Charge conservation and time-varying speed of light

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Abstract

It has been recently claimed that cosmologies with time dependent speed of light might solve some of the problems of the standard cosmological scenario, as well as inflationary scenarios. In this letter we show that most of these models, when analyzed in a consistent way, lead to large violations of charge conservation. Thus, they are severly constrained by experiment, including those where c is a power of the scale factor and those whose source term is the trace of the energy-momentum tensor. In addition, early Universe scenarios with a sudden change of c related to baryogenesis are discarded.

Since one of the key hypothesis of special relativity is the frame independence of the velocity of light c, it is implied in this statement the time and space independence of this velocity. As well established that it may seem, this constancy principle has been recently contested [1, 2] to provide an alternative account of the horizon, flatness and cosmological constant problems present in the standard big bang scenario. Instead of the superluminal expansion of the Universe present in inflationary scenarios, a period in which light traveled much faster than today would explain the homogeneity we see today in the Universe. Some cosmological models have also been analyzed afterwards [3, 4] to test the dynamical viability of this scenario.

These ideas are highly provocative, not only from the observational viewpoint but also from the conceptual one. Indeed one of the key aspects of Einstein equivalence principle is the time-independence of the so called "fundamental constants" of physics [5]. The replacement of these parameters by one or more dynamical fields can lead to time- as well as space-dependent local fundamental constants. Unification schemes such as superstring theories [6] and Kaluza-Klein theories [7] have cosmological solutions in which the low-energy fundamental constants are functions of time. Usually low-energy phenomena are

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used to analize the variation rate of the fundamental constants [8]-[26]. If the cosmological dynamics of a field is such that its large-scale value is invariant under local Lorentz transformations, or if the local coupling with matter is strong enough so that it depends on the local environment (*e.g.* electromagnetism with the absorber condition), then the local field equations will be Lorentz invariant. If on the other hand the cosmological evolution is non-trivial and the field couples softly with the local matter, it will act as an external bath, breaking local Lorentz invariance. A variable speed of light theory may belong to the latter set of theories. Any VSL theory poses an additional problem, namely that c is a dimensional constant, and talking about a varying dimensional constant is not an invariant statement: we can change our units and obtain a different time dependence of such a parameter. Of course, once we fix our units, every claim about a dimensional parameter is an invariant claim, since we are implicitly referring to a dimensionless ratio: that between the parameter and the unit [17, 18].

Any scientific theory has to be stated in clear and precise terms. Beckenstein's theory of a variable fine structure constant was based on Lorentz invariance, explicitly protecting charge conservation. In the case of VSL theories, local Lorentz invariance is relaxed, and the inhomogeneous Maxwell equations are assumed to be [2]

$$\frac{1}{c}\partial_\mu\left(cF^{\mu\nu}\right) = 4\pi j^\nu, \tag{1}$$

where $j^\mu = (\rho, \mathbf{j}/c)$ is the electric charge current. In reference [2] it was suggested that charge is conserved, implying a variation of the fine structure constant $\alpha = \frac{e^2}{\hbar c}$. The constancy of e can be derived, for instance, from Dirac's equation, written in Hamiltonian form:

$$i\hbar\partial_t\psi = -(i\hbar\boldsymbol{\alpha}\cdot\boldsymbol{\nabla} + \text{h.c.})\psi + mc^2\psi$$

which implies that $Q = e\int_V \psi^*\psi d^3x$ is conserved. The above form is, however, not unique since powers of c can be introduced in the equation in several ways, followed by appropriate symmetrization.

However, from equation (1) it can be easily seen that j^μ is no longer a conserved current, but satisfies the equation:

$$4\pi\partial_\mu(cj^\mu) = \partial_\mu\partial_\nu(cF^{\mu\nu}) \tag{2}$$

The right hand side is not null, because the partial derivatives do not commute:

$$[\partial_0, \partial_i] = \frac{c_{,i}}{c^2}\partial_t$$

It is usually assumed that c is a function of a scalar field and that it depends only on time in the comoving cosmological frame. In the local frame of the solar system, moving with a velocity v with respect to the cosmological frame, a small space dependence will arise, with gradients $O(v/c)$ with respect to the time derivative, which can be neglected for the present purposes. So, the right

hand side of equation 2 is effectively zero. The left hand side, however, is not a four divergence, because $\partial_{x^0} = 1/c(t)\partial_t$. The fully expanded expression is:

$$\partial_t \rho + \frac{\dot{c}}{c}\rho + \nabla \cdot \mathbf{j} = 0 \tag{3}$$

If equation (3) is integrated over a volume V containing the charges, we obtain

$$\frac{\dot{Q}}{Q} = -\frac{\dot{c}}{c} \tag{4}$$

where $Q = \int_V \rho d^3 x$ is the total electric charge. This is our main result.

Equation (4) provides very stringent tests of the variation of c, since there have been many experiments to test the conservation of charge [28]. Depending on the details of the theory, several cases arise.

If we assume, as it is usually done in this context [2, 3] that the electron charge e is constant, charge conservation can only be broken by processes that change charge discontinuously, such as the dissapearance of electrons or the transformation of neutrons into protons. A generic model for these processes has been proposed in references [19, 20], under the assumption of total energy conservation. The first three entries of Table 1 show some sample upper limits obtained from these processes with different techniques and hypothesis. These upper limits on $\dot{c}/c$ are much smaller than those obtained by a direct measurement, for instance in reference [21], namely $| \dot{c}/c |< 10^{-13}$ yr^{-1}.

On the other hand, if e varies continuously in such a way that ce is conserved, then $\alpha \propto c^{-3}$ and strict limits can be obtained from geophysical or astronomical data, such as the Oklo phenomenon [15, 16] or the line spectra of distant quasars [13, 14]. Furthermore, evidence for the time variation of the fine structure constant has been claimed by Webb et al. [25] and confirmed while we were correcting this paper [26]. Thus, if the systematic errors are well estimated, the requirement of conservation of charge suggests that, the mechanism for α variation should not be driven by the change in the speed of light.

These limits discard a great number of cosmological models with varying velocity of light. For instance, the family introduced in [3] parameterizes light velocity in the form: $c = c_0 \left(\frac{a}{a_0}\right)^n$ with a the cosmological scale factor. Even though, [3] was not proposing that this behaviour exists at all times, [22] suggests that this solutions could be extended to radiation and matter-dominated universes. It was shown in reference [3] that $n < -1/2$ is necessary to solve the flatness and horizon problems, and $n < -3/2$ solves the cosmological constant problem.

On the other hand, any complete and consistent VSL theory will predict dynamically the value of c via a wave-like equation for $\psi = \ln \frac{c}{c_0}$, whose source term is proportional to the trace of the energy-momentum tensor T. Thus, in the neighborhood of a quasi-static system, such as a star or a virialized galaxy cluster, a generic expresion for a varying c will be [23]:

$$c = c_c(t) \left(1 - \frac{\lambda G M}{c_0^2 r}\right) \tag{5}$$

3

Here λ is a constant that depends on the specific VSL theory. Furthermore, in the limit $r \to \infty$, the expression for c reduces to the cosmological one $(c \to c_c(t))$.

For the purposes of this paper, our interest is focused on violation of charge conservation. Expanding equation(2) we obtain:

$$4\pi \partial_\mu j^\mu = -4\pi \frac{\partial_\mu c}{c} j^\mu - \partial_t c \frac{\vec{\nabla} c . \vec{E}}{c^3} + \frac{\vec{\nabla} . c}{c} \partial_0 \vec{E} \tag{6}$$

In a static situation $\vec{j} = 0$ and $\partial_0 E = 0$, thus the effects of $\vec{\nabla} c$ are of second order and can be neglected. Hence, there are two contributions to the violation of charge conservation, one accounts for time-variations over cosmological time-scales and the other for the motion of the Earth with respect to massive bodies such as clusters or galaxies:

$$\frac{\dot{c}}{c} = \left(\frac{\dot{c}}{c}\right)_{cosmological} + \left(\frac{\dot{c}}{c}\right)_{local} = n\frac{\dot{a}}{a} + \frac{\lambda GM}{c_0^2 r^3}\vec{r}.\vec{v} \tag{7}$$

where $\vec{v}$ is the velocity of the Earth with respect to the lump. ($\vec{v} \sim 1000\frac{km}{seg}$ for the Virgo supercluster).

From the first term of equation (7) we find the limits of table 1 on n, which contradict the above requirements. (We use $H_0 = 65$ km/s/Mpc $= 6.65 \times 10^{-11}$ yr^{-1}). Similar bounds can be obtained for other similar models, such as those studied in reference [22] (See also [32]).

For the Virgo supercluster, $\frac{GM}{c_0^2 r} \sim 2 \times 10^{-7}$. The last three results in table 1 for λ are obtained from the second term of equation (7) . These results rule out essentially any VSL whose source of variation is the trace of the energy-momentum tensor T.

Finally, models similar to the original Albrecht-Magueijo one [2], involving a sudden change of c between two different constant values in the very early Universe, are not affected by the above limits. Orito and Yoshimura [33] observed that if charge conservation is broken in the very early Universe, a large charge excess should have been formed through a mechanism similar to that of baryogenesis: violation of Q, C and CP conservation while the system is out of thermodynamic equilibrium [34, 35]. In the above mentioned models, the net charge excess will be produced by way-out-of-equilibrium production and decay of heavy mesons [35].

Let X be an unstable meson that produces a mean baryon number ϵ_B and a mean charge excess ϵ_Q per decay, and assume that matter is created during the charge transition period. Then, the equations for the evolution of the number densities of X, B, Q will be [2, 3, 35]:

$$
\begin{aligned}
(a^3 n_X)^\bullet + \lambda_X(a^3 n_X) &= \frac{3Kc\dot{c}}{4\pi Gm_X}a \\
(a^3 \dot{n}_B)^\bullet &= \epsilon_B \lambda_X n_X a^3 \\
(a^3 \dot{n}_Q)^\bullet &= \epsilon_Q \lambda_X n_X a^3
\end{aligned}
$$

In these equations, K is the curvature parameter, m_X is the mass of the X meson, and all densities are effective densities (particle - antiparticle). Let us assume that the change in c occurs in a short interval of time $t_c << \tau << \frac{1}{\lambda_x}$. Charge conservation will be broken only during this interval, but the X meson decay will always produce a baryon excess. With these hypothesis, the above equations have the following solutions:

$$a^3 n_X \quad \simeq \frac{3K(c_0^2 - c_P^2)}{8\pi G m_X} a(0) e^{-\lambda t} \quad = a^3(0) n_X^0 e^{-\lambda t}$$

$$a^3 n_B \quad \simeq \epsilon_B a^3(0) n_X^0 \left(1 - e^{-\lambda t}\right) \quad \rightarrow \epsilon_B a^3(0) n_X^0$$

$$a^3 n_Q \quad \simeq \epsilon_Q \lambda_X \tau a^3(0) n_X^0 \quad \simeq \frac{\epsilon_Q}{\epsilon_B} \lambda_X \tau a^3 n_B$$

After the transition, n_Q will be fixed but n_B will be diluted from the above estimate by thermal processes [35]. So we finally get a lower bound on the charge excess:

$$\left| \frac{n_Q}{n_B} \right| \sim \frac{\epsilon_Q}{\epsilon_B} \lambda_X \tau \tag{8}$$

As we have mentioned before, we expect on general grounds that $\tau > t_{Pl}$, while $\epsilon_Q \sim \epsilon_B$, since these fractions depend both on the C and CP breaking terms in the lagrangean. Thus, equation (8) predicts a firm lower limit for the charge excess. Orito and Yoshimura [33] have given limits on any charge excess in the Universe, shown in table 1. These limits are many orders of magnitude below the prediction of equation (8). The last column of the table shows rough estimates of τ taken from the observational limits, assuming $1/\lambda_X \sim t_{GUT}$.

Although these results do not rule out all varying velocity of light theories, they put very stringent bounds on them through the conservation of charge requirement. Moreover, similar bounds will hold for any theory with varying speed of light velocity in the early universe. These bounds, which may be lowered through improvements in the experimental techniques [38], will lead into deeper understanding of these interesting theories.

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Process	Ref.	Datum	$\mid \dot{Q}/Q \mid$ (y^{-1})	Param.
		τ (y)		$\mid n \mid$
$^{71}\mathrm{Ga} \to {}^{71}\mathrm{Ge}$	[29]	$\geq 3.5 \times 10^{26}$	$\leq 2.9 \times 10^{-27}$	$< 5 \times 10^{-17}$
$e \to \nu_e \gamma$	[30]	$\geq 2.4 \times 10^{25}$	$\leq 4.2 \times 10^{-26}$	$< 7 \times 10^{-16}$
$e \to$ any	[31]	$\geq 2.7 \times 10^{23}$	$\leq 3.7 \times 10^{-24}$	$< 6 \times 10^{-14}$
		$\mid \Delta\alpha/\alpha \mid$		
Oklo phenomenon	[16]	$\leq 1.2 \times 10^{-7}$	$\leq 2.0 \times 10^{-16}$	$< 3 \times 10^{-6}$
Quasar absorption systems	[13]	$\leq 3.5 \times 10^{-4}$	$\leq 1.18 \times 10^{-13}$	$< 2 \times 10^{-3}$
Quasar absorption systems	[14]	$\leq 6 \times 10^{-5}$	$\leq 5 \times 10^{-15}$	$< 7 \times 10^{-5}$
		n_Q/n_B		τ/t_{Pl}
Coulomb force smaller than				
Newton force in stars	[33]	$< 10^{-18}$	—	$< 10^{-7}$
CMB anisotropy	[33]	$< 2 \times 10^{-20}$	—	$< 2 \times 10^{-9}$
Cosmic ray isotropy	[33]	$< 10^{-29}$	—	$< 10^{-18}$
		τ (y)		$\mid \lambda \mid$
$^{71}\mathrm{Ga} \to {}^{71}\mathrm{Ge}$	[29]	$\geq 3.5 \times 10^{26}$	$\leq 2.9 \times 10^{-27}$	$< 2 \times 10^{-10}$
$e \to \nu_e \gamma$	[30]	$\geq 2.4 \times 10^{25}$	$\leq 4.2 \times 10^{-26}$	$< 2 \times 10^{-9}$
$e \to$ any	[31]	$\geq 2.7 \times 10^{23}$	$\leq 3.7 \times 10^{-24}$	$< 2 \times 10^{-7}$

Table 1: Upper limits on charge non-conservation. The columns show the process considered, the corresponding references, the observational data, the charge non-conservation upper bound and the limits for the model parameters.

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