Diffusion and the occurrence of hydrogen shell flashes in helium white dwarf stars

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ABSTRACT

In this paper we investigate the effects of element diffusion on the structure and evolution of low-mass helium white dwarfs. Attention is focused mainly on the occurrence of hydrogen shell flashes induced by diffusion processes during cooling phases. Physically sound initial models with stellar masses of 0.406, 0.360, 0.327, 0.292, 0.242, 0.196, 0.169 and 0.161 $M_\odot$ are constructed by applying mass loss rates at different stages of the red giant branch evolution of a solar model up to the moment the model begins to evolve to the blue part of the HR diagram. The multicomponent flow equations describing gravitational settling, and chemical and thermal diffusion are solved and the diffusion calculations are coupled to an evolutionary code. In addition, the same sequences are computed but neglecting diffusion. Results without diffusion are similar to those of Driebe et al. (1998).

We find that element diffusion strongly affects the structure and cooling history of helium white dwarfs. In particular, diffusion induces the occurrence of hydrogen shell flashes in models with masses ranging from 0.18 to 0.41 $M_\odot$, which is in sharp contrast from the situation when diffusion is neglected. In connection with the further evolution, these diffusion-induced flashes lead to much thinner hydrogen envelopes, preventing stable nuclear burning from being a sizeable energy source at advanced stages of evolution. This implies much shorter cooling ages than in the case when diffusion is neglected.

These new evolutionary models are discussed in light of recent observational data of some millisecond pulsar systems with white dwarf companions. We find that age discrepancies between the predictions of standard evolutionary models and such observations appear to be the result of ignoring element diffusion in such evolutionary models. Indeed, such discrepancies vanish when account is made of diffusion.

Key words: stars: evolution - stars: interiors - stars: white dwarfs - pulsar: general

1 INTRODUCTION

Over the last few years, low-mass helium white dwarf (WD) stars have been detected in numerous binary configurations. The theoretical prediction that low mass WDs with helium core would be the result of mass transfer episodes in close binary systems (Paczyński 1976; Iben & Webbink 1989; Iben & Livio 1993) was first place on a firm observational ground by Marsh 1995 and Marsh, Dhillon & Duck 1995. From then on, these objects have been observed in various binary systems containing usually either another WD or a neutron star (see, e.g., Lundgren et al. 1996; Moran, Marsh & Bragaglia 1997; Orosz et al. 1999; Maxted et al. 2000). In addition, several helium WDs have been found in open and globular clusters (Landsman et al. 1997 and Edmonds et al. 1999). Evolutionary models for these WDs with the emphasis on their mass-radius relations have been presented by Althaus & Benvenuto (1997), Benvenuto & Althaus (1998), Hansen & Phinney (1998a) and Driebe et al.(1998).

An important configuration in which helium WDs can be expected are those binary systems involving the presence of millisecond pulsars. In this regard, numerous observations reveal that the majority of WDs having millisecond pulsar

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companions are low mass WDs (Hansen & Phinney 1998b and references cited therein; van Kerkwijk et al. 2000; and Phinney & Kulkarni 1994 for a review). In particular, the presence of a WD in these binary systems offers an opportunity to check assumptions made about the ages of millisecond pulsar. In fact, cooling ages for WDs provide an estimation of the age of the system, independent of spin-down age of the pulsar. This is so because the pulsar spin down begins at nearly the same time as the companion adopts a WD configuration, after the end of mass loss episodes. The PSR J1012+5307 system is the best studied system of this type. Indeed, this system has captured the attention of numerous investigators and atmospheric parameters of the low-mass WD companion are relatively well known (van Kerkwijk, Bergeron & Kulkarni 1996; Callanan, Garnavich & Koester 1998). In particular, the spin-down age of the pulsar is approximately 7 Gyr (Lorimer et al. 1995). This value for the pulsar age can be compared with theoretical predictions of the cooling of its helium WD companion.

In this connection, Sarna, Antipova & Ergma (1999) have presented very detailed calculations of the binary evolution leading to the formation of low-mass, helium WDs with stellar masses less than 0.25 $M_\odot$. They found that, after detachment of the Roche lobe, helium cores are surrounded by a massive hydrogen layer of 0.01 - 0.06 $M_\odot$ with a surface hydrogen abundance by mass of $X_H = 0.35 - 0.50$. Massive hydrogen envelopes for helium WDs have also been derived by Driebe et al.(1998), who have simulated the binary evolution by forcing a 1 $M_\odot$ model at the red giant branch to a large mass loss rate. Driebe et al. (1998) have followed their calculations down to very low stellar luminosities and derived mass-radius relations in order to analyse recent observational data. These studies (see also Webbink 1975 and Alberts et al. 1996) show that thermal flashes do not occur for WDs less massive than approximately 0.2 $M_\odot$ and that the predicted hydrogen-rich envelope is massive enough so that stationary hydrogen shell burning remains dominant for helium WDs even down to effective temperature (T_{eff}) values below 10000K, thus substantially prolonging their cooling times. In particular, these evolutionary models give an age for the helium WD in PSR J1012+5307 in good agreement with the spin-down age of the pulsar.

However, recent observational studies seem to cast some doubt on the thickness of the hydrogen envelopes predicted by the evolutionary calculations mentioned in the foregoing paragraph. Indeed, van Kerkwijk et al. (2000) have detected the WD companion to the millisecond pulsar PSR B1855+09 and determined its T_{eff} to be 4800 ± 800K. Since the mass of the WD is accurately known thanks to the Shapiro delay of the pulsar signal ($0.258^{+0.028}_{-0.016} M_\odot$; see Kaspi, Taylor & Ryba 1994), this low T_{eff} value corresponds to a WD cooling age of 10 Gyr according to the Driebe et al. (1998) evolutionary models. This is in strong discrepancy with the characteristic age of the pulsar, $\tau_c = 5$ Gyr. Interestingly enough, the pulsars PSR J0034-0534 and PSR J1713+0747 have very cool WD companions (Hansen & Phinney 1998b), thus implying cooling ages far grater than 10 Gyr from Driebe et al. models. As discussed by van Kerkwijk et al. (2000), these evolutionary models may overestimate the cooling ages. Finally, Ergma, Sarna & Antipova (2000) have recently discussed the cooling history of PSR J0751+1807 and found that helium

WD models with thick hydrogen envelopes implies cooling ages too high for the estimated T_{eff} of the WD component.

Needless to say, the cooling age of the WD depends on the mass of the hydrogen envelope before entering the cooling track. In fact, less massive hydrogen envelopes prevent hydrogen burning from being a major source of energy during the further evolution of the WD, thus giving rise to low ages over late cooling stages. There exist several ways by which the mass of the hydrogen envelope can be reduced. For instance, it is possible that after a flash episode the WD envelope reaches again giant dimensions, and another mass loss phase can be initiated, causing a substantial reduction of the mass of the hydrogen layers. This possibility has been analysed by Iben & Tutukov (1986), who have presented a detailed treatment of the evolution of a 0.3 $M_\odot$ helium WD remnant in a binary system. These authors find that their model experiences two hydrogen shell flashes which force the model to again fill its Roche lobe. As a result, the mass of the hydrogen envelope is considerably reduced ($M_H = 1.4 \times 10^{-4}$ $M_\odot$), which leads to small cooling ages at late stages of evolution. In this sense the existence of hydrogen shell flashes should play an important role in determining the thickness of the hydrogen layer atop a helium WD, and thus in establishing the actual time-scale for the further evolution of the star. Another possibility has recently been explored by Ergma et al. (2000), who propose that a very low mass, helium pre-WD, after detachment of its Roche lobe, looses a considerable fraction of its hydrogen envelope due to pulsar irradiation.

Here, we undertake the current investigation in order to assess the role played by diffusion in the occurrence of hydrogen flashes in helium WDs and more importantly to investigate whether or not the hydrogen envelope mass can be considerably reduced by enhanced hydrogen burning during flash episodes. As a matter of fact, gravitational settling and chemical diffusion are known to alter the distributions of element abundances below the stellar surface. In particular, chemical diffusion is expected to lead to a tail of hydrogen penetrating downwards through hotter layers (Iben & MacDonald 1985 and Althaus & Benvenuto 2000). In the case of a 0.3 $M_\odot$ helium WD, this feature has been suspected to induce a hydrogen shell flash (Iben & Tutukov 1986), forcing the model to reach giant dimensions again.

Diffusion processes in WDs have captured the attention of numerous investigators since the early work by Schatzman (1958) on the role played by diffusion in the evolution of the superficial chemical composition of WDs. From then on, various studies have shown that diffusion processes cause elements heavier than the main atmospheric constituents to sink below the photosphere over time scales much smaller than the evolutionary time scales, thus providing an explanation for the purity of almost all WD atmospheres (Fontaine & Michaud 1979; Alcock & Illarionov 1980; Muchmore 1984; Iben & MacDonald 1985; Paquette et al. 1986b; Dupuis et al. 1992, amongst others). In addition, sophisticated models invoking an interplay amongst various mechanisms such as diffusion, convection, accretion and wind mass loss have been developed to explain the presence of heavy elements in very small proportions detected in the spectrum of some WDs (see, e.g., Fontaine & Michaud 1979; Vauclair, Vauclair & Greenstein 1979; Pelletier et al. 1986; Paquette et al. 1986b; Iben & MacDonald 1985; Dupuis et

al. 1992; Unglaub & Bues 1998). In the context of helium WD evolution, Iben & Tutukov (1986) have found that diffusion processes carry some hydrogen inwards through hotter layer to such an extent that a second hydrogen shell flash is induced in a 0.3 $M_\odot$ model. Very recently, Althaus & Benvenuto (2000) have explored the effect of diffusion in the mass-radius relation for helium WDs.

In this work, we explore the evolution of helium WDs in a self- consistent way with the evolution of element abundances resulting from diffusion processes. The diffusion calculations are based on the multicomponent treatment of the gas developed by Burgers (1969) and gravitational settling, thermal and chemical diffusion are considered in our calculations. Reliable initial models are obtained from explicit modeling of their pre-WD evolution. To this end, we simulate the mass exchange phases expected in a real situation by forcing the evolution of a 1 $M_\odot$ model to a sufficiently large mass loss rates. We shall see that diffusion is a fundamental ingredient in the evolution of helium WDs that has to be taken into account when assessing general characteristics of binary systems containing a helium WD. In particular, we shall see that several aspects of helium WD evolution are strongly altered, as compared with the situation when diffusion is neglected (Driebe et al. 1998). Amongst these aspects, we mention the fact that nuclear burning remains insignificant throughout the entire cooling phase following the end of flash episodes, thus resulting in considerably shorter evolutionary ages of helium WDs. Needless to say, this feature has important implications when attempt is made in comparing theoretical predictions on the WD evolution with expectations from millisecond pulsars. Details about our evolutionary code, diffusion treatment and initial models are given in Section 2. Results are presented in Section 3 and finally Section 4 is devoted to making some concluding remarks.

2 COMPUTATIONAL DETAILS

2.1 Evolutionary code and diffusion equations

The evolutionary code we used in the present calculation has been described in our previous works on WD evolution (Althaus & Benvenuto 1997, 1998) and we refer the reader to those articles for details. Briefly, our code is based on a very detailed and updated physical description such as OPAL radiative opacities (Iglesias & Rogers 1996) and molecular opacities (Alexander & Ferguson 1994); the equation of state for the low density regime is an updated version of that of Magni & Mazzitelli (1979), while for the high density regime we consider ionic contributions, Coulomb interactions, partially degenerate electrons, and electron exchange and Thomas-Fermi contributions at finite temperature. High-density conductive opacities and the various mechanisms of neutrinos emission are taken from the works of Itoh and collaborators (see Althaus & Benvenuto 1997 for details). Hydrogen burning is taken into account by considering a complete network of thermonuclear reaction rates corresponding to the proton-proton chain and the CNO bicycle. Nuclear reaction rates are taken from Caughlan & Fowler (1988). Electron screening is treated as in Wallace, Woosley & Weaver (1982).

Gravitational settling, and chemical and thermal diffusion have been fully taken into account. To this end, we follow the treatment for multicomponent gases presented by Burgers (1969), thus avoiding the trace element approximation usually invoked in most WD studies. It is worth noting that after mass loss episodes helium WDs are expected to have envelopes made up of a mixture of hydrogen and helium, thus the trace element approximation would clearly not be appropriate for the case we want to study here. Radiative levitation, which is important for determining photospheric composition of hot WDs (Fontaine & Michaud 1979) has been neglected. This assumption is justified since we are interested in the chemical evolution occurring quite deep in the star. In the context of WD evolution, the treatment for diffusion we use here has been employed by Muchmore (1984) and Iben & MacDonald (1985). Recently, it has been applied by MacDonald, Hernanz & José (1999) to address the problem of carbon dredge-up in WDs with helium-rich envelopes.

Under the influence of gravity, partial pressure, thermal gradients and induced electric fields (we neglect stellar rotation and magnetic fields) the diffusion velocities in a multicomponent plasma satisfy the set of diffusion equation ($N - 1$ independent linear equations, Burgers 1969)

$$\frac{dp_i}{dr} - \frac{\rho_i}{\rho}\frac{dp}{dr} - n_i Z_i e E = \sum_{j \neq i}^{N} K_{ij}\left(w_j - w_i\right)$$
$$+ \sum_{j \neq i}^{N} K_{ij}\, z_{ij}\frac{m_j\, r_i\, - m_i\, r_j}{m_i\, + m_j}, \qquad (1)$$

and heat flow equation (N equations)

$$\frac{5}{2}n_i k_B \nabla T = -\frac{5}{2}\sum_{j \neq i}^{N} K_{ij}\, z_{ij}\frac{m_j}{m_i\, + m_j}\left(w_j - w_i\right) - \frac{2}{5}K_{ii}\, z_{ii}''\, r_i$$

$$- \sum_{j \neq i}^{N} \frac{K_{ij}}{(m_i\, + m_j)^2}\left(3m_i^2 + m_j^2 z_{ij} + 0.8 m_i m_j z_{ij}''\right)\, r_i$$
$$+ \sum_{j \neq i}^{N} \frac{K_{ij} m_i m_j}{(m_i\, + m_j)^2}\left(3 + z_{ij}' - 0.8 z_{ij}''\right)\, r_j. \qquad (2)$$

Here, p_i, ρ_i, n_i, Z_i and m_i means, respectively, the partial pressure, mass density, number density, mean charge and mass for species i (N means the number of ionic species plus electron). T, k_B and ∇T are the temperature, Boltzmann constant and temperature gradient, respectively. The unknown variables are the diffusion velocities with respect to the center of mass, w_i, and the residual heat flows r_i (for ions and electrons). In addition the electric field E has to be determined. The resistance coefficients (K_{ij}, z_{ij}, z_{ij}' and z_{ij}'') are from Paquette at al (1986a) and averages ionic charges are treated following an approximate pressure ionization model as given by Paquette et al. (1986b), which is sufficient for our purposes.

To complete the set of equations, we use the conditions for no net mass flow with respect to the center of mass

$$\sum_i A_i n_i w_i = 0, \qquad (3)$$

and no electrical current

Table 1. Stellar mass, envelope mass and hydrogen surface mass fraction X_H at the point of maximum effective temperature.

$M/M_\odot$	$M_{env}/M_\odot$	X_H
0.406	7.3×10^{-4}	0.701
0.360	1.1×10^{-3}	0.701
0.327	1.4×10^{-3}	0.701
0.292	2.0×10^{-3}	0.701
0.242	3.7×10^{-3}	0.694
0.196	6.7×10^{-3}	0.504
0.169	1.0×10^{-2}	0.423

The quoted values correspond to the sequences in which diffusion is neglected

$$\sum_i Z_i n_i w_i = 0. \tag{4}$$

In terms of the gradient in the number density we can transform Eq. (1) to

$$\frac{1}{n_i} \left[\sum_{j \neq i}^N K_{ij} \left(w_i - w_j \right) + \sum_{j \neq i}^N K_{ij} \, z_{ij} \frac{m_i \, r_j \, - m_j \, r_i}{m_i \, + m_j} \right]$$
$$- Z_i e E = \alpha_i - k_B T \, \frac{d \ln n_i}{dr}, \tag{5}$$

where

$$\alpha_i = -A_i m_H g - k_B T \, \frac{d \ln T}{dr}, \tag{6}$$

where A_i, m_H, g and T are the atomic mass number, hydrogen atom mass, gravity and temperature, respectively. Let us write the unknowns w_i, r_i and E in terms of the gradient of ion densities in the form (similarly for r_i and E)

$$w_i = w_i^{gt} - \sum_{ions(j)} \sigma_{ij} \, \frac{d \ln n_j}{dr}, \tag{7}$$

where w_i^{gt} stands for the velocity component due to gravitational settling and thermal diffusion. The summation in Eq. (7) is to be effected over the ions only. With Eqs. (2) and (5) together with (3) and (4) we can easily find the components w_i^{gt} and σ_{ij} by matrix inversions (LU decomposition).

Now, we are in a position to find the evolution of element distribution throughout the star by solving the continuity equation. Details are given in Althaus & Benvenuto (2000) (see also Iben & MacDonald 1985). In particular, we follow the evolution of the isotopes ^1H, ^3He, ^4He, ^{12}C, ^{14}N and ^{16}O. In order to calculate the dependence of the structure of our WD models on the varying abundances self-consistently, the set of equations describing diffusion has been coupled to our evolutionary code. We want to mention that after computing the change of abundances by effect of diffusion, they are evolved according to nuclear reactions and convective mixing. It is worth mentioning that radiative opacities are calculated for metallicities consistent with diffusion predictions. In particular, the metallicity is taken as two times the abundances of CNO elements.
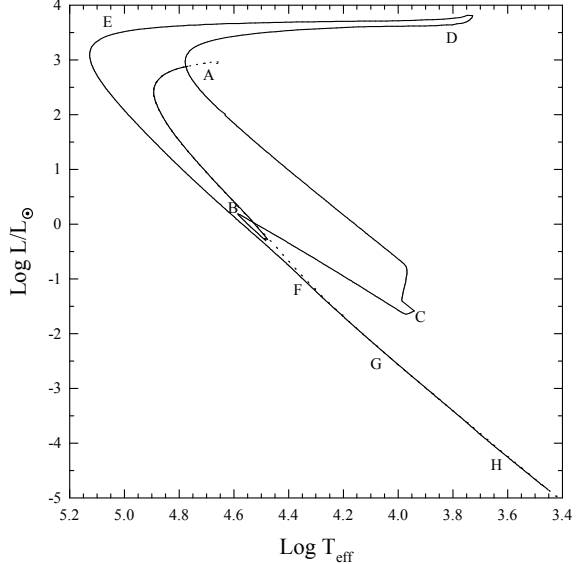

Figure 1. Hertzsprung - Russell (HR) diagram for the evolution of the 0.406 $M_\odot$ helium WD model. Solid line corresponds to the case when diffusion is included and dotted line depicts the situation when diffusion is neglected. Note that diffusion processes lead to the occurrence of a hydrogen shell flash, forcing the model to evolve back to the red giant region (point D). This is in sharp contrast with the situation encountered under the assumption of no diffusion, in which case flash episodes do not occur at all. Characteristics of the models with diffusion labeled by letters are given in Table 2.
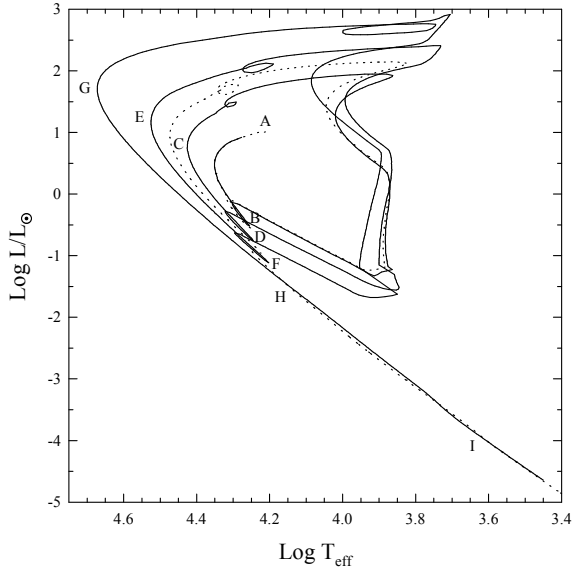

Figure 2. Same as Fig. 1 but for 0.242 $M_\odot$ helium WD models. Here diffusion gives rise to three hydrogen shell flashes, whilst one flash is obtained when diffusion is neglected. Characteristics of the models with diffusion labeled by letters are given in Table 2.
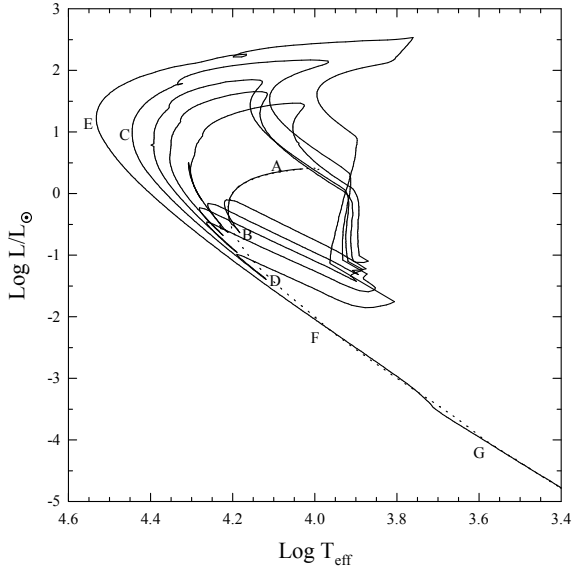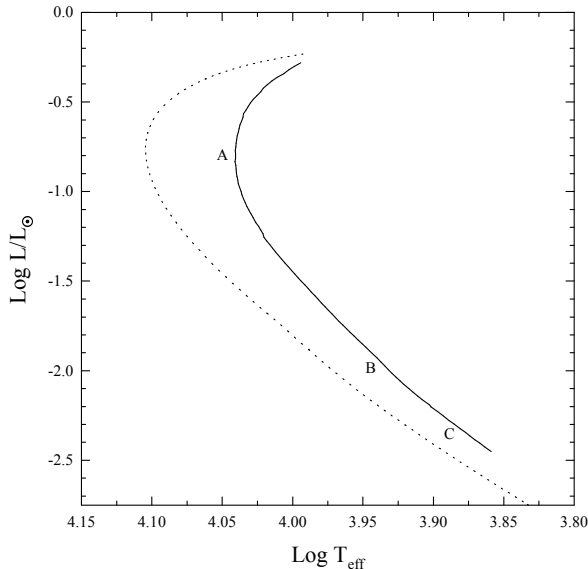
Figure 3. Same as Fig. 1 but for 0.196 $M_\odot$ helium WD models. Here diffusion gives rise to five hydrogen shell flashes, whilst no flash is obtained when diffusion is neglected. Characteristics of the models with diffusion labeled by letters are given in Table 2.

Figure 4. Same as Fig. 1 but for 0.169 $M_\odot$ helium WD models. Unlike more massive models, here no hydrogen shell flash takes place even in the presence of diffusion. This results in quite similar evolutionary ages for both sets of sequences. However note that at low $T_{\rm eff}$ values, models become markedly less compact when diffusion in considered. Characteristics of the models with diffusion labeled by letters are given in Table 2.

2.2 Initial models

In this work we have not modeled in detail the binary evolution leading to the formation of helium WDs (see Sarna et al. 1999 and references therein for details). Rather, we have obtained our initial models by simply abstracting mass from a 1 $M_\odot$ model at appropriate stages of its evolution towards
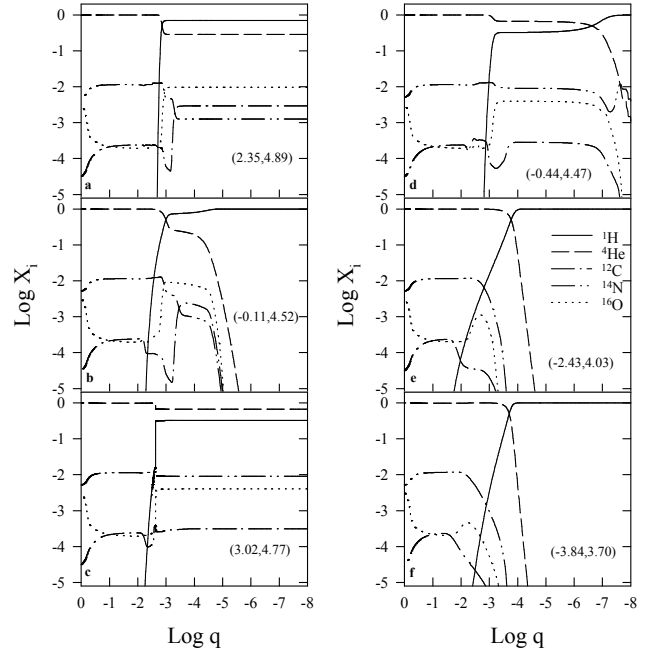

Figure 5. Abundance by mass of ^{1}H, ^{4}He, ^{12}C, ^{14}N and ^{16}O versus the outer mass fraction q ($q = 1 - M_r/M_*$) for 0.406 $M_\odot$ helium WD models at selected evolutionary stages characterized by values of $\log L/\mathrm{L}_\odot$ and $\log T_{\rm eff}$ (numbers given between brackets). Figure **a** shows the initial chemical stratification before the model reaches the cooling branch for the first time. Figure **b** corresponds to the situation short before the occurrence of the hydrogen shell flash and figure **c** depicts the stratification immediately after mixing episodes and before the model returns to the red giant region. Finally, figures **d**, **e** and **f** depict models corresponding to the final cooling branch. Note that diffusion substantially alters the chemical profiles, rapidly leading to pure hydrogen envelopes. Note also the tail of hydrogen extending into hotter and helium-rich regions as a result of chemical diffusion (figures **b** and **e**). At high $T_{\rm eff}$ values, this effect is responsible for the occurrence of hydrogen shell flash episodes.

the red giant branch (see also Iben & Tutukov 1986 and Driebe et al. 1998). In this way, we were able to generate initial WD models with stellar masses of 0.406, 0.360, 0.327, 0.292, 0.242, 0.196 and 0.169 $M_\odot$. We want to mention that the evolution of the 1 $M_\odot$ model has been calculated on the basis of a very detailed physical description, such as the equation of state of OPAL (Rogers, Swenson & Iglesias 1996). In Table 1, we tabulate the main characteristic of our initial models. We list, in particular, for each stellar mass, the envelope mass and the surface abundance of hydrogen at maximum $T_{\rm eff}$. Note that our resulting envelopes and hydrogen surface abundance are in good agreement with those quoted by Driebe et al (1998), particularly for massive models. For less massive models, however, our envelopes are somewhat larger than that of the mentioned authors. Interestingly, very large envelopes have recently been derived by Sarna et al. (1999) on the basis of detailed binary evolution calculations.

We want to stress here that the approach we follow to obtain our initial models represents a simplification as compared with the real situation in which a common envelope phase is expected. However, after the termination of mass
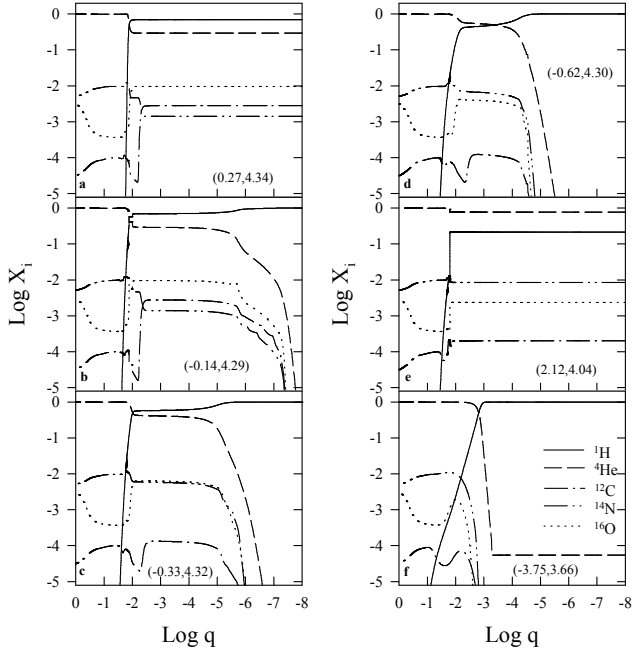
Figure 6. Same as Fig. 5 but for 0.242 $M_\odot$ helium WD models. Here, figures **b**, **c** and **d** correspond to evolutionary phases just before the occurrence of each flash episode. Figure **e** shows the chemical stratification immediately after mixing episodes and before the model returns to the red giant region for the last time. Finally, figure **f** corresponds to an advanced stage of evolution where some helium is present in the outer layers as a result of convective mixing. See text for details.

Figure 7. Same as Fig. 5 but for 0.169 $M_\odot$ helium WD models. Figure **a** depicts the chemical stratification before the model reaches the cooling branch, while the other figures correspond to models on the cooling branch. Note that diffusion is very efficient in modifying the outer layer composition even at high $T_{\rm eff}$ values due to the large evolutionary times characterizing this evolutionary phase.

loss episodes, the further evolution of the model does not depend on the details of how most of the envelope was lost. In particular, the mechanical and thermal structure of the models are consistent with the predictions of binary evolution calculations (see Driebe et al. 1998)

3   RESULTS

In this section we describe the main results of our calculations. Using the evolutionary code and the treatment for diffusion described in the preceding section, we have calculated helium WD models with stellar masses of 0.406, 0.360, 0.327, 0.292, 0.242, 0.196, 0.169 and 0.161 $M_\odot$, which amply covers the range of stellar masses expected for these objects. Realistic starting models were obtained by abstracting mass from a 1 $M_\odot$ giant star up to the moment it begins to evolve to the blue part of the HR diagram. From then on, evolution was followed down to very low stellar luminosities and assuming a constant value for the stellar mass. As stated in the introduction, this investigation is aimed at exploring the influence that diffusion has on the evolution of helium WDs. Thus, in addition to evolutionary sequences in which diffusion is considered, we have computed sequences for which diffusion is neglected. This has enabled us to clearly identify the effect induced by diffusion on the evolution of these objects. We want to mention that the results we have obtained for the standard treatment of no diffusion are in good agreement with those of Driebe et al. (1998).
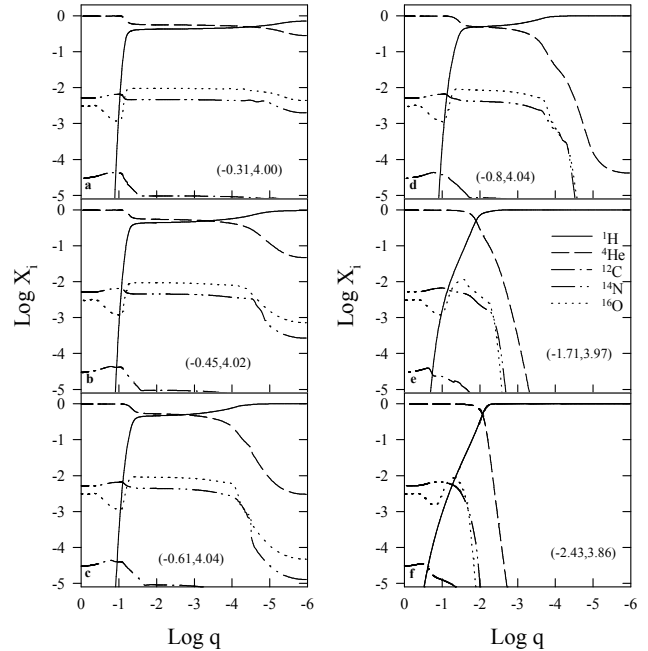
We begin by examining Figs 1 to 4 in which we show the evolution in the HR diagram of helium WD models with masses of 0.406, 0.242, 0.196 and 0.169 $M_\odot$ considering (solid lines) and neglecting (dotted lines) diffusion. Characteristics of the models with diffusion labeled by letters are given in Table 2 in which, from left to right, we list the stellar mass and photon luminosity (both in solar units), the effective temperature, the age (in million years), the surface gravity (g), the nuclear luminosity (in solar units), the abundance by mass of surface hydrogen and the hydrogen envelope mass. The most outstanding feature illustrated by these figures is that element diffusion plays a very important role in inducing thermonuclear flashes, which, as we shall see, are critical regarding the subsequent evolution of these objects even during their final cooling phase. During flash episodes, the star evolves very rapidly and in the majority of the cases makes an excursion to the red giant part of the HR diagram. Note that if diffusion is neglected, only the 0.242 $M_\odot$ model experiences thermal instabilities, in agreement with the predictions of Driebe et al. (1998), who found that hydrogen flashes occur for two of their sequences: 0.234 and 0.259 $M_\odot$ models. On the contrary, we find that models with masses greater than ≈ 0.18 $M_\odot$ experience thermonuclear flashes when account is made of diffusion. This is true even for the most massive model we have analysed here. This can be understood on the basis that there is a tail in the hydrogen distribution that chemically diffuses inwards where temperature is high enough to burn it, triggering a thermal runaway. We shall see that this will be responsible for the fact that at late stages of evolution ages are very different

Table 2. Selected stages for 0.406, 0.242, 0.196 and 0.169 $M_\odot$ helium WD models considering diffusion

$M_*/M_\odot$	$Log(L/L_\odot)$	$LogT_{\rm eff}$	Age (10^6 yr)	$Log(g)$	$Log(L_{\rm nuc}/L_\odot)$	X_H	$Log(M_H/M_*)$
0.406 (A)	2.9465	4.7001	0.050	4.8559	2.9426	0.7007	-2.764
" (B)	0.4136	4.6003	0.208	6.9897	-0.6597	0.9690	-2.990
" (C)	-1.6085	3.950	13.47880	6.4105	6.2738	0.9999	-3.112
" (D)	3.6399	3.800	13.47882	0.5621	3.4811	0.3217	-3.118
" (E)	3.4490	5.0547	13.48295	5.7717	3.4427	0.3217	-3.454
" (F)	-0.7533	4.400	16.782	7.3555	-2.4647	0.9997	-3.599
" (G)	-2.1305	4.100	351.53	7.5339	-3.7283	0.9999	-3.628
" (H)	-4.0503	3.650	8334.90	7.6527	-7.5068	1.0000	-3.634
0.242 (A)	1.0199	4.2099	17.00	4.5962	1.0193	0.6936	-1.892
" (B)	-0.5299	4.2538	33.057	6.3219	-0.6130	0.9999	-2.035
" (C)	0.7534	4.4258	40.209	5.7263	0.7457	0.9996	-2.155
" (D)	-0.7527	4.2454	45.827	6.5107	-0.8618	1.0000	-2.168
" (E)	1.0765	4.5238	54.70	5.7951	1.0684	0.9820	-2.347
" (F)	-1.1160	4.2025	71.695	6.7027	-1.2617	0.9997	-2.363
" (G)	1.7033	4.6722	95.974	5.7620	1.6940	0.4798	-2.676
" (H)	-1.3322	4.1809	100.59	6.8326	-1.9306	0.9997	-2.725
" (I)	-3.8149	3.6506	4222.8	7.1942	-5.8365	0.9997	-2.743
0.196 (A)	0.3393	4.1000	39.53	4.7470	0.3367	0.5042	-1.679
" (B)	-0.6347	4.1828	235.43	6.0509	-0.6521	1.0000	-1.910
" (C)	1.0140	4.4448	315.95	5.4503	1.0088	0.7750	-2.311
" (D)	-1.4105	4.1146	386.99	6.5538	-1.4687	0.9997	-2.376
" (E)	1.2206	4.5322	418.34	5.5932	1.2068	0.2387	-2.645
" (F)	-2.0401	4.0004	495.09	6.7266	-2.4071	0.9997	-2.718
" (G)	-3.9494	3.6011	4520.7	7.0388	-6.9530	0.9403	-2.727
0.169 (A)	-0.7204	4.0425	480.75	5.5118	-0.72463	0.9996	-1.685
" (B)	-2.0476	3.9250	4629.6	6.3691	-2.05169	1.0000	-1.875
" (C)	-2.3589	3.8750	11211	6.4804	-2.36304	1.0000	-1.956

Ages are counted from the end of mass transfer (at $T_{\rm eff} = 5000K$ for $M_*= 0.406$ and 0.242 $M_\odot$, and $T_{\rm eff} = 10000K$ for $M_*= 0.196$ and 0.169 $M_\odot$)

in both set of calculations. Note that less massive models do not experience hydrogen flashes but they become markedly less compact when diffusion is considered.

The effect of diffusion on the element distributions within the star is appreciated in Figs 5 to 7, where the abundances of ^{1}H, ^{4}He, ^{12}C, ^{14}N and ^{16}O are shown as a function of the outer mass fraction q ($q = 1 - M_r/M_*$) for 0.406, 0.242 and 0.169 $M_\odot$ helium WD models at various epochs. In each case, figure **a** shows the initial chemical stratification (as given by the pre-WD evolution) before the model reaches the cooling branch for the first time and short after the end of mass loss episodes (for the 0.169 $M_\odot$ model diffusion has already modified the composition of the very outer layers). Note that for the least massive model shown, initial abundances in the outer layers are different from those assumed for the interstellar medium. This is because mass loss exposed layers in which hydrogen burning occurred during the pre-WD evolution and essentially all the initial ^{12}C was processed into ^{14}N. The effect that diffusion has on the element distributions during the further evolution is clearly noticeable in all of the models analysed in these figures. Note that diffusion rapidly causes hydrogen to float to the surface, leading to pure hydrogen envelopes even in the case of low stellar masses. At the same time, the tail of the hydro-

gen distribution chemically diffuses inward to hotter layers, making hydrogen nuclear reactions ignite there. This fact is eventually responsible for the occurrence of additional thermonuclear flashes. During flash episodes, models develop an outer convection zone thick enough so as to mix hydrogen and helium layers, thus modifying the composition of the outer layers (see figures **c** and **e** corresponding to figures 5 and 6, respectively). After mixing episodes, models rapidly evolve back to the red giant regime and finally to the cooling branch (at constant luminosity), at which stage evolutionary time scales become longer and diffusion begins to alter the chemical composition again. This is in sharp contrast with the results predicted by neglecting diffusion, in which case the outer layer chemical composition is established by the last episode of convective mixing and the pre-WD evolution. Another feature worthy of comment is that illustrated by figure **f** of Fig. 6, which shows that at low luminosities some helium is dredged up to the surface as a result of convective mixing. In fact, the hydrogen envelope remaining after the end of flash episodes is thin enough that convection is able to mix it with the underlying helium layers when the star reaches low luminosities. Thus, over a considerable time interval, the star will be characterized by outer layers made up of hydrogen and helium. More specifically, this stage of
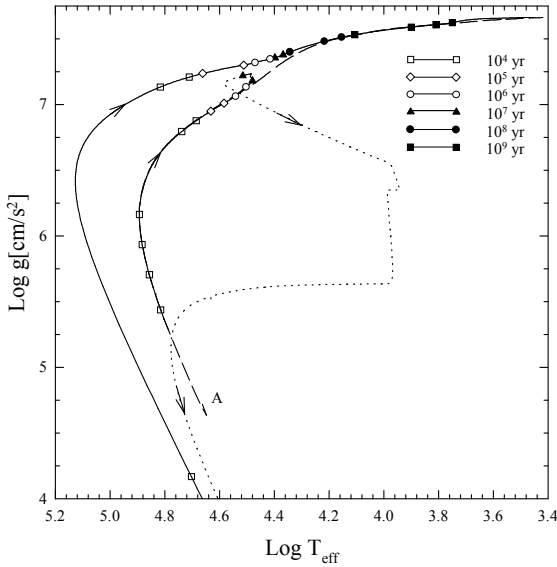
Figure 8. Surface gravity vs. effective temperature for 0.406 $M_\odot$ helium WD models. Solid line corresponds to the case when diffusion is included and dashed line to the situation when diffusion is neglected. Both set of sequences start at point A. The fast evolution during the flash phase is shown by dotted lines and arrows indicate the course of evolution. Also, evolutionary time intervals for models with diffusion are indicated by means of adjacent symbols along the curves. The effect of the thermonuclear flash resulting from element diffusion upon surface gravities is apparent at high $T_{\rm eff}$ values. Except for such stages, where evolution proceeds very fast, surfaces gravities are not appreciably changed by the inclusion of diffusion processes. See text for details.

mixed envelope lasts for about 2 Gyr and it is found to occur only in the mass interval 0.18 - 0.25 $M_\odot$. We will return to this point later in this section. It is important to remark that models without diffusion also experience thermonuclear flashes, but the important point to be made is that diffusion is able to carry some hydrogen to inner and hotter regions and thus to induce flash episodes even in models as massive as 0.41 $M_\odot$. We shall see that this fact produces a different cooling history even at very late stages of evolution.

The role played by diffusion on surface gravities can be seen in Figs 8 to 11. As we mentioned, diffusion causes the bulk of hydrogen to float and helium and heavier elements to sink down, making the star inflate. In addition, the stellar radius depends on the amount of hydrogen left after flash episodes, which will be different according whether diffusion is neglected or not. In more massive models and at high $T_{\rm eff}$ values where evolution proceeds very fast, surface gravities result very different if diffusion is considered. At later stages however, where evolution is very slow, surface gravities are quite similar. In fact, because the inclusion of diffusion reduces the mass of the hydrogen envelope considerably (thus leading to smaller stellar radius), surface gravities are not appreciably changed from the situation when diffusion is neglected. On the contrary, for the 0.169 and 0.161 $M_\odot$ models (Fig. 11) surface gravities are considerably smaller when diffusion is considered. Here, models do not experience thermonuclear flashes even in the presence of diffusion and evolution is very slow during almost the entire cooling phase.

The location of the WD companion to the millisecond pulsar PSR J1012 + 5307 according to Callanan et al. (1998) determination is also indicated in Fig. 11. The fact that models in which diffusion is allowed to operate are characterized by lower surface gravities is due in part to that models with diffusion are characterized by more massive hydrogen envelopes (see Fig. 13) because nuclear reactions are less effective in reducing the hydrogen envelope of these models. We find for instance that for models with $M_* < 0.18$ $M_\odot$ the surface gravity is reduced by almost 80 per cent when diffusion is considered. This has the effect to increase the stellar mass derived from atmospheric parameters and mass-radius relations. Clearly, this effect should be taken into account when assessing stellar masses from mass-radius relations for these objects. In this connection, we see from Fig. 11 that a stellar mass of 0.17 $\pm$ 0.01 $M_\odot$ is derived for the WD companion to PSR J1012 + 5307 when diffusion is considered, in contrast with the estimation of 0.15 $\pm$ 0.01 $M_\odot$ derived from the assumption of no diffusion (Driebe et al. 1998 and our results without diffusion).

As mentioned earlier, diffusion is an important mechanism in establishing the outer layer composition even in low mass, helium WDs. In this context, we show in Fig. 12 the hydrogen abundance by mass (X_H) at a mass depth of $10^{-9}M_*$ below the stellar surface as a function of age for the 0.196 $M_\odot$ helium WD sequence. During each flash episode, the outer convection zone digs deep into the star and reaches helium-rich layers, thus leading to envelopes made up by hydrogen and helium (for the 0.406 $M_\odot$ model this takes place at point C in Fig. 1). At the last flash for instance, the surface hydrogen abundance drops to $X_H = 0.083$. Note that the time intervals during which the envelope remains helium-enriched are indeed extremely short. In fact, the purity of the outer layers is established by diffusion as the star rapidly returns to the cooling branch. However, at advanced stages of evolution and over a time interval of 2 Gyr, models are characterized by helium-rich outer layers ($X_H \approx 0.75$). This is because the hydrogen envelope remaining after the end of flash episodes is thin enough that convection is able to mix it with the underlying helium layers at low $T_{\rm eff}$ values. As evolution proceeds, convection reaches the domain of degeneracy and begins to retreat outwards (the maximum depth reached by the convection zone is at $M_{conv}/M_* = 10^{-2.76}$). Diffusion time scales at the bottom of this convection zone are comparable to evolutionary time scales and thereby helium is depleted from the entire outer convection zone slowly, ultimately leading to pure hydrogen envelope again, as it is apparent from Fig. 12. It is nevertheless worth mentioning that elements heavier than helium remain out of reach of the convection zone (only traces of carbon are present, $X_{12C} \approx 2 \times 10^{-7}$), so the presence of metals in these stars should be explained basically in terms of accretion episodes.[§] These results are in sharp contrast with the situation in which diffusion is neglected. In this case, the hydrogen surface abundance remains unchanged

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[§] Because of the long diffusion time scales found at the base of the convection zone, metals accreted from interstellar clouds could be maintained in the outer layers of cool, low-mass helium WDs for a long time, thus favouring their detection; see Althaus & Benvenuto (2000)
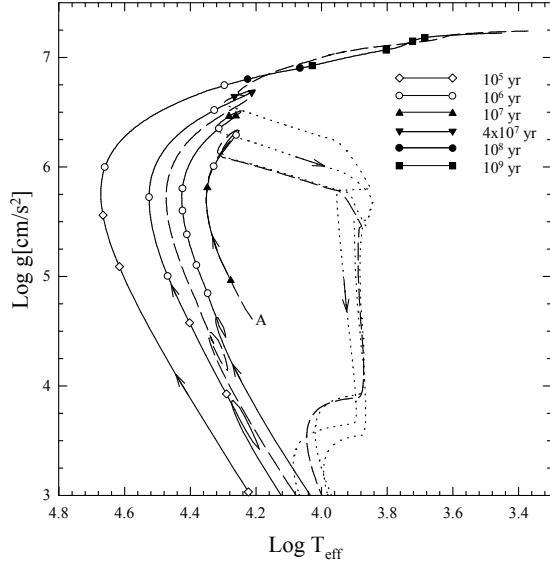
Figure 9. Same as Fig. 8 but for 0.242 $M_\odot$ helium WD models. Here, models with diffusion experience three thermonuclear flashes, the effect of which upon surface gravities is clearly noticeable at high $T_{\rm eff}$ values where evolution proceeds very fast.
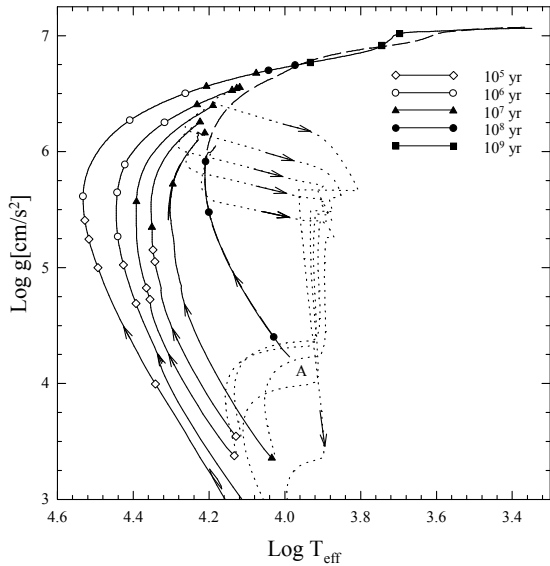

Figure 10. Same as Fig. 8 but for 0.196 $M_\odot$ helium WD models. Here, models with diffusion experience five thermonuclear flashes, the effect of which upon surface gravities is clearly noticeable at high $T_{\rm eff}$ values where evolution proceeds very fast.

$(X_H = 0.504)$ throughout the whole evolution and it is fixed by the pre-WD phases during mass transfer (the 0.196 $M_\odot$ model does not experience thermonuclear flashes in the absence of diffusion). We mention that we found the range of stellar masses for which the presence of helium in the envelope is expected at low $T_{\rm eff}$ to be 0.18 - 0.25 $M_\odot$, though the abundance of helium decreases considerably for more massive models (for instance, the helium content reaches 10^{-4} for the 0.242 $M_\odot$ models; see figure **f** of Fig. 6).

Let us analyse the amount of hydrogen left atop the WD
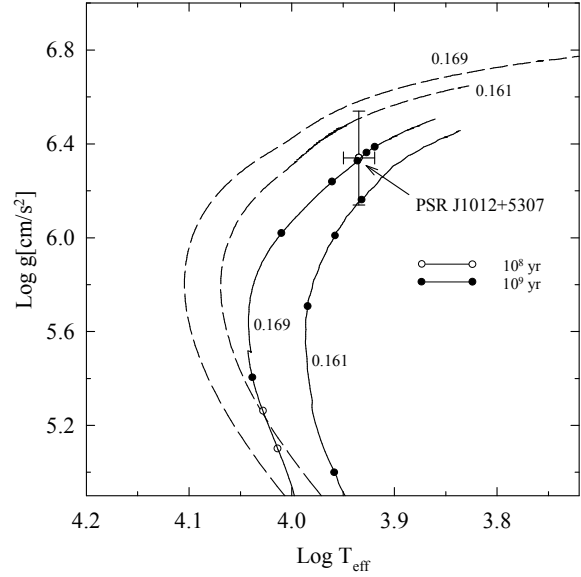

Figure 11. Same as Fig. 8 but for 0.169 and 0.161 $M_\odot$ helium WD models. Here, models do not experience thermonuclear flashes even in the presence of diffusion. The location of the WD companion to the millisecond pulsar PSR J1012 + 5307 according to Callanan et al. (1998) determination is also indicated. Note that models in which diffusion is allowed to operate are characterized by substantially lower surfaces gravities.
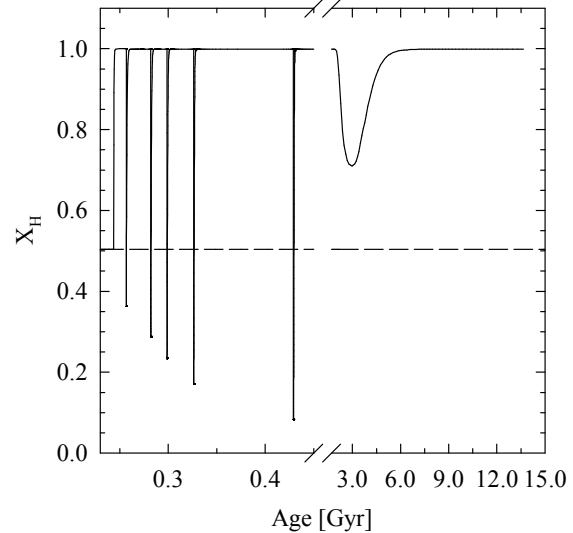

Figure 12. Hydrogen abundance by mass (at a mass depth of $10^{-9} M_*$ below the stellar surface) as a function of age for 0.196 $M_\odot$ helium WD models. Solid line corresponds to the case with diffusion and dashed line when diffusion is neglected. During each flash episode, hydrogen abundance is strongly reduced as a result of convective mixing but the purity of the outer layers is rapidly established by diffusion. Note that at advanced stages of evolution and over a time interval of 2 Gyr models present surface layers made up of hydrogen and helium. In fact, the hydrogen envelope remaining after the end of flash episodes is thin enough that convection is able to mix it with the underlying helium layers. See text for details.
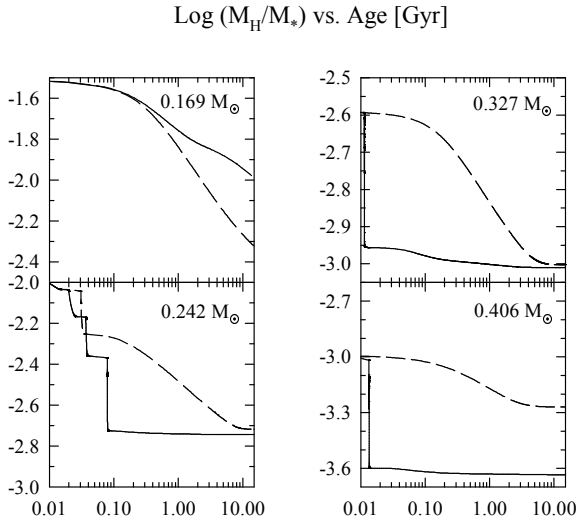
Log (M_H/M_*) vs. Age [Gyr]

Figure 13. Hydrogen envelope mass against age. Stellar mass values are indicated in each figure. Solid line corresponds to the case when diffusion is included and dashed line to the situation when diffusion is neglected. Except for the smallest stellar mass which does not experience thermonuclear flashes, the amount of hydrogen left short after the end of flash episodes is markedly lower when diffusion is considered. Note that models without diffusion burnt an appreciable fraction of its hydrogen content on the cooling track.

after the end of flash episodes. This is an important subject concerning the final cooling behaviour of the helium WD. To this end, we show in Fig. 13 the mass of the hydrogen envelope as a function of age for 0.169, 0.242, 0.327 and 0.406 $M_\odot$ WD models. The most important feature illustrated by this figure is that, except for the 0.169 $M_\odot$ model, the amount of hydrogen remaining short after hydrogen flashes have ceased is markedly lower when diffusion is considered. More specifically, in the case of models without diffusion, the hydrogen mass at entering the final cooling track is about 2 - 4 times as large as in the case with diffusion. As mentioned, the 0.242 $M_\odot$ model is the only one for which we found flashes in the absence of diffusion. Note that in this situation, an appreciable amount of hydrogen is also burnt during the flash but the fact that chemical diffusion is able to carry some hydrogen to hotter layers (thus triggering thermal instabilities) ultimately implies that the sequence with diffusion must be left with a much thinner hydrogen envelope at entering the final cooling branch. For less massive models, the situation is different because they do not experience thermonuclear flashes and thus the amount of hydrogen at the beginning of the cooling branch is the same irrespective of whether diffusion is considered or not. It is noticeable that the final hydrogen content for this model is larger when diffusion is allowed to operate because diffusion makes the star inflate, leading to lower temperatures at the bottom of the envelope where hydrogen is burnt. Here, most of the hydrogen content is burnt during the final cooling phase. This is true even in the presence of diffusion. On the contrary, for more massive models, only when diffusion is not considered an appreciably amount of hydrogen is processed over the final evolution. In

view of these results, it is expected that the role played by nuclear burning during the final WD cooling phase is different in both sequences, as it is indeed borne out by the results shown in Fig. 14. Here, we show the ratio of nuclear to photon luminosities as a function of age for the same stellar masses as analysed in Fig. 13. Solid line corresponds to the case when diffusion is included and dashed line to the situation when diffusion is neglected. Only the evolution on the final cooling branch is depicted in the figure. Except for the 0.169 and 0.161 $M_\odot$ models, for which nuclear burning is dominant in both sequences, nuclear energy release is negligible for models with diffusion. On the contrary, in agreement with Driebe et al. (1998) predictions, for models without diffusion hydrogen burning is the dominant energy source even at advanced stages of evolution. So we arrive at a very important result: *diffusion prevents hydrogen burning from being a main source of energy for most of the evolution of helium WDs with stellar masses greater than* ≈ 0.18 $M_\odot$.

The implications of the results discussed in the foregoing paragraph for the evolutionary ages are illustrated by Fig. 15 in which T_{eff} versus age relation is shown for all of our models together with the observational data for the WD companions to the millisecond pulsars PSR J1012+5307 and B1855+09 (Callanan et al. 1998 and van Kerkwijk et al. 2000, respectively). Only the evolution corresponding to the final cooling branch is depicted in the figure. First, we wish to emphasize that our results for the standard treatment of neglecting diffusion are in very good agreement with the results derived by Driebe et al. (1998) on the basis of the same assumption. In this case, evolution is dictated by residual hydrogen burning, giving rise to very long cooling ages. In particular, these evolutionary models predict an age for the helium WD in PSR J1012+5307 in good agreement with the spin-down age of the pulsar (≈ 7 Gyr). However, for the WD companion to PSR B1855+09 such models predict ages above 10 Gyr, in strong discrepancy with the pulsar age of 5 Gyr.

Now, let us examine the cooling times when diffusion is allowed to operate. We saw that in this case models with stellar masses greater than ≈ 0.18 $M_\odot$ suffer from hydrogen flashes and that the hydrogen mass left before they enter the final cooling branch is so small that the nuclear luminosity released by the object is negligible. Thereby, the WD has to obtain energy from its relic thermal content, with the consequent result that cooling ages become substantially smaller as compared with the case in which diffusion is neglected. This is clearly illustrated by Fig. 15. In particular, for the PSR B1855+09 companion, models with diffusion predict an age of 4 ± 2 Gyr in good agreement with the pulsar age. For masses lower than 0.18 $M_\odot$ nuclear burning is dominant and cooling ages resemble those derived from models without diffusion. Thus, evolutionary models taking into account element diffusion predict cooling ages consistent with the ages of the above mentioned pulsars. This naturally solves the age discrepancy for PSR B1855+09 system, making it unnecessary to invoke ad-hoc mass loss episodes, as proposed recently by Schönberner, Driebe and Blöcker (2000).
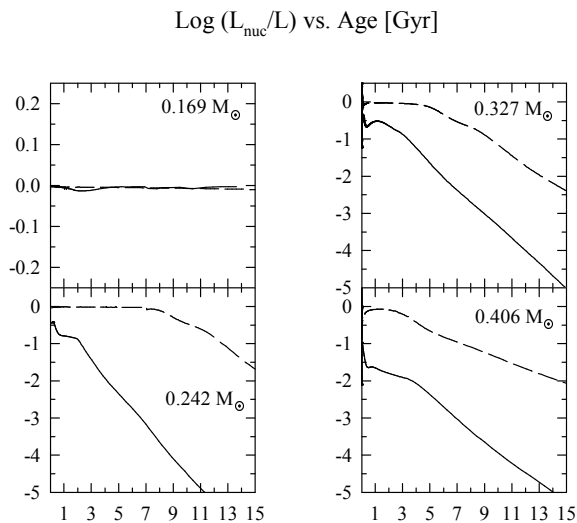
Log (L$_{nuc}$/L) vs. Age [Gyr]

Figure 14. Ratio of nuclear to photon luminosities as a function of age. Stellar mass values are indicated in each figure. Solid line corresponds to the case when diffusion is included and dashed line to the situation when diffusion is neglected. Except for the smallest stellar mass which does not experience thermonuclear flashes, nuclear energy release is negligible for models with diffusion on the cooling branch. However, for models without diffusion, hydrogen burning is the dominant energy source even at advanced stages of evolution, with the consequent result that evolution is considerably slowed down.

4 DISCUSSION AND CONCLUSIONS

Motivated by recent observational data of low-mass white dwarf (WD) companions to millisecond pulsar (van Kerk-wijk et al. 2000), which seem to cast doubts on the thickness of the hydrogen envelopes predicted by standard evolutionary calculations (Driebe et al. 1998; Sarna et al. 1999), we have undertaken the current investigation in order to assess the role played by element diffusion in the occurrence of hydrogen flashes in helium WDs and more importantly to investigate whether or not the hydrogen envelope mass can be considerably reduced by enhanced hydrogen burning during flash episodes. To this end, we have explored the evolution of 0.406, 0.360, 0.327, 0.292, 0.242, 0.196, 0.169 and 0.161 M$_\odot$ helium WD models in a self-consistent way with the evolution of element abundances resulting from diffusion processes. The diffusion calculations are based on the multicomponent treatment of the gas developed by Burgers (1969) and gravitational settling, and thermal and chemical diffusion have been considered in our calculations. Reliable initial models have been obtained by abstracting mass to a 1 M$_\odot$ model at appropriate stages along its red giant branch evolution. In the interests of a consistent comparison, the same stellar masses were evolved but neglecting diffusion. In this regard, our results are similar to those of Driebe et al. (1998) on the basis of the same assumption. We want to mention that in the calculations presented in this paper we have not invoked additional mass transfer when models return to the red giant region of the HR diagram as a result of a thermonuclear flash. In fact, the aim of the work has been to assess the possibility that diffusion alone can
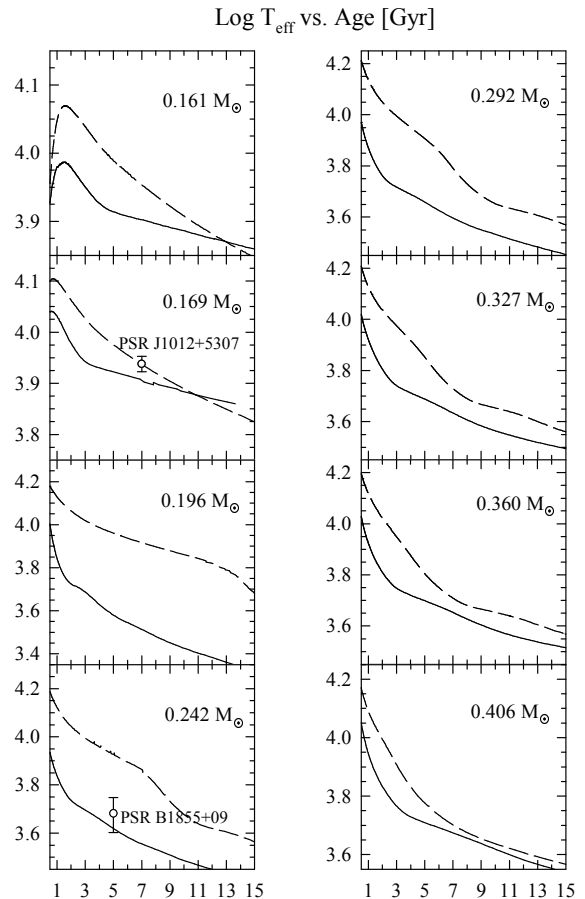
Log T$_{eff}$ vs. Age [Gyr]

Figure 15. Effective temperature as a function of age for the whole set of our models. Solid line corresponds to the case when diffusion is included and dashed line to the situation when diffusion is neglected. The observational data for the WD companion to the millisecond pulsars PSR B1855+09 and PSR J1012+5307 are included. At low T_{eff} values and for stellar masses greater than ≈ 0.18 M$_\odot$, models with diffusion evolve significantly faster as compared with the standard treatment of no diffusion. Note that models without diffusion predict ages in good agreement with the spin - down age for PSR J1012+5307 but not for PSR B1855+09. On the contrary, when diffusion is considered cooling ages are consistent with the ages of both pulsars.

eventually lead to hydrogen envelope thin enough so as to prevent nuclear burning from being an important source of energy at advanced cooling stages. In this sense, our treatment differs from that of Iben & Tutukov (1986) for the case of a 0.3 M$_\odot$ model. In their model, high mass loss causes a markedly reduction of the hydrogen envelope mass and this is responsible for the fact that the object cools down rapidly during the final cooling phase.

From the results presented in this work it is clear that element diffusion considerably affects the structure and evolution of helium WDs. In fact, diffusion produces a different cooling history for helium WDs depending on the mass of the models. We have found that models with stellar masses ranging from ≈ 0.18 M$_\odot$ to the most massive model we analyse (0.406 M$_\odot$) experience at least one thermonuclear flash when account is made of diffusion. This is in sharp contrast with the predictions of the standard treatment of no diffusion for which only the sequence with 0.242 M$_\odot$ un-

dergoes flash episodes (note that Driebe et al. 1998 found thermal instabilities only for their 0.259 and 0.234 models). In connection with the further evolution, these diffusion-induced flashes lead to much thinner hydrogen envelopes, preventing stable nuclear burning from being a sizeable energy source. Because the star has a much lower amount of available energy, it implies much shorter evolutionary time scales as compared with the situation when diffusion is neglected, in which case evolution is delayed to very long ages by the active hydrogen burning zone that characterizes such models.

These results have important implications when attempt is made in comparing theoretical predictions on the WD evolution with expectations from millisecond pulsars. This is particularly important in view of recent observational data on the helium WD companion to the pulsar PSR B1855+09. In fact, van Kerkwijk et al. (2000) determined the T_{eff} of the WD companion to be 4800 ± 800K. Since the mass of the WD is accurately known thanks to the Shapiro delay of the pulsar signal ($0.258^{+0.028}_{-0.016} M_{\odot}$), this low T_{eff} value corresponds to a WD cooling age of 10 Gyr according to the Driebe et al. evolutionary models or our models without diffusion. This is in strong discrepancy with the spin-down age of the pulsar (5 Gyr). On the contrary, we found that when diffusion is properly accounted for in stellar models of helium WD, evolution is accelerated to such an extent that this age discrepancy vanishes completely. In fact, we found that our models predict an age of 4 ± 2 Gyr for the WD companion to PSR B1855+09. This naturally solves the age discrepancy for PSR B1855+09 system, making unnecessary invoke ad-hoc mass loss episodes, as proposed recently by Schönberner et al. (2000), or a non standard braking index. We also found that for masses lower than 0.18 $M_{\odot}$, nuclear burning is dominant even in the presence of diffusion and cooling ages resemble those derived from models without diffusion.

In light of the new results presented in this work, it is worthwhile to re-examine other millisecond pulsar systems with WD companions for which cooling and spin-down ages appear to be discrepant. PSR J0034-0534 system is particularly noteworthy in this regard. In fact, the WD in this system is very cool ($T_{eff} < 3500$K) and the pulsar age is 6.8 ± 2.4 Gyr (Hansen & Phinney 1998b). The WD mass according to the relation between the orbital period of the system and the WD mass (Tauris & Savonije 1999) results $\approx 0.21 M_{\odot}$(see Schönberner et al. 2000). Helium WD models that neglect diffusion predict ages well above 10 Gry for this WD companion. On the contrary, evolutionary calculations including element diffusion give for this helium WD an age of 6 and 8 Gyr at $T_{eff} = 3500$ and 3000, respectively, which is in excellent agreement with the pulsar age. It is clear that consistency between pulsar and WD ages in PSR J0034-053 system can be obtained with helium WD models and not only with WD models having a carbon-oxygen core with stellar mass of 0.5 $M_{\odot}$, as argued by Schönberner et al. (2000) on the basis of evolutionary models in which diffusion is neglected. Note that the mass of the WD derived by Schönberner et al. is clearly at odds with that inferred from the relation of Tauris & Savonije (1999).

Other millisecond pulsar system with a cool WD companion of relevance in the context of the new results presented here and which is also a discrepant case is PSR J1713+0747. For the WD companion to this pulsar, Hansen & Phinney (1998b) give an effective temperature of $T_{eff} < 3800$K and a spin-down age for the pulsar of 9.2 ± 0.4 Gyr. The WD mass according to the relation of Tauris & Savonije (1999) is ≈ 0.33 $M_{\odot}$. Models that neglect diffusion predict an age for the WD greater than 13 Gyr at that T_{eff} value, whilst models with diffusion give an age of 9.1 Gyr in very good agreement with the pulsar age.

In closing, it is clear that element diffusion plays a very important role in the cooling history of helium WDs. From the results of the present study, we conclude that age discrepancies between the predictions of standard evolutionary calculations and recent observational data of millisecond pulsar systems with WD companions appear to be the result of ignoring element diffusion in existing evolutionary calculations. Indeed, such discrepancies completely vanish when account is made of diffusion.

Finally, we wish to mention that because of the employment of gray atmospheres in our calculation, cooling ages at very low T_{eff} values should be taken with some caution. As discussed recently by Hansen (1999) and Salaris et al. (2000) in the context of carbon-oxygen WDs, an adequate treatment of the atmosphere is required at advanced stages of evolution where the gray assumption becomes a poor approximation. We are planning to include in our helium WD evolutionary calculations a non gray atmosphere as an outer boundary condition appropriate for very old helium WDs.

Complete tables containing the results of the present calculations are available at http://www.fcaglp.unlp.edu.ar/~althaus/ or upon request to the authors at their e-mail addresses

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