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The impact of element diffusion on the formation and evolution of helium white dwarf stars

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ABSTRACT

The aim of this work is to investigate the effect of element diffusion on the evolution of helium white dwarfs. To this end, we couple the multicomponent flow equations that describe gravitational settling, chemical and thermal diffusion to an evolutionary code. We compute the evolution of a set of helium white dwarf models with masses ranging from 0.169 to 0.406 $M_{\odot}$. In particular, several low-mass white dwarfs have been found in binary systems as companion to millisecond pulsars. In these systems, pulsar emission is activated by mass transfer episodes so that, if we place the zero-age point at the end of such mass transfer, then the pulsar and the white dwarf ages should be equal. Interestingly enough, available models of helium white dwarfs neglect element diffusion. Using such models, good agreement has been found between the ages of the components of the PSR J1012+5307 system. However, recent observations of the PSR B1855+09 system cast doubts on the correctness of such models, which predict a white dwarf age twice as long as the spin-down age of the pulsar. In this work, we find that element diffusion induces thermonuclear hydrogen shell flashes for models in the mass interval $0.18 \lesssim M/M_{\odot} \lesssim 0.41$. We show, in particular, that the occurrence of these diffusion-induced flashes eventually leads to white dwarf models with hydrogen envelope masses too small to support any further nuclear burning, thus implying much shorter cooling ages than in the case when diffusion is neglected. In particular, excellent agreement is found between the ages of PSR B1855+09 system components, solving the age discrepancy from first principles.

Key words: diffusion – stars: evolution – stars: interiors – pulsars: general – pulsars: individual: PSR B1855+09 – white dwarfs.

1 INTRODUCTION

Binary systems composed of a millisecond pulsar and a white dwarf (WD) star have captured the attention of many researchers (Phinney & Kulkarni 1994 for a review; Alberts et al. 1996; Hansen & Phinney 1998a and references cited therein; van Kerkwijk et al. 2000). This has been motivated in part by the fact that, from the study of one of the components of the pair, we can infer characteristics of the other one. This is important in connection with pulsar mass determinations and the structure and evolution of WD stars. The majority of the companions to millisecond pulsars detected so far have masses below the threshold value for helium ignition ($\sim$0.5 $M_{\odot}$) and thus are currently believed to be WDs with helium cores. The formation of these objects within a Hubble time can be expected only if mass loss episodes in a close binary system lead to a large removal of the hydrogen envelope of the WD progenitor. In particular, the accretion of mass from this progenitor on to the pulsar spins it up (recycled pulsar) (Bhattacharya & van den Heuvel 1991), so that the pulsar will begin to spin-down at almost the same time as the WD progenitor shrinks within its Roche lobe. Thus, setting the zero-age point shortly after mass transfer episodes, the spin-down age of the pulsar and the cooling age of the WD should be nearly the same. The former can be assessed from the knowledge of the current spin period P of the pulsar and its rate of change $\dot{P}$ as

$$t_{\mathrm{PSR}} = \frac{P}{(n-1)\dot{P}}\left[1 - \left(\frac{P_0}{P}\right)^{n-1}\right], \tag{1}$$

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where P_0 is the initial period. If $P_0 \ll P$ and the radiation is dipolar ($n = 3$) then

$$t_{\rm PSR} = P/2\dot{P}. \tag{2}$$

On the other hand, the WD age can be derived from evolutionary calculations (Althaus & Benvenuto 1997; Benvenuto & Althaus 1998; Hansen & Phinney 1998b; Driebe et al. 1998).

In particular, the amount of hydrogen that remains when the helium WD is formed represents a key quantity in determining its evolutionary time-scale. In this connection, Sarna, Antipova & Ergma (1999) have presented very detailed calculations of the binary evolution that leads to the formation of helium WDs with stellar masses less than $0.25\,M_{\odot}$. They found that hydrogen envelopes are quite massive, ranging from 0.01 to $0.06\,M_{\odot}$. In addition, Driebe et al. (1998) have simulated binary evolution by subjecting a 1-$M_{\odot}$ model to a large mass loss rate during its evolution along the red giant branch, and found massive hydrogen envelopes, as well. They have followed their calculations down to low stellar luminosities and found that thermal flashes occur only for two model sequences, 0.234 and $0.259\,M_{\odot}$. In particular, the resulting hydrogen-rich envelope is massive enough to make stable hydrogen shell burning the dominant energy source for helium WDs even down to effective temperatures well below 10 000 K, thus substantially prolonging their cooling times.

These evolutionary models predict an age for the helium WD companion to PSR J1012+5307 of 6 ± 1 Gyr. This age is in good agreement with the spin-down age of $t_{\rm PSR} \approx 7$ Gyr derived for the pulsar of this system (Lorimer et al. 1995). However, recent optical observations of the PSR B1855+09 system cast doubts on the correctness of such evolutionary calculations. In fact, the spin-down age of this pulsar is about 5 Gyr. For this system, the mass of the WD has been determined by measuring the Shapiro delay of the pulsar signal (Ryba & Taylor 1991; Kaspi, Taylor & Ryba 1994), which leads to a value of $0.258^{+0.028}_{-0.016} M_{\odot}$. Very recently, van Kerkwijk et al. (2000) have detected this WD optically and determined its effective temperature to be $T_{\rm eff} = 4800 \pm 800$ K. Using the models of Driebe et al. (1998), they estimated the age of the WD to be about 10 Gyr, i.e. much older than the pulsar, showing an appreciable discrepancy with the spin-down age. Notably, evolutionary calculations leading to this age discrepancy neglect element diffusion.

Needless to say, a key feature in determining the life of a star is its internal chemical profile. Standard stellar evolution considers nuclear reactions and convection as the only processes that modify such profile. However, element diffusion also changes the distribution of isotopes. In this work we present calculations of the formation and evolution of helium WDs considering gravitational settling, and chemical and thermal diffusion for a wide range of masses (from 0.169 to $0.406\,M_{\odot}$). This mass range includes the masses of the helium WDs discovered as companions to PSR J1012+5307 and B1855+09. We find that, in the case of objects with masses ranging from approximately 0.18 to $0.41\,M_{\odot}$ (the largest mass value analysed in this work), diffusion induces thermonuclear flashes. During these flashes a sizeable amount of hydrogen is burnt, with the consequent result that evolution is strongly accelerated, compared with the case in which diffusion is neglected. In this way, diffusion is called to play a very important role in determining the ages of helium WDs. In particular, we find that the inclusion of diffusion naturally solves the age discrepancy for the PSR B1855+09 system from first principles.

2 NUMERICAL METHODS

For our purpose, we have employed a detailed and updated stellar code. It includes an updated version of the equation of state of Magni & Mazzitelli (1979) for the low-density regime, while for high densities we consider ionic contributions, Coulomb interactions, partially degenerate electrons, and electron exchange and Thomas–Fermi contributions at finite temperature. High-density conductive opacities and the various mechanisms of neutrinos emission are taken from the works of Itoh and collaborators (see Althaus & Benvenuto 1997 for details). Detailed nuclear reaction rates for CNO and PP cycles are taken from Caughlan & Fowler (1988), while screening treatment is from Wallace, Woosley & Weaver (1982). Radiative opacities are from the OPAL group (Iglesias & Rogers 1996), supplemented by the molecular opacities of Alexander & Ferguson (1994). Gravitational settling, and chemical and thermal diffusion have been included following the treatment presented by Burgers (1969) for multicomponent gases, thus avoiding the trace element approximation usually considered in most WD studies. This approximation would clearly not be appropriate for the case we want to study here because after mass loss episodes the model envelope is made up of a mixture of hydrogen and helium. The resulting flow equations for ^{1}H, ^{3}He, ^{4}He, ^{12}C, ^{14}N and ^{16}O have been integrated by using a semi-implicit, finite-difference scheme. Details are given in Althaus & Benvenuto (2000) (see also Iben & MacDonald 1985). The set of equations describing diffusion has been coupled to our evolutionary code in order to calculate the influence of this process in a self-consistent way. We want to mention that the first authors in considering gravitational settling and chemical diffusion during WD evolution were Iben & MacDonald (1985) and Iben & Tutukov (1986) for the cases of carbon–oxygen and a 0.3-$M_{\odot}$ helium WD, respectively. The coupling between abundances changes produced by diffusion and nuclear reactions was taken into account by using the operator splitting technique. It is worth mentioning that opacity was computed self-consistently with diffusion predictions for the heavy element composition; in particular, metallicity was taken as twice the abundances of CNO elements.

In this work we have not modelled in detail the binary evolution leading to the formation of helium WDs (see Sarna et al. 1999 and references therein for details). In fact, in order to obtain reliable initial models, we have simulated close binary evolution by subjecting an evolving 1-$M_{\odot}$ model (we assume solar metallicity) at different locations on the red giant branch to a large mass loss rate. This procedure enables us to obtain initial models in good agreement with expectations from binary evolutionary calculations (Driebe et al. 1998). In this way, we have been able to generate initial helium WD models with stellar masses of 0.406, 0.360, 0.327, 0.292, 0.242, 0.196 and $0.169\,M_{\odot}$. We mention that our resulting envelope masses and surface hydrogen abundance (see Table 1) are in good agreement with those of Driebe et al. (1998), particularly for the more massive models.

3 RESULTS

In this section, we present the main results we have obtained. As mentioned, we have calculated the evolution of helium WD models with masses of 0.406, 0.360, 0.327, 0.292, 0.242, 0.196 and $0.169\,M_{\odot}$, which cover most of the range of masses expected for these objects. Let us note that we are primarily interested in exploring the influence that diffusion has on the evolution of

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Table 1. Stellar mass, envelope mass and hydrogen surface mass fraction X at the point of maximum effective temperature. The quoted values correspond to the sequences in which diffusion is neglected.

$M/M_{\odot}$	$M_{env}/M_{\odot}$	X
0.406	7.3×10^{-4}	0.700
0.360	1.1×10^{-3}	0.700
0.327	1.4×10^{-3}	0.700
0.292	2.0×10^{-3}	0.700
0.242	3.7×10^{-3}	0.694
0.196	6.7×10^{-3}	0.504
0.169	1.0×10^{-2}	0.423

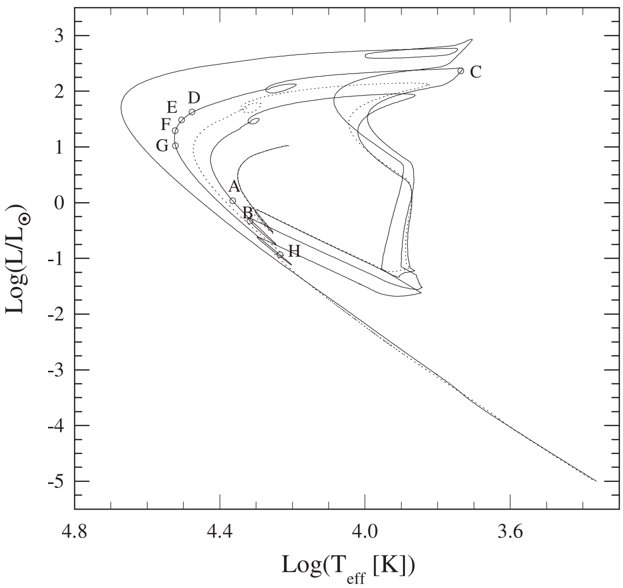

Figure 1. The evolution of a 0.242-$M_{\odot}$ WD model in the HRD. Full curve, results computed under the assumption of diffusion and dotted curve, the case in which diffusion is neglected. Letters denote selected stages of the evolution. Note that the number of flash episodes (each associated to extensive loops in the HRD) depends on whether diffusion is considered or not.

helium WDs. Thus, besides the models in which diffusion is considered, we have computed a set of models in which diffusion is neglected. This has allowed us to undoubtedly identify the effects that diffusion induces on the evolution of helium WDs.

After the end of mass loss episodes, diffusion becomes an important process. Widely speaking, it makes hydrogen float and the other elements sink, giving rise to pure hydrogen envelopes as evolution proceeds. However, not all the hydrogen diffuses outwards. Indeed, chemical diffusion causes some hydrogen to move inwards to hotter layers, ultimately triggering at least one thermonuclear flash. This is true for all of our models, except for the 0.169-$M_{\odot}$ one. In contrast, when diffusion is neglected, we find that only the 0.242-$M_{\odot}$ model experiences a thermonuclear flash. Interestingly enough, Driebe et al. (1998), who have neglected the effect of diffusion, found that only their 0.234- and 0.259-$M_{\odot}$ sequences experience flash episodes, in agreement with our predictions of no diffusion. Another feature worthy of comment is the fact that as the stellar mass is decreased, the number of flash episodes becomes larger. Indeed, we note that our 0.196-$M_{\odot}$ sequence experiences up to five thermonuclear flashes.

In what follows, we shall discuss the results for the 0.242-$M_{\odot}$ model, which is representative of our set of stellar masses that experience flash episodes, and for the 0.169-$M_{\odot}$ models as well, which do not suffer from flashes even in the presence of diffusion. It is worth noting that these mass values are very close to the mass estimation for the WD companion to the millisecond pulsars PSR B1855+09 and PSR J1012+5307, respectively.

We begin by examining Fig. 1 in which we show the evolutionary track in the Hertzsprung–Russell diagram (HRD) corresponding to the 0.242-$M_{\odot}$ model. The outstanding feature shown by this figure is that when diffusion is neglected the model experiences only one thermonuclear flash, but if we allow diffusion to operate, the star undergoes three flashes. As a result of these flash episodes, the model rapidly reaches giant dimensions again. As we mentioned, the presence of additional flashes is a result of the fact that there is a tail in the hydrogen distribution that chemically diffuses inwards, where temperature is high enough to burn it, leading to a thermal runaway. This is illustrated by Fig. 2, in which the evolution of the hydrogen profile at the bottom of the hydrogen envelope is depicted as a function of the outer mass fraction q ($q = 1 - M_r/M_*$). The results shown here correspond to those evolutionary stages before and after the onset of the second

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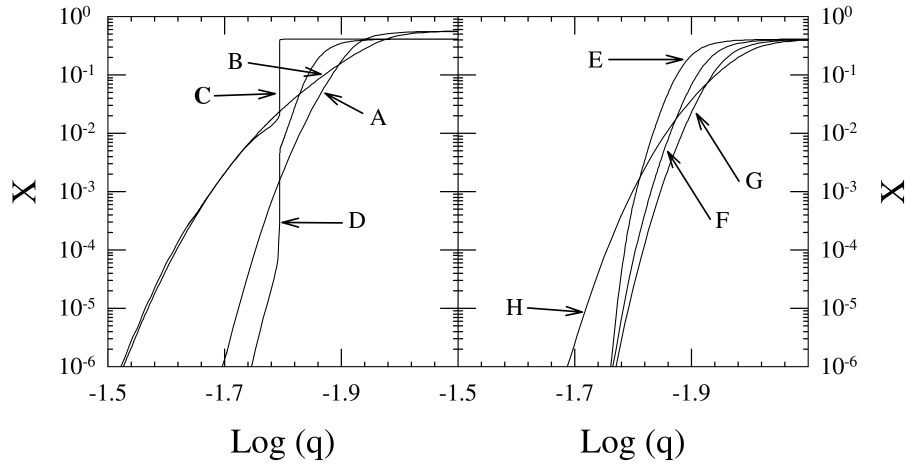

Figure 2. Hydrogen abundance at the bottom of the hydrogen-rich envelope versus the outer mass fraction for the 0.242-$M_{\odot}$ model for the case in which diffusion is considered. Curves correspond to the evolutionary stages indicated by letters in Fig. 1. Note the tail of hydrogen extending into helium-rich layers (from A to B and from G to H) as a result of chemical diffusion. At high effective temperatures, this effect is responsible for the occurrence of additional flash episodes.

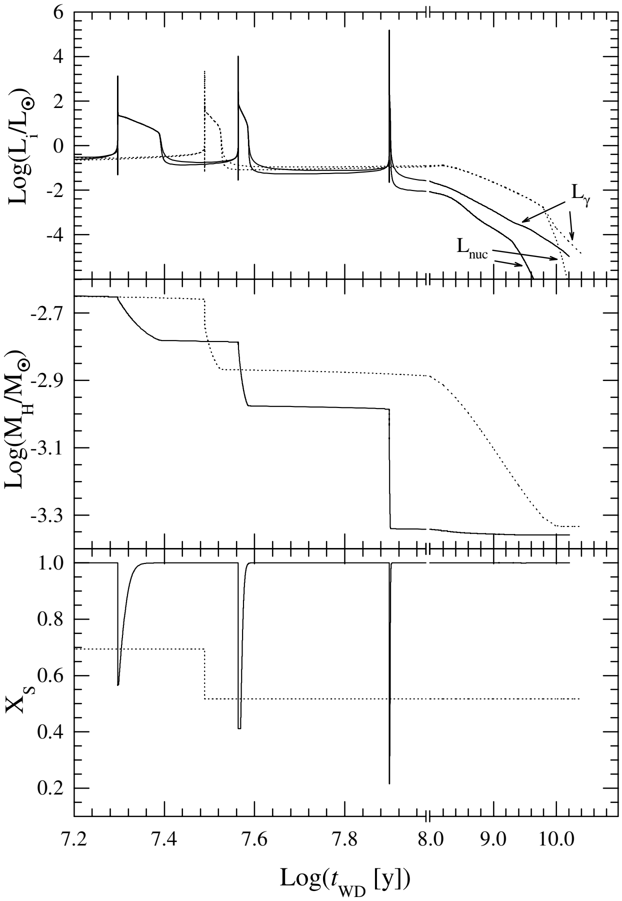

Figure 3. Some relevant characteristics of the 0.242-M$_\odot$ WD model as a function of age for the cases with (full curves) and without (dotted curves) diffusion. Top panel, the evolution of photon and nuclear luminosities (denoted by L_γ and $L_{\rm nuc}$, respectively). The spikes are caused by the occurrence of thermonuclear flashes. Note that at advanced ages, in the case of diffusion, the role played by nuclear burning is markedly less important from the situation when diffusion is neglected. Middle panel, the evolution of the hydrogen envelope mass. Note that after flashes, the mass of the hydrogen envelope is strongly reduced when account is made of element diffusion. Bottom panel, abundance by mass fraction of hydrogen at $1 - M_r/M_* = 10^{-9}$. Shortly after flashes, models develop an outer convection zone, which leads to hydrogen and helium mixing, abruptly reducing the surface abundance of hydrogen. However, diffusion rapidly causes the bulk of hydrogen to float again. This is in sharp contrast with the situation in which diffusion is neglected.

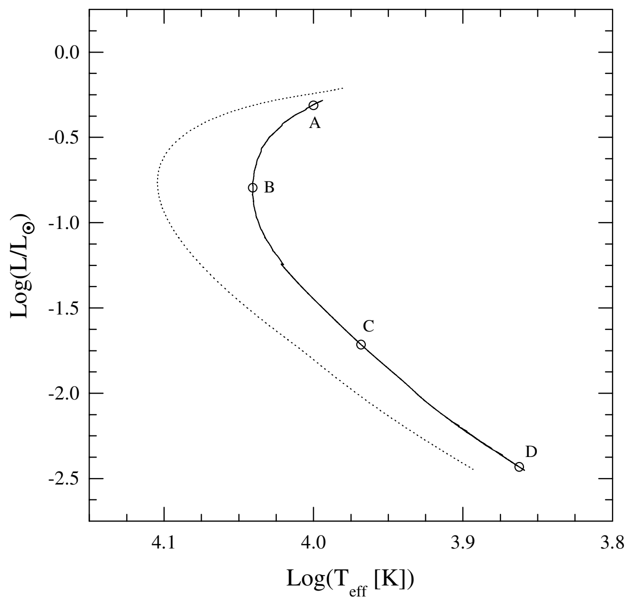

Figure 4. The evolution of the 0.169-M$_\odot$ WD model in the HRD for the case with diffusion (full curve) and without diffusion (dotted curve). Letters denote selected stages of the evolution. In contrast with the 0.242-M$_\odot$ results presented in Fig. 1, this model does not experience any thermonuclear flash. Note that, for a given luminosity value, models with diffusion have lower effective temperature (and thus larger radii) owing to the fact that hydrogen diffuses upwards during the course of evolution.

flash episode. Clearly, it is noticeable the tail of hydrogen diffusing downwards as evolution proceeds from A to B shortly before the onset of the second flash. It is worthwhile remarking that this effect is not the only mechanism for inducing a flash in a helium WD star. In fact, the 0.242-M$_\odot$ model also undergoes a flash even when diffusion is not considered, but after this, there is no mechanism capable of fuelling hydrogen to hot layers. This is a key difference between both sequences of models.

Just after the onset of a flash, the object develops an outer convection zone, which mixes hydrogen and helium, producing the step-like profile corresponding to model C. The profile of model D clearly shows the effects of the subsequent nuclear burning which makes the profile propagate outwards. Afterwards, the hydrogen profile continues propagating outwards until hydrogen burning becomes strongly decreased (Fig. 2, right-hand panel). After such epoch, chemical diffusion dominates again, carrying some fresh hydrogen to deeper layers (curve H). This, in turn, seeds these layers for undergoing the last thermonuclear flash.

In the top panel of Fig. 3 we show the photon and nuclear luminosity versus time. Note the spikes in nuclear luminosity which correspond to the very short evolutionary stages during thermonuclear flashes. The increase of flash intensity is caused by the increasing degeneracy of matter in the layers where hydrogen is burnt. We note that after the model undergoes flash episodes, and before it enters the final cooling stage, the amount of hydrogen left is markedly lower compared with the case in which diffusion is not considered (middle panel), thus preventing any further nuclear burning from being a major source of energy for the model. This is in contrast with the case of models without diffusion for which stable nuclear reactions are the dominant energy source during most of their evolution. Because the star has a much lower amount of available energy, it will be forced to evolve over a much shorter time-scale. In the bottom panel we show the surface hydrogen abundance. Here, the effect of diffusion is noteworthy. After each flash episode convection reaches deep enough to dredge helium up, producing a steep decline in the surface hydrogen abundance. Afterwards, diffusion takes over again, leading to pure hydrogen envelopes. This is again in sharp contrast with the predictions of models without diffusion.

Let us discuss the results for the 0.169-M$_\odot$ model, which is representative of the PSR J1012+5307 companion. In Fig. 4 we show its evolutionary track in the HRD for the cases of diffusion and no diffusion. For this stellar mass value we find that thermonuclear flashes do not occur even in the presence of diffusion. Clearly, diffusion leads to a larger radius at a given $T_{\rm eff}$. This effect has previously been found by Althaus & Benvenuto (2000), and it is expected to affect mass–radius relations for low-mass helium WDs. A detailed discussion of this topic is deferred to

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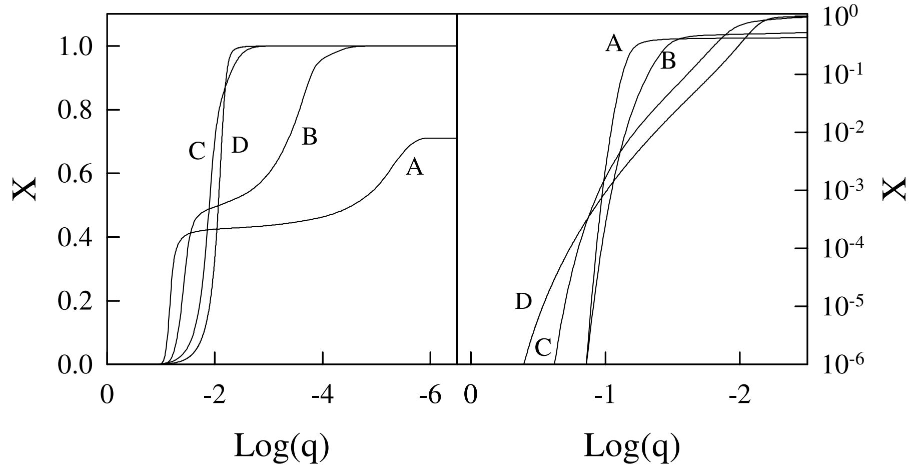

Figure 5. Hydrogen abundance versus the outer mass fraction for the 0.169-$M_{\odot}$ model for the case in which diffusion is considered. Curves correspond to the evolutionary stages indicated by letters in Fig. 4.

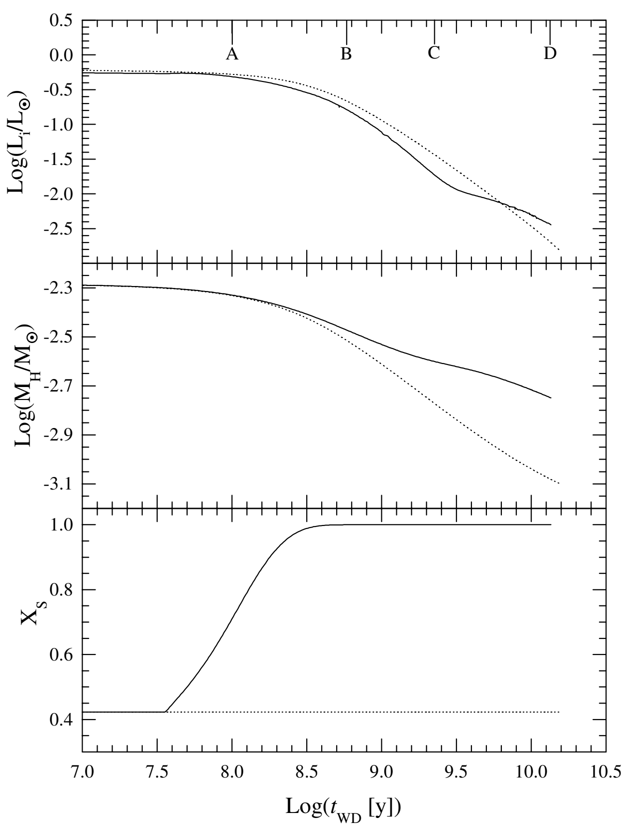

Figure 6. Same as Fig. 3 but for the 0.169-$M_{\odot}$ model for the cases with (full curve) and without (dotted curve) diffusion. In the top panel we do not include $L_{\rm nuc}$ (as done in Fig. 3) because it is almost coincident with photon luminosity throughout the entire evolution. Note that no thermonuclear flash is found even in the presence of diffusion. From the middle panel we note that when account is taken of diffusion, the final amount of hydrogen content is substantially larger than in the case without diffusion. This is in contrast with the situation found in more massive models. Because of the lower surface gravities characterizing these models, diffusion proceeds on a much longer time-scale than in more massive models. However, pure hydrogen envelopes are developed within a time period much shorter than the age of the Universe (bottom panel).

a forthcoming paper; suffice it to mention here that surface gravity is reduced by ~ 80 per cent in comparison to models without diffusion. The role played by diffusion in the chemical evolution of this model can be appreciated in Fig. 5. In spite of the very low gravity values, note that diffusion is able to produce pure hydrogen envelopes within a few hundred million years. As in the case of more massive models, a tail in the hydrogen distribution is diffusing downwards (see the right-hand panel of Fig. 5). Unlike the situation found in more massive models, this will not lead to thermonuclear flashes at least for ages lower than 13 Gyr. The lack of thermonuclear flashes forces the evolution of the star to be dominated by stable nuclear burning throughout the entire evolution of this model, and this fact is independent of whether or not diffusion is considered. This is illustrated in the top panel of Fig. 6, in which the photon luminosity (here coincident with nuclear luminosity) is depicted as a function of age. Note that at advanced stages of evolution ages are very large in both cases. The behaviour of the evolution of the photon luminosity deserves some comment. First, it is clear that when diffusion is considered, evolution proceeds on somewhat shorter time-scales compared with the case of no diffusion (from time marks A to C). This is expected because the model considerably inflates as a result of diffusion. However, note that the trend of the cooling curve is reversed from time mark C on. Indeed, as we mentioned, diffusion carries some hydrogen downwards to hotter layers (see the right-hand panel Fig. 5). In the case of low-mass models, which do not suffer from thermonuclear flashes, the tail of the hydrogen distribution reaches hot enough layers for stable nuclear burning to be ignited there and this is responsible for the change in the slope of the cooling curve at advanced ages. The mass of the hydrogen envelope is shown in the middle panel of Fig. 6. It is noticeable that the mass of the hydrogen envelope for this model is larger when diffusion is allowed to operate. Finally, from the bottom panel of Fig. 6 we note that, although diffusion proceeds here much more slowly than in more massive models, the stellar surface tends to be of pure hydrogen on time-scales shorter than evolutionary time-scales.

4 DISCUSSION AND CONCLUSIONS

As stated above, diffusion produces a different cooling history for WDs depending on the mass of the models. We have found that, in

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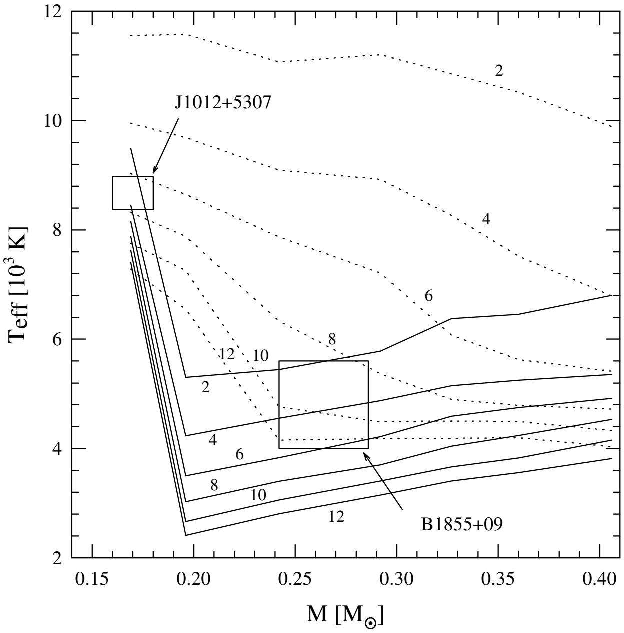

Figure 7. Isochrones in the effective temperature–mass plane for our suite of stellar models. Full curves are the predictions of models including diffusion, whilst dotted curves are for the case in which diffusion is neglected. Also, we include the observational data for the WD companions to pulsars PSR B1855+09 and PSR J1012+5307 with their corresponding error boxes. Isochrones are labelled with age values given in Gyr. Models without diffusion predict ages in agreement with the spin-down age for PSR J1012+5307 but not for PSR B1855+09. In contrast, models calculated under the assumption of diffusion give cooling ages consistent with spin-down ages of both pulsars. This is a result of the existence of a mass threshold value for the occurrence of diffusion-induced thermonuclear flashes located in between the masses of the pulsar companions cited above. This is translated into an abrupt change of slope in the isochrones.

contrast with predictions of models without diffusion (Driebe et al. 1998, also this work), models with masses greater than $\approx 0.18\,M_\odot$ undergo at least one thermonuclear flash. On the other hand, models with masses below this limit do not experience any flash.

Notably, the threshold mass value for the occurrence of flashes lies somewhere in between the masses of PSR B1855+09 and PSR J1012+5307 companions. In Fig. 7 we show the resulting isochrones and the observational data for PSR B1855+09 and PSR J1012+5307 companions. Isochrones without diffusion are rather smooth while those corresponding to models with diffusion show two distinctive features. Indeed, for masses above the threshold value for thermonuclear flashes, models cool down very rapidly; whereas for lower masses, ages are comparable to those derived from models without diffusion. From this figure we note that the WD ages expected from models with diffusion are in good agreement with the ages of the above-cited pulsars. In particular, for the PSR B1855+09 companion, such models predict an age of 4 ± 2 Gyr, while for the same object, models without diffusion are clearly at odds with the spin-down age of 5 Gyr of this pulsar (van Kerkwijk et al. 2000). Interestingly, both sets of calculations give rise to age values for the PSR J1012+5307 companion in accord with the pulsar age.

In summary, we have found that models with stellar masses greater than $\approx 0.18\,M_\odot$ experience at least one thermonuclear flash when diffusion is considered. This is in contrast with the situation in which diffusion is neglected where only the 0.242-$M_\odot$ model suffers from a thermonuclear flash. In relation with the subsequent evolution, these flashes lead to much thinner hydrogen envelopes, preventing stable nuclear burning from being an appreciable energy source for the star along the cooling track. Thus, for these models cooling ages are markedly lower. Indeed, the results we present here prove that the inclusion of diffusion in evolutionary calculations solves the age discrepancy cited above in a natural way, making it unnecessary to invoke a non-standard braking index (van Kerkwijk et al. 2000) or ad hoc mass loss episodes (Schönberner, Driebe & Blöcker 2000). In addition, we have found that diffusion considerably affects the mass–radius relation for low-mass helium WDs.

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REFERENCES

Alberts F., Savonije G. J., van den Heuvel E. P. J., Pols O. R., 1996, Nat, 380, 676
Alexander D. R., Ferguson J. W., 1994, ApJ, 437, 879
Althaus L. G., Benvenuto O. G., 1997, ApJ, 477, 313
Althaus L. G., Benvenuto O. G., 2000, MNRAS, 317, 952
Benvenuto O. G., Althaus L. G., 1998, MNRAS, 293, 177
Bhattacharya B., van den Heuvel E. P. J., 1991, Phys. Rep., 203, 1
Burgers J. M., 1969, Flow Equations for Composite Gases. Academic, New York
Caughlan G. R., Fowler W. A., 1988, At. Data Nucl. Data Tables, 40, 290
Driebe T., Schönberner D., Blöcker T., Herwig F., 1998, A&A, 339, 123
Hansen B. M. S., Phinney E. S., 1998a, MNRAS, 294, 569
Hansen B. M. S., Phinney E. S., 1998b, MNRAS, 294, 557
Iben I., Jr, MacDonald J., 1985, ApJ, 296, 540
Iben I., Jr, Tutukov A. V., 1986, ApJ, 311, 742
Iglesias C. A., Rogers F. J., 1996, ApJ, 464, 943
Kaspi V. M., Taylor J. H., Ryba M. F., 1994, ApJ, 428, 713
Lorimer D. R., Festin L., Lyne A. G., Nicastro L., 1995, Nat, 376, 393
Magni G., Mazzitelli I., 1979, A&A, 72, 134
Phinney E. S., Kulkarni S. R., 1994, ARA&A, 32, 951
Ryba M. F., Taylor J. H., 1991, ApJ, 371, 739
Sarna M. J., Antipova J., Ergma E., 1999, in Solheim J.-E., Meistas E. G., eds, ASP Conf. Ser. Vol. 169, 11th European Workshop on White Dwarfs. Astron. Soc. Pac., San Francisco, p. 400
Schönberner D., Driebe T., Blöcker T., 2000, A&A, 356, 929
van Kerkwijk M. H., Bell J. F., Kaspi V. M., Kulkarni S. R., 2000, ApJ, 530, L37
Wallace R. K., Woosley S. E., Weaver T. A., 1982, ApJ, 258, 696

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