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Icarus 191 (2007) 394–396

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Note

Envelope instability in giant planet formation

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Received 30 March 2007; revised 22 June 2007

Abstract

We compute the growth of isolated gaseous giant planets for several values of the density of the protoplanetary disk, several distances from the central star and two values for the (fixed) radii of accreted planetesimals. Calculations were performed in the frame of the core instability mechanism and the solids accretion rate adopted is that corresponding to the oligarchic growth regime. We find that for massive disks and/or for protoplanets far from the star and/or for large planetesimals, the planetary growth occurs smoothly. However, notably, there are some cases for which we find an envelope instability in which the planet exchanges gas with the surrounding protoplanetary nebula. The timescale of this instability shows that it is associated with the process of planetesimals accretion. The presence of this instability makes it more difficult the formation of gaseous giant planets.

Keywords: Accretion; Planetary formation; Planetesimals

The currently favored scenario to account for the formation of gaseous giant planets is the so-called core instability mechanism (Mizuno, 1980; Bodenheimer and Pollack, 1986; Pollack et al., 1996). In the frame of this hypothesis, planets are initially formed by accretion of solid material onto a very small solid core. As the core grows, it binds an increasing amount of gas from the protoplanetary nebula inside which it is immersed. When the core mass has grown to few tens of Earth masses and bound a similar amount of gas, runaway gas accretion begins, making the protoplanet to grow up to its final mass. Here we shall work assuming the core instability mechanism.

In most calculations of giant planet formation by the core instability mechanism, the rapid growth regime for the core of Greenzweig and Lissauer (1992) is assumed (e.g., Pollack et al., 1996; Hubickyj et al., 2005; Alibert et al., 2005). In our model (Fortier et al., 2007) we adopt the growth model of Ida and Makino (1993) where the protoplanet–planetesimals interaction is considered to be in the dispersion-dominated regime and the particle-in-a-box approximation is valid to calculate the solids accretion rate. The planetesimal r.m.s. velocities are assumed to be in an equilibrium state, given by the balance between the excitation due to the protoplanet gravity and the damping by the drag in the nebular gas (as in Thommes et al., 2003). Therefore, the formation timescale of the planet's core is that of the oligarchic growth regime (Fortier et al., 2007) being longer than the one found, e.g., in Pollack et al. (1996).

The calculations to be presented below were performed with the code described in Benvenuto and Brunini (2005) that has been updated to include the oligarchic growth regime for the core by Fortier et al. (2007). Our code employs the Henyey technique with an adaptive grid to compute the envelope structure of the planet. Usually, our models have few thousand mesh points and the typical time step is of a hundred years. We adopt the equation of state of Saumon et al. (1995), grain opacities from Pollack et al. (1985) and, at higher temperatures, we consider data from Alexander and Ferguson (1994) and Rogers and Iglesias (1992). We include energy deposition by planetesimals accretion in the planetary envelope and the effective capture cross section of the protoplanet is calculated considering the enhancement produced by gas drag. The bulk density of planetesimals is taken to be 1.5 $\mathrm{g\,cm^{-3}}$ and the density of the core is held constant throughout the calculation and equal to 3 $\mathrm{g\,cm^{-3}}$. The planet's feeding zone is assumed to extend four Hill's radii on each side of the planet's orbit. The total mass of planetesimals in the feeding zone is assumed to equal the initial mass of planetesimals in the (current) accretion zone minus the amount that has already been accreted by the protoplanet. In our computations, planetesimals are not allowed to migrate in and out of the feeding zone, planetary migration caused by disk torques is not addressed and ejection of planetesimals by the forming planet is not taken into account.

We computed the growth of planets for several values of the surface density of the disk and distances from the central star of the system. Specifically, we performed runs considering four values for the density of the protoplanetary disk: 10, 7, 5, and 3 times that corresponding to the model of the Minimum Mass Solar Nebula (hereafter MMSN) of Hayashi (1981). We have adopted

$$\Sigma_s = f_d 40 (a/1\ \mathrm{AU})^{-3/2}\ \mathrm{g\,cm^{-2}} \quad (a > a_{\mathrm{snow}}),$$

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0019-1035/$ – see front matter
doi:10.1016/j.icarus.2007.06.019

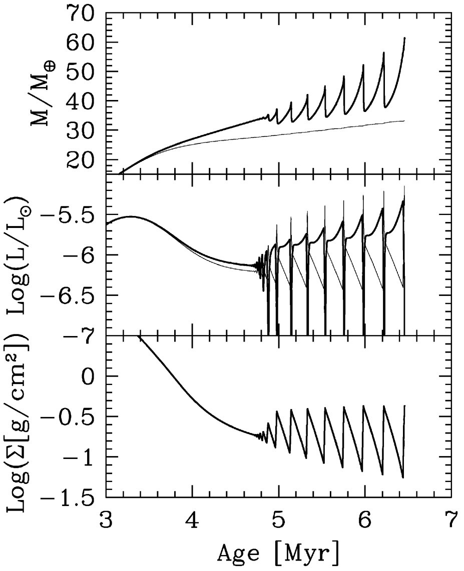

Fig. 1. Main characteristics of the growth of a protoplanet immersed in a 5 MMSN at a 5.2 AU from the central star of the system. Upper panel shows the total (thick solid line) and the core (thin solid line) mass of the protoplanet. The difference between these curves corresponds to the amount of gas bound to the protoplanet. Mid panel depicts the total (thick solid line) and accretion (thin solid line) luminosities. Lower panel displays the surface abundance of planetesimals inside the feeding zone of the protoplanet. In this case, the planetesimal have $r = 10$ km. For further discussion see main text.

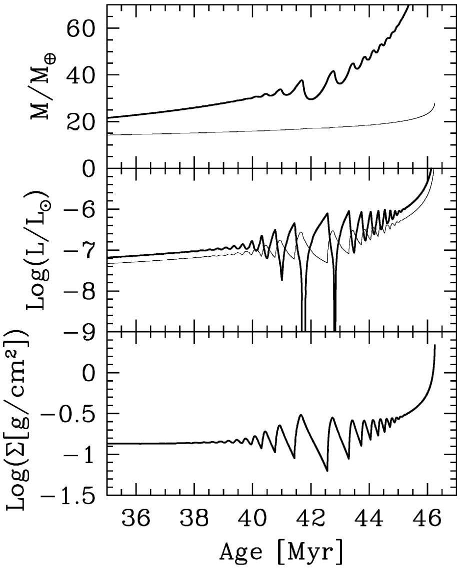

Fig. 2. Same as Fig. 1 but for the case of a protoplanet growing immersed in a 3 MMSN at a 5.2 AU from the central star of the system. In this case we considered planetesimals with $r = 100$ km. Notice the qualitative differences of these results as compared to those of Fig. 1. In this case, the instability sets in but after few cycles, further accretion reaches the final runaway conditions suppressing the instability.

$$\Sigma_g = f_d 2 \times 10^3 (a/1\ \mathrm{AU})^{-3/2}\ \mathrm{g\,cm^{-2}},$$

where Σ_s (Σ_g) is the solid (gas) surface density, f_d is the enhancement factor with respect to the MMSN, a is the distance from the central star and a_{snow} is the location of the snow line, set at 2.7 AU. In all the cases considered here the feeding zone always remains behind the snow line. We computed models at four different distances from the central star: 4, 5.2, 6, and 8 AU. Also, we considered two fixed values for the radius of planetesimals: 10 and 100 km.

Apart from the difference in our accretion model (the oligarchic growth regime for the accretion rate of planetesimals) with respect to the one used in previous works (Pollack et al., 1996; Hubickyj et al., 2005; Alibert et al., 2005) that imply (a priori) expectable differences in the evolution of the main characteristics of the planetary growth (see Fortier et al., 2007), in some cases we find an unexpected unstable behavior of the gaseous envelope. For example, in Fig. 1 we show the main characteristics of the growth of a planet immersed in a 5 MMSN, populated by planetesimals with a radius of 10 km at a distance of 5.2 AU. The upper panel shows the evolution of the core and total mass of the planet, mid panel displays the total and solids accretion luminosities while lower panel depicts the planetesimals surface density inside the feeding zone. In this case, at a time of 4.7 Myr, when the planetesimals surface density Σ in the feeding zone reaches a (critical) value of $\Sigma = 0.19\ \mathrm{g\,cm^{-2}}$ and the core mass is of $\approx 28\ M_\oplus$, we find that the envelope starts to undergo a kind of oscillatory instability in which the mass of the gaseous envelope initially varies by few percents but tends to grow to a very large amplitude. For the cases in which no instability occurs, the whole process of protoplanet growth is rather standard, and we shall not discuss them any further in this paper.

In Fig. 2 we show the growth of a planet immersed in a 3 MMSN with planetesimals of 100 km at a distance of 5.2 AU. This case is qualitatively different from the results shown in Fig. 1. Here, the instability sets in, but the continuous growth of the core makes it the amplitude of the oscillations to reach a maximum and, afterward, to damp and even to vanish. Subsequently, a standard runaway growth establishes. Even though the age at which this transition occurs is unrealistically large, this shows that at least in some cases, the envelope instability delays the planetary formation but does not inhibit it at all.

We find that there is some kind of competition between this envelope instability and the onset of the runaway gas accretion. Let us define the isolation mass, M_{iso}, as the core mass of the planet in the limit that it accretes all the solid mass in its feeding zone. M_{iso} is given by

$$M_{\mathrm{iso}} = 0.65 f_d^{3/2} \left(\frac{a}{1\ \mathrm{AU}}\right)^{3/4} (1+\alpha)^{1/2} M_\oplus, \tag{1}$$

where α is the mass of the envelope in terms of the core mass. Near depletion of the feeding zone (when the mass of the core is close to the isolation mass), the energy released by the accreted planetesimals is not enough to sustain the gaseous envelope against contraction of the envelope.[4] Then, the envelope contracts and the planet increases its mass. As the mass of the envelope increases, the feeding zone expands, embracing regions of the disk that are still not depleted of planetesimals. This forces an increase in the accretion luminosity. As a consequence, the envelope expands, and a fraction of the gas becomes unbound from the planet. Hence, as the mass of the protoplanet decreases so do the available planetesimals, the luminosity due to accretion falls down, the envelope contracts and the planet again increases its mass, and the cycle is restarted. Then, the envelope instability of the growing planet has been established. On the other hand, if the feeding zone is able to provide enough solid material to reach the core mass for the onset of the runaway growth of the envelope before depletion, then the planet gets the runaway conditions and the whole formation process is completed in stable conditions.

In Table 1, we summarize the behavior of the models as a function of the disk density and the distance from the central star, for planetesimals with radii of 10 and 100 km. Notice that large planetesimals tend to favor the stability of envelope. Nevertheless, the major contribution to the solid mass in a real disk will be given by small planetesimals which, in turn, are the most efficiently accreted. In the oligarchic regime the solid accretion rate depends on planetesimal size as large planetesimals have higher relative velocities than small ones. Also, the enhancement of the capture cross section of the planet by the envelope is smaller for large planetesimals. Hence, the accretion luminosity due to large planetesimals is lower than for the small ones. This favors the onset of the instability in the case of small planetesimals. On the other hand, as M_{iso} grows with the distance from the central star (see Fig. 3), this naturally accounts for the fact

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[4] Notice that such a contraction due to a deficit in the accretion rate of planetesimals has been previously found by other researchers; see Hubickyj et al. (2005).

Table 1
Behavior of the models as a function of the density of the disk and the distance from the central star, for planetesimals with radii of 10 km and 100 km. S (U) means a stable (unstable) behavior of the envelope

Disk [MMSN]	4 AU		5.2 AU		6 AU		8 AU	
	10 km	100 km	10 km	100 km	10 km	100 km	10 km	100 km
10	S	S	S	S	S	S	S	S
7	U	S	S	S	S	S	S	S
5	U	U	U	S	U	S	S	S
3	U	U	U	U	U	S	U	S

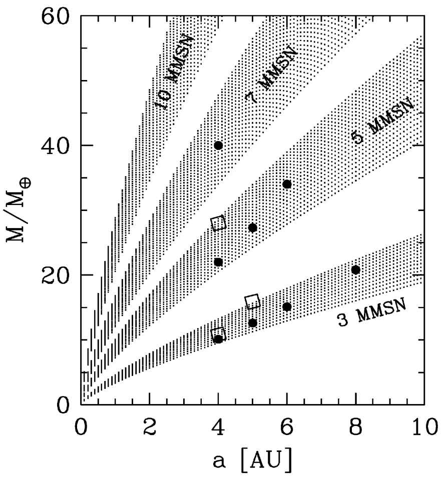

Fig. 3. The isolation masses for several values of the disk densities for protoplanets with gaseous envelopes corresponding to α in the interval $[0, 1]$ (see Eq. (1)). For comparison, we include the values of the core mass at which the onset of the envelope instability occurs. Filled dots (hollow squares) represents the case of models computed with planetesimals with a radius of 10 km (100 km).

that the models are stable at large distances from the center of the disk. In addition, we have made runs with exactly the same conditions of models J1 and J3 of Pollack et al. (1996). In the J1 case we get a behavior similar to the one shown in Fig. 2 whereas for the case J3 the envelope shows a stable behavior.

Remarkably, the envelope instability is *not* a free radial pulsation like that found in some variable stars (e.g., Cepheids). The timescale of $\approx 10^5$ yr is far larger than that expected for a radial pulsation which is of the order of $R_p/C_s \sim$ hours (where R_p and C_s are the planetary radius and the velocity of sound). This is due to the fact that the envelope instability is a forced oscillation, and its timescale is associated with the accretion process.

As previously remarked, the envelope instability is not periodic. Thus, in order to study the excitation and damping mechanisms we must be careful. For this purpose, it is fairly standard to compute the work integral (see, e.g., Clayton, 1968, Section 6.10)

$$W = \int \mathrm{d}M_r \oint_{\text{cycle}} P\,\mathrm{d}V. \tag{2}$$

In the P–V plane, if a portion of material evolves along cycles in clockwise (counterclockwise) sense, it excites (damps out) the instability. We find that internal layers of the planetary envelope excite the instability while those located outside damps it out. The fact that internal layers of the envelope excite the instability is expectable, because this is where most of the planetesimal accretion energy is released. Notably, the integration of the work done over the whole planet gives a negative result. This means that the planet tends to *damp* the instability.

The instability of the envelope should be considered as a process that tends to make the formation of giant planets more difficult when considering low mass disks and/or protoplanets close to the central star of the system and/or accretion of small sized planetesimals. In any case, we should remark that there are several effects operating in nature that have been neglected here. For example, planetesimal migration will affect the population of the feeding zone of the protoplanet and consequently, depending on the configuration of the system, it should be able to start or suppress the instability. Also, the presence of other objects growing simultaneously will also affect the population of the feeding zone significantly. The same kind of effect should be expected if we consider planetary migration. Also a distribution of sizes of planetesimals should have important effects because of the expected differences in the effective capture radius of the protoplanet. An exploration of these points is in order and will be the subject of future research.

Acknowledgment

A.B. acknowledges ANPCyT.

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