Towards an Abstract Characterization of the Subargument Relation

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Abstract

Dung's classic framework is formed by abstract arguments and a binary relation denoting attacks between arguments. Several semantic elaboration and extensions based on this framework are present in the literature. The notion of subargument, however, was not widely studied as an abstract concept although it is an important part of fully implemented argument systems. In this paper we introduce the characterization of properties of a sensible subargument relation in abstract argumentation frameworks.

1 Introduction

Since Dung's seminal work [6], a broad community has grown, exploring the capabilities of its abstract framework, and providing new semantics for accepting arguments. A classical argumentation framework is formed by a pair (A, R) where A is a set of arguments and R is a set of attacks between arguments in A. In this fashion, arguments are an inference black box that provides a reason for a particular claim from certain premises. The consideration of arguments as abstract entities, with no reference to their inner structure, proved to be a suitable basis to study high-level properties of general argumentation. Since its introduction in [6], this framework was used in its original form by several authors [8, 3, 12]. Nonetheless, a part of the abstract argumentation community went even further, by extending Dung's framework to encompass new capabilities [1, 2, 10, 14]. An important element in defeasible argumentation, which has not been widely explored as an extension in the abstract framework, is the consideration of *subarguments*, *i. e.,* inner portions of an argument that are arguments by themselves. subarguments constitute a vital part of their enclosing structure, called *superarguments*. As subarguments are also arguments, they verify the same properties as any other argument in the system.

Formalizing subarguments in an abstract level is not an easy task. The main difficulty is that subarguments relate to the inner structure of arguments, and thus some degree of abstraction in the treatment of arguments is lost. In this paper we address the problem of defining an abstract subargument relation towards an extended argumentation framework. The formalization is centered in a set of properties and characterizations of a sensible subargumentation.

In the following section we discuss the general notion of subarguments. In Section 3 basic postulates about argumentation are introduced. In Section 4 the closure of sets of arguments regarding the subargument relation is presented. In Section 5 several semantics notions are considered in the context of subarguments. Finally the conclusions and related work are shown in Section 6

2 Subarguments

An abstract formalization of the notion of subargument is interesting because this concept is already present in several argumentation theories [15, 17, 16, 14] and in fully specified argumentation systems [7, 13]. The abstract approach leads to a general study of this special structural relation between arguments. Perhaps the most general definition of subargument is as follows:

Definition 1 (Subargument) *Let $\mathcal{A}$ be an argument. A subargument of $\mathcal{A}$ is an inner structural piece of $\mathcal{A}$ that can be considered an argument by itself.*

The most important open issue in the previous definition is the meaning of the term *"inner structural piece"*. Clearly, the notion of *piece* is strongly related to how the arguments are constructed, which in turn is related to the underlying logic. Thus, given the desired level of abstraction, the concept of *piece* will not be formally specified in this work. It is sufficient to state that a subargument $\mathcal{A}$ is somehow included in (or taking part of) other argument $\mathcal{B}$, with two important considerations:

- the inclusion is morphologic (*i. e.,* argument $\mathcal{A}$ is a structural piece of $\mathcal{B}$)
- the inclusion is complete, in the sense that no part of $\mathcal{A}$ is not included in $\mathcal{B}$ (*i. e.,* argument $\mathcal{A}$ is an inner piece of $\mathcal{B}$)

Throughout this paper we will use the following binary relation to denote subarguments, by the symbol commonly used in [17, 10].

Definition 2 (Subargument Relation) *A subargument relation is a binary relation $\sqsubseteq$ between arguments, such that $\mathcal{A}\sqsubseteq\mathcal{B}$ if $\mathcal{A}$ is a subargument of $\mathcal{B}$. It is also said that argument $\mathcal{B}$ is a superargument of $\mathcal{A}$.*

In previous works by the authors, two forms of representing subarguments are used, as shown in Figure 1, meaning that $\mathcal{B}$ is a subargument of $\mathcal{A}$. The first one is used in complex argument graphs, while the second (triangles resembling derivation trees) is used to emphasize the structural relation between subarguments.

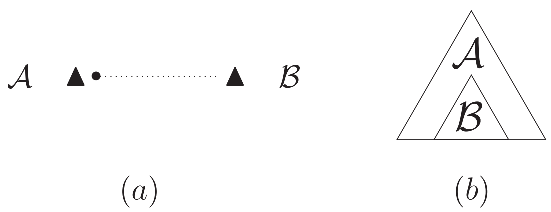

Figure 1: Argument $\mathcal{B}$ is a subargument of $\mathcal{A}$

Definition 3 (Direct subargument) *Let $\mathcal{A}$ be an argument. An argument $\mathcal{B}$ is a direct subargument of $\mathcal{A}$ if $\mathcal{B} \sqsubseteq \mathcal{A}$ and there is no other argument $\mathcal{C}$ such that $\mathcal{B} \sqsubseteq \mathcal{C} \sqsubseteq \mathcal{A}$.*

We will often refer to all the subarguments of a given argument. This is formalized in the following definition.

Definition 4 (Set of subarguments) *Let $\mathcal{A}$ be an argument. The set of all subarguments of $\mathcal{A}$ is denoted $\texttt{subs}(\mathcal{A})$. The set of all direct subarguments of $\mathcal{A}$ is denoted $\texttt{subs}_d(\mathcal{A})$.*

As an argument is considered a subargument of itself, then clearly $\mathcal{A} \in \texttt{subs}(\mathcal{A})$. Thus, it is never the case that $\texttt{subs}(\mathcal{A}) = \{\}$ for any argument $\mathcal{A}$. In the same way, an argument is a superargument of itself. The notion of *direct subarguments* of a given $\mathcal{A}$ refers to a somehow minimal set of arguments needed to support $\mathcal{A}$. Therefore, $\mathcal{A} \notin \texttt{subs}_d(\mathcal{A})$.

Definition 5 (Atomic argument) *An argument $\mathcal{A}$ is said to be atomic if $\texttt{subs}(\mathcal{A}) = \{\mathcal{A}\}$.*

An atomic argument has no subarguments other than itself. This notion denotes an elemental piece of reasoning and there are several examples in the literature. The difference, of course, lies on the underlying logic and how the arguments are structurally composed. For instance, when the construction of arguments is based on deductive logic as in [13, 17], an atomic argument may be a simple fact. In that case, an argument is atomic if it is a particular element in the language. Thus, atomicity of arguments can be identified by examining the structure of individual arguments and then this property is independent of the overall set of arguments, *i. e.,* the total framework. On the other hand, in DeLP [7], an argument $\mathcal{A}$ may include several defeasible rules (*i. e.,* is not a simple fact and it includes several steps of reasoning). However, if there is no other argument using a subset of those rules, then $\mathcal{A}$ is an atomic argument. Thus, in DeLP atomicity of arguments is determined by the lack of arguments sharing rules, not by the solely structure of single arguments. Clearly then, atomic arguments are not necessarily "*small*" arguments. The existence of atomic arguments is addressed by the following proposition.

Proposition 1 *Given a non-empty set of arguments $\mathbb{A}$ and a subargument relation defined over $\mathbb{A}$. If $\mathbb{A}$ is finite, then the set of atomic arguments is non-empty.*

Proof: Let assume an empty set of atomic arguments from a non-empty finite set of arguments $\mathbb{A}$. If the set of atomics is empty, then every argument in $\mathbb{A}$ has at least one subargument, otherwise the remaining ones will be atomic. Each of these subarguments is an argument in $\mathbb{A}$. In order for none of them to be atomic, they have to have at least one subargument, again. And each one will be also in $\mathbb{A}$. This analysis yields an infinite set $\mathbb{A}$, which violates the hypothesis. The contradiction arose from the sole extra assumption of an empty set of atomics $\square$

The applicability of an infinite set of arguments can be put under discussion, in the same way logical systems with an infinite set of rules could. However, the reader might think of many logic-based argumentation systems that define arguments as a particular kind of minimal consistent derivation upon an instance of the rules used, and since rules might contain functions, the range of instances is potentially infinite. This line of reasoning can be misleading, because at any time the argumentation system is going to handle a finite set of instances, and this is what our abstract representation is aiming for. Following the usual fashion in abstract frameworks, we assume a finite set of arguments. Next, we define the basic notion of *subargumentation-capable argumentation framework*, which extends the usual abstract argumentation framework to be able to handle subarguments.

Definition 6 (Subargumentation-capable argumentation framework) *A subargumentation-capable argumentation framework (ScAF) is a triplet* $(\mathbb{A}, \mathbb{R}, \sqsubseteq)$ *where* $\mathbb{A}$ *is a set of arguments,* $\mathbb{R}$ *is a binary relation defined over* $\mathbb{A}$ *denoting attacks between arguments, and* $\sqsubseteq$ *is the subargument relation between arguments of* $\mathbb{A}$*.*

The following proposition establishes the relation between the ScAF and Dung's framework, showing that the former is an extension of the latter.

Proposition 2 *Let* $S = (\mathbb{A}, \mathbb{R}, \sqsubseteq)$ *be a ScAF. If every argument in* $\mathbb{A}$ *is atomic (*i. e., $\sqsubseteq = \emptyset$*), then* $S' = (\mathbb{A}, \mathbb{R})$ *is a Dung's classical framework.*

A high level of abstraction is desirable towards universality, and then no strong reference to the structural composition of arguments can be made. It is possible, however, to characterize properties and postulates that a good, sensible subargument relation should satisfy. This is the objective of this article, and the corresponding analysis is made in the following sections.

3 Subargumentation postulates

As stated before, the notion of subargumentation can be modeled by its behaviour in the system and by its structural composition. This composition, however, cannot be referenced in an abstract approach. Therefore, in this section we provide a set of properties a *sensible* subargument relation should satisfy. We begin with a set of basic postulates that takes into account that subargumentation is naturally a binary relation between arguments.

Postulate 1 *The subargument relation is a partial order, therefore meeting the properties of reflexivity, antisymmetry and transitivity.*

This is a natural postulate. The subargument relation is reflexive, because any argument is considered a subargument of itself. It is antisymmetric, because if $\mathcal{A}$ is a subargument of $\mathcal{B}$, argument $\mathcal{B}$ can be a subargument of $\mathcal{A}$ only if $\mathcal{A} = \mathcal{B}$. It is transitive because if argument $\mathcal{A}$ is a subargument of $\mathcal{B}$ which in turn is a subargument of $\mathcal{C}$, then clearly also $\mathcal{A}$ is taking part of the structure of argument $\mathcal{C}$ and thus $\mathcal{A}$ is a subargument of $\mathcal{C}$.

Proposition 3 *As the subargument relation is transitive, then* $\mathtt{subs}(\mathcal{B}) \subseteq \mathtt{subs}(\mathcal{A})$ *for any argument* $\mathcal{B} \sqsubseteq \mathcal{A}$*.*

Proposition 3 makes explicit the fact that an argument may have several levels of subarguments. Some traditional properties of arguments can be formalized in terms of subarguments, as stated in the following postulate.

Postulate 2 *Let* $\mathcal{A}$ *be an argument in* $\mathbb{A}$ *and let* $\mathbb{R}$ *be an attack relation between arguments such that* $\mathcal{A}\mathbb{R}\mathcal{B}$ *if* $\mathcal{A}$ *attacks* $\mathcal{B}$*. An argument is required to verify* ***consistency:*** $\mathcal{A}$ *is consistent if there are no two subarguments* $\mathcal{A}_1 \sqsubseteq \mathcal{A}$ *and* $\mathcal{A}_2 \sqsubseteq \mathcal{A}$ *such that* $\mathcal{A}_1 \mathbb{R} \mathcal{A}_2$*.*

Note that the consistency postulate trivially eliminates the possibility of having a self-defeating argument (*i. e.,* $\mathcal{A}\mathbb{R}\mathcal{A}$), since an argument is a subargument of itself.

Other postulates and characterizations are related to a particular element in argumentation, the preference criteria. This preference is used to compare arguments when convenient, usually because a conflict is present.

Comparison criteria and subarguments

Comparison of arguments is an essential part of argumentation systems and some authors explicitly add preference orders in their frameworks [1, 2, 11]. It is very likely that at some point, arguments will be evaluated and compared to each other, mostly because a conflict needs to be solved. An appropriate abstract definition for a comparison criterion is as follows:

Definition 7 *[9] Given a set of arguments $\mathbb{A}$, an* argument comparison criterion $\succeq$ *is a binary relation on $\mathbb{A}$. If $\mathcal{A}\succeq\mathcal{B}$ but not $\mathcal{B}\succeq\mathcal{A}$ then $\mathcal{A}$ is preferred to $\mathcal{B}$, denoted $\mathcal{A} \succ \mathcal{B}$. If $\mathcal{A}\succeq\mathcal{B}$ and $\mathcal{B}\succeq\mathcal{A}$ then $\mathcal{A}$ and $\mathcal{B}$ are arguments with equal relative preference, or indistinguishable strength, denoted $\mathcal{A} \equiv \mathcal{B}$. If neither $\mathcal{A}\succeq\mathcal{B}$ or $\mathcal{B}\succeq\mathcal{A}$ then $\mathcal{A}$ and $\mathcal{B}$ are incomparable arguments, denoted $\mathcal{A} \bowtie \mathcal{B}$.*

Note that we are speaking of preference without reference to conflict. When comparing arguments, there are some interesting properties that can be considered under the presence of a subargument relation. For instance, an argument should not be stronger than any of its subarguments, since every subargument represents another level of defeasibility within the argument at issue. In other words, the more subarguments an argument has, the more likely is to be attacked. This was initially proposed in [17].

Definition 8 (Monotonic preference) *Let $\succeq$ be a preference order defined over $\mathbb{A}$. Then $\succeq$ is monotonically non-increasing, if $\forall \mathcal{A}, \mathcal{B} \in \mathbb{A}$ such that $\mathcal{B}\succeq\mathcal{A}_1$, $\mathcal{A}_1\sqsubseteq\mathcal{A}$ then $\mathcal{B}\succeq\mathcal{A}$.*

This is the only preference property we consider relevant for the topics addressed in this work. We do not, however, impose restrictions on specific properties. It could be the case in which the preference order should be transitive. That is, there is no sequence $\mathcal{A}\succeq\mathcal{B}\succeq \ldots \mathcal{A}$. However, this is not imposed as a requirement in this paper, since there are applications in which the preference order is not supposed to meet the property of transitivity. For instance, in decision making, a situation like the following is considered possible: $\mathcal{B}\succeq\mathcal{A}$ and $\mathcal{C}\succeq\mathcal{B}$, but $\mathcal{C}\succeq\mathcal{A}$ on a completely different basis than the principles for comparing these two arguments with $\mathcal{B}$. Furthermore, when taking a set of equally strong arguments, the transitivity of preference may also broken.

Definition 8 has an important implication. When a monotonically non-increasing preference order is used, then attackers are inherited through superarguments, as stated in the following proposition.

Proposition 4 *If $\succeq$ is monotonically non-increasing and attacker arguments are always stronger than attacked arguments, then every attacker of an argument $\mathcal{A}_1$ is also an attacker of any argument $\mathcal{A}$, such that $\mathcal{A}_1\sqsubseteq\mathcal{A}$.*

Proof: Let $\mathcal{B}$ be an argument, attacker of $\mathcal{A}_1$. This means that $\mathcal{B}$ is considered stronger than $\mathcal{A}_1$. As the preference order is monotonically non-increasing, then $\mathcal{B}$ is also considered stronger than $\mathcal{A}$. Because of this, and the fact that $\mathcal{B}$ and $\mathcal{A}$ cannot be accepted simultanously (the former is attacking a part of the latter), then $\mathcal{B}$ implicitly attacks $\mathcal{A}$ $\square$

Proposition 4 establishes a sufficient condition for an interesting property regarding an argument and its subarguments. If the preference order is monotonically non-increasing, then every attacker of an argument $\mathcal{A}$ is also an attacker of any superargument of $\mathcal{A}$. We will refer to this condition as *conflict inheritance*.

In the following section we present an elemental kind of cohesion of sets containing arguments and subarguments. We also present some operations to make sound modifications to sets of arguments.

4 Closed sets of arguments

For a set of arguments following a sensible subargument relation, a notion of consequence has to be associated, as an analogy of the logical consequence. However, there is a significant difference that we address next. The subargument relation allows us to "derive" superarguments when every subargument is present, but it also lets us pose a strong requirement: whenever an argument belongs to a set, all of its subarguments should be also in it. This is not something that could be expected on any logical system, but it does make sense in an argumentation system including subarguments, since they are a necessary, structural piece of the argument they belong to. This is formalized by the following definition.

Definition 9 (Set closed under subargumentation) *A set of arguments S is said to be closed under subargumentation, if for every $\mathcal{A} \in S$, then $\texttt{subs}(\mathcal{A}) \subseteq S$.*

Any set of arguments presented as a semantic extension should be closed under subargumentation, since subarguments are vital parts of their superarguments. If an argument $\mathcal{A}$ belongs to a certain set, *i. e.,* it is collectively accepted under some particular criteria, then all of its subarguments should also belong to that set, because an argument is accepted as a whole and all its parts are accepted. Therefore, there are arguments that could be incorporated into a set of arguments without violating closure. These arguments are those that have every subargument within that set. We say that an argument verifying this property can be derived from the set, as formalized next.

Definition 10 (Argument Derivation) *Let S be a set of arguments and $\mathcal{A}$, an argument. Set S is able to derive $\mathcal{A}$, denoted $S \overset{\triangle}{\rightarrow} \mathcal{A}$, if and only if $\texttt{subs}_d(\mathcal{A}) \subseteq S$.*

Definition 11 (Minimal Argument Derivation) *An argument derivation $S \overset{\triangle}{\rightarrow} \mathcal{A}$ is said to be minimal if there is no $S' \subset S$ such that $S' \overset{\triangle}{\rightarrow} \mathcal{A}$.*

Minimal derivations allow us to isolate the set of arguments needed to derive an argument; thus, no superfluous arguments are considered. It is interesting to note that the inverse for a minimal proper derivation is the $\texttt{subs}_d$ function.

Proposition 5 *If $\mathcal{A}$ is an atomic argument then $\{\} \overset{\triangle}{\rightarrow} \mathcal{A}$.*

That is, by definition an atomic argument lacks subarguments and thus the minimal derivation is minimally achieved from the empty set.

Proposition 6 *If a set of arguments S is closed under subargumentation, then for every $\mathcal{A} \in S$ it holds $S \overset{\triangle}{\rightarrow} \mathcal{A}$.*

Proof: As S is closed under subargumentation, then all the subarguments of any $\mathcal{A} \in S$ are in S. In particular, every direct subargument of $\mathcal{A}$ is in S and thus $S \overset{\triangle}{\rightarrow} \mathcal{A}$ $\square$

Proposition 7 $\texttt{subs}(\mathcal{A}) \overset{\triangle}{\rightarrow} \mathcal{A}$.

Proof: Clearly, for any argument $\mathcal{A}$, it holds that $\texttt{subs}_d(\mathcal{A}) \subseteq \texttt{subs}(\mathcal{A})$*, and thus* $\texttt{subs}(\mathcal{A}) \stackrel{\triangle}{\rightarrow} \mathcal{A}$ $\square$

Closure under subargumentation means that all the subarguments of every argument in a set are also included in that set. We are also interested in another form of closure, based on argument derivation. We call this property *completeness*.

Definition 12 (Complete set of arguments) *A set of arguments S is said to be complete, if for every $\mathcal{A}$ such that $S \stackrel{\triangle}{\rightarrow} \mathcal{A}$ then $\mathcal{A} \in S$.*

The union of sets closed under subargumentation is also a set closed under subargumentation. The union of complete sets may not be a complete set.

It is possible to define derivation as an function over sets of arguments, based on Definition 10.

Definition 13 (Set Derivation) *Let S be a set of arguments. The set derivation operator $\stackrel{\triangle}{\Rightarrow}$ is defined as $S \stackrel{\triangle}{\Rightarrow} S_1$ if and only if $S_1 = \{\mathcal{A} : S \stackrel{\triangle}{\rightarrow} \mathcal{A}\}$.*

Proposition 8 *The union of sets closed under subargumentation is also a set closed under subargumentation.*

Definition 14 (Sensible Argumentation Framework) *A subargumentation-capable argumentation framework $AF = (\mathbb{A}, \mathbb{R}, \sqsubseteq)$ is said to be **sensible** iff AF is closed under subargumentation and verifies conflict inheritance.*

Change over the set of arguments

Argument semantics is about characterizing sets of arguments according to different rational positions. We have previously defined properties that a set of arguments should satisfy, towards a sensible subargument notion. In order to maintain closure of the subargument relation, we should perform safe operations over any set of arguments. For instance, when subtracting an argument, the set should be kept closed, and so it should be when adding an argument. To understand the reason behind these change operations, recall that, for any argument in a sensible subargumentation setting, we require all of its subarguments to hold along with it.

Definition 15 (Argument Safe Subtraction) *Let S be a set of arguments and $\mathcal{A}$ an argument in S. The operator $\triangleq$ is defined as $S \triangleq \mathcal{A} = S - Sp(\mathcal{A})$ where $Sp(\mathcal{A})$ is the set of all superarguments of $\mathcal{A}$.*

Definition 15 states that when removing (for semantic reasons) an argument $\mathcal{A}$ from a set S, then also every superargument of $\mathcal{A}$ should be removed. This is because an argument is accepted as a whole, and its acceptance naturally implies the acceptance of every subargument. Thus, if an argument $\mathcal{A}$ is not taken into account under a semantic notion, then every argument that includes $\mathcal{A}$ should not be considered under the same semantic notion.

Remark 1 *If $\mathcal{A}$ has no superarguments, then $S \triangleq \mathcal{A} = S \setminus \{\mathcal{A}\}$.*

The argument subtraction as defined in Definition 15 preserves closure under subargumentation. This is formalized in the following proposition.

Proposition 9 *If S is closed under subargumentation, then $S \triangleq \mathcal{A}$ is closed under subargumentation.*

Proof: Suppose $S \triangleq \mathcal{A}$ is not closed under subargumentation. This means that, for an argument $\mathcal{B} \in S \triangleq \mathcal{A}$, $\mathtt{subs}(\mathcal{B}) \nsubseteq S \triangleq \mathcal{A}$. Argument $\mathcal{B}$ cannot be a superargument of $\mathcal{A}$ as it belongs to $S \triangleq \mathcal{A}$. If $\mathcal{B}$ is a subargument of $\mathcal{A}$ then it is not affected by the subtraction of $\mathcal{A}$ and thus $\mathtt{subs}(\mathcal{B}) \subseteq S \triangleq \mathcal{A}$, which is a contradiction. Thus, $\mathcal{B}$ cannot be related to $\mathcal{A}$. But then $\mathtt{subs}(\mathcal{B}) \subseteq S \triangleq \mathcal{A}$ which is a contradiction. Therefore, $S \triangleq \mathcal{A}$ is closed under subargumentation $\square$

This proposition states an important result: when changing a closed under subargumentation set of arguments through the argument safe subtraction, it maintains the closure of the set. Now that the argument safe subtraction is defined, we can use it to define the *framework safe subtraction*; that is, now we can remove a subset of the arguments in the framework, and propagate that change over the attack relation.

Definition 16 (Framework Safe Subtraction) *Let $AF = (\mathbb{A}, \mathbb{R}, \sqsubseteq)$ be a framework and $\mathbb{A} \triangleq \mathcal{A}$, an argument safe subtraction of $\mathcal{A}$ over $\mathbb{A}$. The* ***framework safe subtraction*** *of AF wrt. $\mathcal{A}$ is $AF - \mathcal{A} = (\mathbb{A} \triangleq \mathcal{A}, \mathbb{R}^-)$, where $\mathbb{R}^- = \{(\mathcal{B}, \mathcal{C}) \in \mathbb{R} \mid \mathcal{B} \neq \mathcal{A}, \mathcal{C} \neq \mathcal{A}\}$.*

Analogously, when we add an argument to a set, we should include all of its subarguments, in order to preserve the closure. Remember that an argument is accepted as a whole, and then also its subarguments are all accepted. This motivates the following definition.

Definition 17 (Argument Safe Addition) *Let S be a set of arguments and $\mathcal{A}$, an argument. The operator $\overset{\triangle}{+}$ for an argument safe addition is defined as $S \overset{\triangle}{+} \mathcal{A} = S \cup \mathtt{subs}(\mathcal{A})$.*

Proposition 10 *$S \overset{\triangle}{+} \mathcal{A} = S \cup \{\mathcal{A}\}$ iff:*

- *$\mathcal{A} \in S$ and S is closed under subargumentation, or*
- *$S \overset{\triangle}{\rightarrow} \mathcal{A}$ and $\mathcal{A}$ is atomic.*

Proposition 11 *If S is closed under subargumentation, then $S \overset{\triangle}{+} \mathcal{A}$ is closed under subargumentation.*

Again, this important result ensures that a closed set of arguments will remain that way after performing a safe addition.

As the subtraction operator removes subarguments and the addition operator adds superarguments in a set, the order in which both operations are applied is relevant.

Proposition 12 *$(S \triangleq \mathcal{A}) \overset{\triangle}{+} \mathcal{A} = S$ if and only if $\mathcal{A}$ is an atomic argument without superarguments.*

The definition for the argument safe addition allows us to build a framework safe addition, analogously to the framework safe subtraction. Framework addition, however, implies the inclusion of new arguments and the definition of new attack relations.

Definition 18 (Framework Safe Addition) *Let $AF = (\mathbb{A}, \mathbb{R}, \sqsubseteq)$ be a framework and $\mathbb{A} \overset{\triangle}{+} \mathcal{A}$, an argument safe addition of $\mathcal{A}$ over $\mathbb{A}$. The* ***framework safe addition*** *of AF wrt. $\mathcal{A}$ is $AF + \mathcal{A} = (\mathbb{A} \overset{\triangle}{+} \mathcal{A}, \mathbb{R})$.*

Note that the framework safe addition only adds a new argument looking for the closure of the resulting set. It would be interesting to perform similar operations, *i. e.,* additions and subtractions, of attacks between arguments. Moreover, this would motivate change operations at a framework level, like the merge introduced in [4]. These mechanisms, however, fall beyond the scope of this article.

5 Argumentation semantics and subarguments

Abstract argumentation is a pathway to study argumentation semantics, *i. e.,* the understanding of the consequences of a set of attacking arguments. These consequences (or outcome) of an attack scenario may obbey different rational principles, according to specific purposes. The most elemental collectively acceptable sets of arguments, called *extensions*, were proposed in [5], but other authors proposed new semantic elaborations.

Regarding acceptance of arguments, there is an important premise that any subargumentation-capable argumentation system must satisfy. It is related to the fact that arguments are accepted as a whole, including any information exposed by them.

Postulate 3 *Let $\mathcal{S}$ be a semantic notion, and let $E_{\mathcal{S}}$ be an argument extension under $\mathcal{S}$. For any argument $\mathcal{A} \in E_{\mathcal{S}}$, then $\mathtt{subs}(\mathcal{A}) \subseteq E_{\mathcal{S}}$.*

Any extension, under any semantic notion, must include all the subarguments of its members. This is because when accepting an argument (*i. e.,* by including it in semantic extensions), every piece of information of that argument should also be accepted under the same criteria. Thinking it by opposition, the non-acceptance of an argument will affect every superargument of it, since a piece of their own reasoning structure is rejected. This is particulary evident when the preference order is monotonically non-increasing, as an argument cannot be stronger than any of their subarguments. Because of this, the argument cannot survive to the attacks to its subarguments. The previous postulate has an interesting consequence.

Proposition 13 *Let $\mathcal{S}$ be a semantic notion, let $E_{\mathcal{S}}$ be an argument extension under $\mathcal{S}$ and let $\mathcal{A}$ be an argument. If $\mathcal{A} \notin E_{\mathcal{S}}$, then any superargument of $\mathcal{A}$ does not belong to $E_{\mathcal{S}}$.*

This means that any evaluation process of argument acceptance may discard superarguments of rejected arguments, despite any further conflict that may exists on these subsequent superarguments, as they will be rejected anyway. Proposition 13 states an important result towards the definition of algorithms for generalized calculation of semantics, while implying the following observation.

Observation 1 *Let $\mathcal{S}$ be a semantic notion, and let $E_{\mathcal{S}}$ be an argument extension under $\mathcal{S}$. Let S be a set of arguments, and $\mathcal{A}$ an argument such that $S \xrightarrow{\triangle} \mathcal{A}$. If $S \subseteq E_{\mathcal{S}}$, then not necessarily $\mathcal{A} \in E_{\mathcal{S}}$.*

5.1 Computing subargumentation-closed extensions

In the literature, semantics are defined on top of conflict-free sets, defense sets (in which every argument that attacks an argument in the set is attacked by the set), and admissible sets of arguments (conflict-free defense sets). For instance, a preferred extension is a maximally admissible set of arguments. All of these notions remain the same in a subargumentation-capable argumentation framework when speaking of atomic arguments. However, if we intend to include non-atomic arguments in any of these sets of arguments while keeping the set closed under subargumentation, we need to calculate them by iteratively deriving new arguments, being careful not to violate any of the conditions we are seeking. Next, we provide a way to build an admissible set of arguments closed under subargumentation, in order to define the subargumentation version of the preferred extension.

Definition 19 (Conflict-free set of arguments closed under subargumentation) *The set C of conflict-free arguments closed under subargumentation is the least fixed point of the operator cf:*

- $cf^0 = \emptyset$;
- $cf^k = cf^{k-1} \cup \{\mathcal{A} \mid cf^{k-1} \overset{\triangle}{\rightarrow} \mathcal{A}, \nexists \mathcal{B} \in cf^k$ *such that* $\mathcal{B}\mathbb{R}\mathcal{A}$ *or* $\mathcal{A}\mathbb{R}\mathcal{B}\}$

There are two points worthwhile to mention: first, note that atomic arguments are derived from the empty set; second, the conflict-free set is not unique: according to the order in which arguments are added to the set, different conflict-free sets can be obtained.

Example 1 *Let* $S_1 = (\{\mathcal{A}, \mathcal{B}_1, \mathcal{B}, \mathcal{C}_1, \mathcal{C}, \mathcal{D}, \mathcal{E}\}, \{(\mathcal{A}, \mathcal{D}), (\mathcal{D}, \mathcal{A}), (\mathcal{A}, \mathcal{B}_1), (\mathcal{B}_1, \mathcal{C}), (\mathcal{D}, \mathcal{C})\}, \{(\mathcal{B}_1, \mathcal{B}), (\mathcal{C}_1, \mathcal{C})\})$ *be a ScAF, as depicted in Figure 2. Then, three possible maximal outcomes of the conflict-free fixed-point operator are:*

$C1$	$C2$	$C3$
$cf^0 = \emptyset$	$cf^0 = \emptyset$	$cf^0 = \emptyset$
$cf^1 = \{\mathcal{E}, \mathcal{D}, \mathcal{B}_1\}$	$cf^1 = \{\mathcal{E}, \mathcal{A}, \mathcal{C}_1\}$	$cf^1 = \{\mathcal{D}, \mathcal{E}, \mathcal{C}_1\}$
$cf^2 = \{\mathcal{E}, \mathcal{D}, \mathcal{B}_1, \mathcal{B}\}$	$cf^2 = \{\mathcal{E}, \mathcal{A}, \mathcal{C}_1, \mathcal{C}\}$	$cf^2 = cf^1$
$cf^3 = cf^2$	$cf^3 = cf^2$	

The first iteration always includes conflict-free atomic arguments, while beginning from the second iteration derived arguments are added. In particular, note that in set $C3$ argument $\mathcal{C}$ is not derived because $\mathcal{D}$ is attacking it.

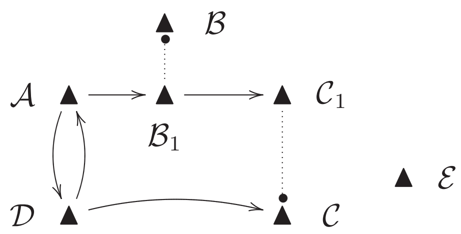

Figure 2: Example 1

In the same fashion as the conflict-free closed set was defined, an admissible set of arguments (also closed under subargumentation) can be calculated.

Definition 20 (Admissible set of arguments closed under subargumentation) *The admissible set A of arguments closed under subargumentation is the least fixed point of the operator adm:*

- $adm^0 = \emptyset$;
- $adm^k = adm^{k-1} \cup \{\mathcal{A} \mid adm^{k-1} \overset{\triangle}{\rightarrow} \mathcal{A}, \nexists \mathcal{B} \in adm^k$ *such that* $\mathcal{B}\mathbb{R}\mathcal{A}$ *or* $\mathcal{A}\mathbb{R}\mathcal{B}$ *and if* $\exists \mathcal{D} \notin adm^k$ *such that* $\mathcal{D}\mathbb{R}\mathcal{A}$*, then* $\exists \mathcal{C} \in adm^k$ *such that* $\mathcal{C}\mathbb{R}\mathcal{D}\}$

Example 2 *From Example 1 we have that $C1$ and $C2$ are maximally admissible sets of arguments in S_1, whereas $C3$ is not, since $\mathcal{C}_1$ is attacked by $\mathcal{B}_1$, and there is no argument in the set that attacks $\mathcal{B}_1$; the inclusion of $\mathcal{A}$ is out of the question, due to $\mathcal{D}$ belonging to $C3$.*

The subargumentation-closed admissible set is also non-univocally determined and several can be obtained. Finally, it becomes trivial to define the *preferred extension closed under subargumentation*.

Definition 21 (Preferred extension closed under subargumentation) *Given a framework $AF = (\mathbb{A}, \mathbb{R}, \sqsubseteq)$, a set of arguments $E \subseteq \mathbb{A}$ is a preferred extension of AF iff E is a maximally (wrt. set inclusion) admissible set of arguments closed under subargumentation from AF.*

Since there are possibly multiple closed admissible sets of arguments, the property held in usual argumentation frameworks of having at least one preferred extension is maintained. In order to be compatible with the abstract argumentation community, a definition for the grounded and stable semantics should be given, but it is left as future work due to space limitations –although when looking to the definition for the preferred semantics they seem to be a rather straightforward analogy.

6 Conclusions and future work

In this article, we have illustrated a preliminary formalization for a subargumentation-capable argumentation framework. The first definitions cope with the subargument relation *per se*, its meaning and usage. Afterwards, a definition for closure is given, in concordance with the expected behavior of a system defined to use subargumentation. The closure under subargumentation of a given set of arguments states a requirement expected to be satisfied by any set of arguments compliant with our definitions for a sensible subargument relation. Then, the matter of argumentation semantics is addressed, analyzing the role of subarguments when computing the set of acceptable arguments, independently of the chosen semantics. Finally, a fixed-point operator is defined to obtain conflict-free and admissible sets of arguments, both of them closed under subargumentation. Thus, it is trivial to define the preferred semantics, which, along with the grounded and the stable semantics, is one of the better known semantics in the literature of computational abstract argumentation.

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