

Defining the structure of well-formed argumentation lines in abstract frameworks

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ABSTRACT

The area of Knowledge Representation and Reasoning has been enriched during the past two decades with the addition of Argument-Based Reasoning Systems. Defeat between arguments is established by a combination of two basic elements: a conflict or defeat relation, and a preference relation on the arguments involved in this conflict. The research activities are centered in our abstract framework for argumentation, where two kinds of defeat are present, depending on the outcome of the preference relation. This framework also takes subarguments into account, leading to the formalization of well formed argumentation lines.

Keywords: artificial intelligence, abstract argumentation, dialogues.

1 INTRODUCTION

The area of Knowledge Representation and Reasoning has been enriched during the past two decades with the addition of Argument-Based Reasoning Systems [10, 12, 5, 8, 15, 2] to mention a few. The study of the acceptability of arguments is one of the main concerns in Argumentation Theory. In formal systems of defeasible argumentation, arguments for and against a proposition are produced and evaluated to test the acceptability of that proposition following a dialectical process [13]. The main idea in these systems is that a proposition will be accepted as true if there exists an argument that supports it, and this argument is acceptable according to an analysis between it and its counterarguments. This analysis requires a process of comparison of conflicting arguments in order to decide which one is preferable [12, 11, 1]. After this dialectical analysis is performed over the set of arguments in the system, some of them will be *acceptable*, *justified* or *warranted* arguments, while others will be not. Argumentation is used as a form of non-monotonic or defeasible reasoning [9] and it is suitable for modeling dialogues between intelligent agents.

Abstract argumentation systems [5, 15, 7] are formalisms for argumentation where some components remain unspecified. Usually, the actual structure of an argument is abstracted away. In this kind of system, the emphasis is put on the semantic notion of finding the set of accepted arguments. Most of them are based on the single abstract concept of the *attack* represented as an abstract relation, and extensions are defined as sets of possibly accepted arguments. The task of comparing arguments to establish a preference is not always successful. However, finding a preferred argument is essential to determine a defeat relation.

2 ARGUMENTATION FRAMEWORK

Our argumentation framework is formed by four elements: a set of arguments, a binary conflict relation over this set, a subargument relation and a function used to decide which argument is preferred given any pair of arguments.

Definition 1 *An abstract argumentation framework Φ is a quartet $\langle \text{Args}, \sqsubseteq, \mathbf{C}, \mathbf{R} \rangle$, where Args is a finite set of arguments, \sqsubseteq is the subargument relation, \mathbf{C} is a symmetric and anti-reflexive binary conflict relation between arguments, $\mathbf{C} \subseteq \text{Args} \times \text{Args}$, and $\mathbf{R} : \text{Args} \times \text{Args} \rightarrow 2^{\text{Args}}$ is a preference function among arguments.*

Here, arguments are abstract entities [5] that will be denoted using calligraphic uppercase letters. No reference to the underlying logic is needed since we are abstracting the structure of the arguments. The symbol \sqsubseteq denotes subargument relation: $\mathcal{A} \sqsubseteq \mathcal{B}$ means “ \mathcal{A} is a subargument of \mathcal{B} ”. Any argument \mathcal{A} is considered a superargument and a subargument of itself. Any subargument $\mathcal{B} \sqsubseteq \mathcal{A}$ such that $\mathcal{B} \neq \mathcal{A}$ is said to be a non-trivial subargument. Non-trivial subargument relation is denoted by symbol \sqsubset . The following notation will be also used: given an argument \mathcal{A}

then \mathcal{A}^- will represent a subargument of \mathcal{A} , and \mathcal{A}^+ will represent a superargument of \mathcal{A} . When no confusion may arise, subscript index will be used for distinguishing different subarguments or superarguments of \mathcal{A} .

The conflict relation between two arguments \mathcal{A} and \mathcal{B} denotes the fact that these arguments cannot be accepted simultaneously since they contradict each other. For example, two arguments \mathcal{A} and \mathcal{B} that support complementary conclusions l and $\neg l$ cannot be accepted together. The set of all pairs of arguments in conflict on Φ is denoted by \mathbf{C} . Given a set of arguments S , an argument $\mathcal{A} \in S$ is said to be in conflict in S if there is an argument $\mathcal{B} \in S$ such that $(\mathcal{A}, \mathcal{B}) \in \mathbf{C}$. The set $Conf(\mathcal{A})$ is the set of all arguments $\mathcal{X} \in Args$ in conflict with \mathcal{A} .

Conflict relations have to be propagated to superarguments, that is, if an argument \mathcal{A} is in conflict with an argument \mathcal{B} then $(\mathcal{A}, \mathcal{X}) \in \mathbf{C}$ para todo \mathcal{X} tal que $\mathcal{A} \sqsubseteq \mathcal{X}$. The constraints imposed by the conflict relation lead to several sets of possible accepted arguments. For example, if $Args = \{\mathcal{A}, \mathcal{B}\}$ and $(\mathcal{A}, \mathcal{B}) \in \mathbf{C}$, then $\{\mathcal{A}\}$ is a set of possible accepted arguments, and so is $\{\mathcal{B}\}$. Therefore, some way of deciding among all the possible outcomes must be devised. In order to accomplish this task, the relation \mathbf{R} is introduced in the framework and it will be used to evaluate arguments, modelling a preference criterion based on a measure of strength.

Definition 2 Given a set of arguments $Args$, an argument comparison criterion \mathbf{R} is a binary relation on $Args$. If $\mathcal{A}\mathbf{R}\mathcal{B}$ but not $\mathcal{B}\mathbf{R}\mathcal{A}$ then \mathcal{A} is preferred to \mathcal{B} , denoted $\mathcal{A} \succ \mathcal{B}$. If $\mathcal{A}\mathbf{R}\mathcal{B}$ and $\mathcal{B}\mathbf{R}\mathcal{A}$ then \mathcal{A} and \mathcal{B} are arguments with equal relative preference, denoted $\mathcal{A} \equiv \mathcal{B}$. If neither $\mathcal{A}\mathbf{R}\mathcal{B}$ or $\mathcal{B}\mathbf{R}\mathcal{A}$ then \mathcal{A} and \mathcal{B} are incomparable arguments, denoted $\mathcal{A} \bowtie \mathcal{B}$.

As the comparison criterion is treated abstractly, we do not assume any property of \mathbf{R} . Any concrete framework may establish rationality requirements for decision making.

Example 1 $\Phi = \langle Args, \sqsubseteq, \mathbf{C}, \mathbf{R} \rangle$ is an argumentation framework where $Args = \{\mathcal{A}, \mathcal{B}, \mathcal{C}, \mathcal{D}, \mathcal{E}\}$, $\mathbf{C} = \{\{\mathcal{A}, \mathcal{B}\}, \{\mathcal{B}, \mathcal{C}\}, \{\mathcal{C}, \mathcal{D}\}\}$ and $\mathcal{A} \succ \mathcal{B}$, $\mathcal{B} \bowtie \mathcal{C}$, $\mathcal{A} \succ \mathcal{E}$ and $\mathcal{C} \equiv \mathcal{D}$.

For two arguments \mathcal{A} and \mathcal{B} in $Args$, such that the pair $(\mathcal{A}, \mathcal{B})$ belongs to \mathbf{C} , according to definition 2 there are four possible outcomes:

- $\mathcal{A} \succ \mathcal{B}$. In this case a *defeat* relation is established. Because \mathcal{A} is preferred to \mathcal{B} , in order

to accept \mathcal{B} it is necessary to analyze the acceptance of \mathcal{A} , but not the other way around. It is said that argument \mathcal{A} *defeats* argument \mathcal{B} , and \mathcal{A} is a *proper defeater* of \mathcal{B} .

- $\mathcal{B} \succ \mathcal{A}$. In a similar way, argument \mathcal{B} *defeats* argument \mathcal{A} , and therefore \mathcal{B} is a *proper defeater* of \mathcal{A} .
- $\mathcal{A} \equiv \mathcal{B}$. Both arguments are equivalent, *i. e.* there is no relative difference of conclusive force, so \mathcal{A} and \mathcal{B} are said to be *indistinguishable* regarding the preference relation \mathbf{R} . No *proper defeat relation* can be established between these arguments.
- $\mathcal{A} \bowtie \mathcal{B}$. Both arguments are *incomparable* according to \mathbf{R} , and no *proper defeat relation* is inferred.

In the first two cases, a concrete preference is made between two arguments, and therefore a defeat relation is established. The preferred arguments are called *proper defeaters*. In the last two cases, no preference is made, either because both arguments are indistinguishable from each other or because they are incomparable. These cases are slightly different. If the arguments are indistinguishable, then according to \mathbf{R} they have the *same* relative conclusive force. For example, if the preference criterion establishes that smaller¹ arguments are preferred, then two arguments of the same size are indistinguishable. On the other hand, if the arguments are *incomparable* then \mathbf{R} is not able to establish a relative difference of conclusive force. For example, if the preference criterion states that argument \mathcal{A} is preferred to \mathcal{B} whenever the premises of \mathcal{A} are included in the premises of \mathcal{B} , then arguments with disjoint sets of premises are incomparable. This situation seems to expose a limitation of \mathbf{R} , but must be understood as a natural behaviour. Some arguments just can not be compared.

Some authors leave the preference criteria unspecified, even when it is one of the most important components in the system. However, in many cases it is sufficient to establish a set of properties that the criteria must exhibit. A very reasonable one states that an argument is as strong as its weakest subargument [15]. We will assume from now on that this property is present in the criterion \mathbf{R} included in Φ . This is important because any argument \mathcal{A} defeated by another argument \mathcal{B} should also be defeated by another argument \mathcal{B}^+ .

When two conflictive arguments are indistinguishable or incomparable, the conflict between these two

¹In general, the size of an argument may be defined on structural properties of arguments, as the number of logical rules used to derive the conclusion or the number of propositions involved in that process.

arguments remains unresolved. Due to this situation and to the fact that the conflict relation is a symmetric relation, each of the arguments is *blocking* the other one and it is said that both of them are *blocking defeaters* [10, 14]. An argument \mathcal{B} is said to be a *defeater* of an argument \mathcal{A} if \mathcal{B} is a blocking or a proper defeater of \mathcal{A} . In example 1, in the context of argumentation framework Φ_3 , argument \mathcal{A} is a proper defeater of argument \mathcal{B} , while \mathcal{C} is a blocking defeater of \mathcal{D} and vice versa, \mathcal{D} is a blocking defeater of \mathcal{C} .

Abstract frameworks can be depicted as graphs, with different types of arcs. We use the arc $(\cdots\cdots\bullet)$ to denote the subargument relation. An arrow (\longrightarrow) is used to denote proper defeaters and a double-pointed arrow (\longleftrightarrow) connects blocking defeaters. In figure 1, a simple framework is shown. Argument \mathcal{C} is a subargument of \mathcal{A} . Argument \mathcal{B} is a proper defeater of \mathcal{C} and \mathcal{D} is a blocking defeater of \mathcal{B} and viceversa.

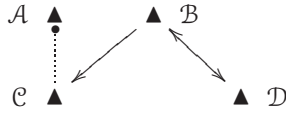


Figure 1: Defeat graph

3 ARGUMENTATION SEMANTICS

Well known semantics for abstract argumentation frameworks are based on defeat relations, usually called *attack* relations [5, 15, 3]. These formalisms assume the existence of a binary relation of attack (not necessarily symmetric) defined over the set of all possible arguments, such that if $(\mathcal{A}, \mathcal{B})$ are in the attack relation then in order to accept \mathcal{B} it is necessary to find out if \mathcal{A} is accepted or not, but not the other way around. The acceptance relation should be derived from a conflict relation between arguments and a suitable comparison criterion, and that criterion usually remains unspecified in the abstract system. This remark on the attack relation is seldom made.

We formalize these concepts using a preference relation to derive defeat relations. However, as stated in our framework, the comparison method used to evaluate pairs of arguments may not always establish a preference on conflicting arguments, and that characteristic deserves more attention. In fact, a method which is able to always decide preference between any pair of arguments in the system is desirable albeit not very realistic. Clearly, in many cases there will be incomparable arguments [15]. Therefore, another special type of defeat must be taken into account: the one derived when no preference can be established between conflictive arguments.

It is our contention that an extended semantics for argumentation will be useful. This semantics will be

based on the two defining characteristics of an argumentation system: the conflict relation between arguments and the comparison criterion used to evaluate such arguments.

The notion of conflict between two arguments \mathcal{A} and \mathcal{B} establishes that these arguments cannot be accepted together. Therefore, any set of accepted arguments could not contain arguments in conflict. A set of arguments of this kind is said to be *coherent*.

Arguments can be classified as *accepted* arguments or *non-accepted* or *rejected* arguments according to their context in the framework. Given a set of arguments S , two kinds of arguments are easily identified as accepted arguments: first, those arguments not involved in any conflict in S ; second, those arguments actually involved in a conflict, but preferred to the arguments that are in conflict with them, according to function \mathbf{R} . Both kinds of special arguments are called *defeater free* arguments. Defeater-free arguments must be accepted, since no (preferred) contradictory information is provided in the framework. Note that this classification is relative to the set in which the argument is included.

As noted before, the semantics of a conflict relation states that when an argument \mathcal{A} is accepted, any argument in $\text{Conf}(\mathcal{A})$ should be rejected. The following definition captures a subset of arguments that should be rejected in the framework. They are called *suppressed arguments*.

Definition 3 Let S be a set of arguments in $\langle \text{Args}, \mathbf{C}, \mathbf{R} \rangle$. An argument $\mathcal{A} \in S$ is said to be *suppressed* in S if one of the following cases hold: (a) there is a defeater-free argument \mathcal{B} in S such that \mathcal{B} is a proper defeater of \mathcal{A} , or (b) there is a blocking defeater \mathcal{B} of \mathcal{A} , and there is no other argument \mathcal{C} ($\mathcal{C} \neq \mathcal{A}$) in S such that \mathcal{C} is a defeater of \mathcal{B} .

Given a set S of arguments it is as easy to identify obviously suppressed arguments as it is to identify inevitably accepted ones. The function $\Upsilon : 2^{\text{Args}} \rightarrow 2^{\text{Args}}$ characterizes the set of arguments not directly suppressed in a given set, and it is defined as

$$\Upsilon(S) = \{\mathcal{A} : \mathcal{A} \in S \text{ and } \mathcal{A} \text{ is not suppressed in } S\}$$

The least fixpoint of this function, is denoted Υ^ω . By definition, no argument is suppressed in Υ^ω . Therefore, if Υ^ω is a coherent set, then any argument in Υ^ω is an *accepted* argument. The set of this arguments in Υ^ω is denoted $\Upsilon^{\omega+}$.

However, the set Υ^ω may still not be a coherent set. This is related to the presence of some special arguments involved in a *fallacy*, as discussed in the next section.

4 CONTROVERSIAL SITUATIONS AND FALLACIES

It is possible that the repeated application of operator Υ will not lead to a coherent set, as shown in [4]. In that case, some controversial situations may be found in the argumentation framework. It can be proved that if Υ^ω is not a coherent set of arguments, then there exists a cycle of defeaters. This cycle is called a *fallacy*, and the comparison criterion plays a very important role in its existence.

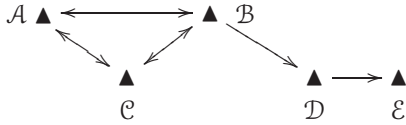


Figure 2: Controversial situation

Definition 4 An argumentation framework Φ is said to contain a fallacy if Υ^ω is not a coherent set of arguments.

The presence of fallacies is related to the lack of decision in the preference function, under some common sense conditions [15, 4]. Several preference relations between arguments are used in different argumentation systems. Most of them are based on properties observed on the structure of arguments. It has been noted that preference relations should be a total or partial pre-order on the set of arguments.

The most important premise in defeasible argumentation is that an argument must be *accepted* only when none of its defeaters are. However, no fallacious argument can exhibit this property, because at least one of its defeaters is also a fallacious argument². Therefore, any argument of this kind should not be accepted. In Figure 2 a simple argumentation framework is depicted, where five arguments are shown interacting with each other. Note that, in this framework, $\Upsilon^\omega = \Upsilon^0$, because no defeater-free arguments are present and there is a cycle of blocking defeaters, and condition 2 from definition 3 is therefore not satisfied. The progressive elimination of suppressed arguments modeled by Υ leads to a non-coherent set. Thus, we have $\Upsilon^{\omega+} = \emptyset$. Note that not every argument in Υ^ω is a fallacious argument: arguments \mathcal{D} and \mathcal{E} are not involved in the fallacy. However, they are not included in $\Upsilon^{\omega+}$ because both of them are directly or indirectly related to fallacious arguments.

The main structure analyzed in this research line is the *defeat path*.

²Because any non-fallacious defeater has been previously suppressed.

Definition 5 (Defeat path) A defeat path λ of an argumentation framework $\langle \text{Args}, \sqsubseteq, \mathbf{C}, \mathbf{R} \rangle$ is a finite sequence of arguments $[A_1, A_2, \dots, A_n]$ such that argument A_{i+1} is a defeater of argument A_i for any $0 < i < n$. The number of arguments in the path is denoted $|\lambda|$. A defeat path for \mathcal{A} is any defeat path starting with \mathcal{A} $[A, \mathcal{D}_1, \mathcal{D}_2, \dots, \mathcal{D}_n]$.

A defeat path is a sequence of defeating arguments. The length of the defeat path is important for acceptance purposes, because an argument \mathcal{A} defeated by an argument \mathcal{B} may be reinstated by another argument \mathcal{C} . In this case, it is said that argument \mathcal{C} *defends* \mathcal{A} against \mathcal{B} . Note that three arguments are involved in a defense situation: the attacked, the attacker and the defender.

If the length of a defeat path for argument \mathcal{A} is odd, then the last argument in the sequence is playing a *supporting* or *defender* role. If the length is even, then the last argument is playing an *interfering* or *attacker* role [13, 6].

The notion of defeat path is very simple and only requires that any argument in the sequence must defeat the previous one. Under this unique constraint, which is the basis of argumentation processes, it is possible to obtain some controversial structures, when pieces of information are reinserted in the sequence [4]. The initial idea of restricting the inclusion of arguments previously considered in the sequence is not enough. Some well-formed structure must be devised. In the next section we explore these ideas.

5 PROGRESSIVE DEFEAT PATHS

Conflict relations are propagated through superarguments: if \mathcal{A} and \mathcal{B} are in conflict, then \mathcal{A}^+ and \mathcal{B} are also conflictive arguments. On the other hand, whenever two arguments are in conflict, it is always possible to identify conflictive subarguments. This notion can be extended to defeat relations. Let \mathcal{A} and \mathcal{B} be two arguments such that \mathcal{B} is a defeater of \mathcal{A} . Then both arguments are in conflict and $\mathcal{A} \not\sqsubseteq \mathcal{B}$. There may exist a non-trivial subargument $\mathcal{A}_i \sqsubseteq \mathcal{A}$ such that $(\mathcal{B}, \mathcal{A}_i) \in \mathbf{C}$. It is clear, as \mathbf{R} is monotonic, that $\mathbf{R}(\mathcal{B}, \mathcal{A}_i) \neq \{\mathcal{A}_i\}$, and therefore \mathcal{B} is also a defeater of \mathcal{A}_i . Thus, for any pair of conflictive arguments $(\mathcal{A}, \mathcal{B})$ there is always a pair of conflictive arguments $(\mathcal{C}, \mathcal{D})$ where $\mathcal{C} \sqsubseteq \mathcal{A}$ and $\mathcal{D} \sqsubseteq \mathcal{B}$. This underlying cause of a conflict relation between two arguments due to the inheritance property is called *core conflict*. This leads to the notion of *disputed subargument*.

Definition 6 (Disputed subargument) Let \mathcal{A} and \mathcal{B} be two arguments such that \mathcal{B} is a defeater of \mathcal{A} . A subargument $\mathcal{A}_i \sqsubseteq \mathcal{A}$ is said to be a *disputed subargument* of \mathcal{A} with respect to \mathcal{B} if \mathcal{A}_i is a *core conflict* of \mathcal{A} and \mathcal{B} .

The notion of *disputed subargument* is very important in the construction of defeat paths in dialectical processes. Suppose argument \mathcal{B} is a defeater of argument \mathcal{A} . It is possible to construct a defeat path $\lambda = [\mathcal{A}, \mathcal{B}]$. If there is a defeater of \mathcal{B} , say \mathcal{C} , then $[\mathcal{A}, \mathcal{B}, \mathcal{C}]$ is also a defeat path. However, \mathcal{C} should not be a disputed argument of \mathcal{A} with respect to \mathcal{B} , as circularity is introduced in the path. Even more, \mathcal{C} should not be an argument that *includes* that disputed argument, because that path can always be extended by adding \mathcal{B} again. It is possible to define a *defeat domain*, a set of valid arguments to add to a defeat path, discarding controversial arguments. The function $D^k(\lambda)$ [4] denotes the set of arguments that can be used to extend the defeat path λ at stage k , *i. e.*, to defeat the argument \mathcal{A}_k . Choosing an argument from $D^k(\lambda)$ avoids the introduction of previous disputed arguments in the sequence. It is important to remark that if an argument including a previous disputed subargument is reintroduced in the defeat path, it is always possible to reintroduce its original defeater. Therefore, in order to avoid controversial situations, any argument \mathcal{A}_i of a defeat path λ should be in $D^{i-1}(\lambda)$. Selecting an argument outside this set implies the repetition of previously disputed information. Those defeat paths in which $\mathcal{A}_i \in D^{i-1}(\lambda)$ for all $1 \leq i \leq n$. The following definition characterizes well structured sequences of arguments, called *progressive defeat paths*. Progressive defeat paths are free of circular situations and guarantees progressive argumentation, as desired on every dialectical process. One of the important consequences of this formalization is that it is possible to include a subargument of previous arguments in the sequence, as long as it is not a disputed subargument. This is allowed in abstract argumentation, as subarguments not involved in any conflict can always participate several times in any argumentation line.

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