and compared. The insights provided by this comparison have leaded us to propose a new method to solve the impulse noise reduction problem in color images. Its comparative performance has been assessed with respect to several alternative proposals, both in quantitative and qualitative terms. The results of these experiments show that our method is able to reduce the impulse noise significantly and reliably, while at the same time it preserves the details and edges of the original image.

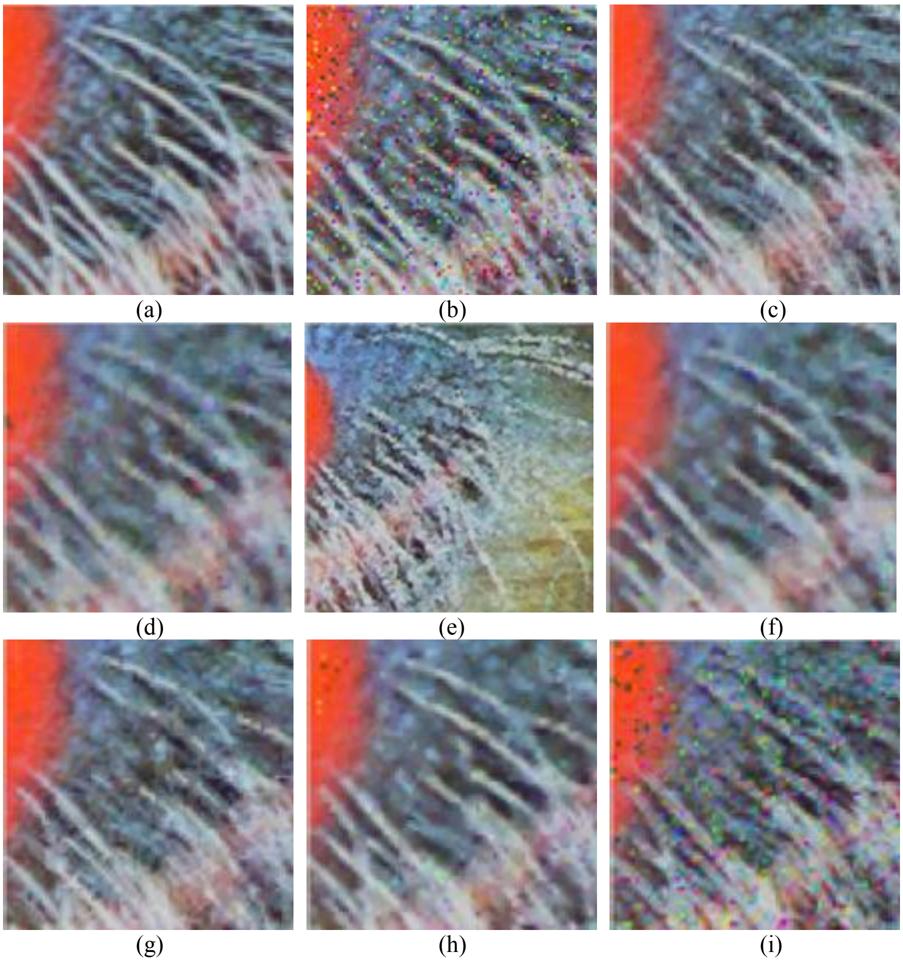


Fig. 3. Detail of the Baboon image (uniform noise). (a) Original image, (b) Image corrupted by 10% uniform impulsive noise, (c) Proposed output, (d) VMF output, (e) BVDF output, (f) DDF output, (g) SVMF output, (h) HVF output, (i) RASVMF output.

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