On the Logic of Theory Change: Incision Functions from Selection Functions

Marcelo A. Falappa Guillermo R. Simari

Artificial Intelligence Research and Development Laboratory
Department of Computer Science and Engineering
Universidad Nacional del Sur
Av. Alem 1253 - (B8000CPB) Bahía Blanca - Argentina
PHONE/FAX: (+54) 291 459 5136
E-MAIL: [mfalappa,grs]@cs.uns.edu.ar

Key Words: Belief Revision, AGM Theory, Partial Meet Contractions, Kernel Contractions.

Abstract

This work present an connection between partial meet contractions (AGM: Alchourrón, Gärdenfors and Makinson) and kernel contractions (Hansson). We present a way to define incision functions (used in kernel contractions) from selection functions (used in partial meet contractions). We present some properties of both kinds of functions and their relations.

1 Introduction

Belief Revision systems are logical frameworks for modelling the dynamics of knowledge. That is, how we modify our beliefs when we receive new information. The main problem arises when that information is inconsistent with the beliefs that represent our epistemic state. In order to accept the new information we have to give up some information while preserving as much of the old information as possible.

There are a lot of frameworks to model the dynamic of knowledge but AGM is the most popular one. It defines three kinds of changes: expansion, contraction and revision. Contractions in AGM are called *partial meet contractions* and they are based on a selection among subsets of the original set that do not imply the information to be retracted. In order to make that, partial meet contractions use *selection functions*. On the other hand, *kernel contractions* [Han93] are based on a selection among the sentences that imply the information to be retracted. In order to make that, kernel contractions use *incision functions*. However, contraction is not the one kind of operator can be defined in partial meet and kernel way. It is possible to define *prioritized revisions*, that make a kind of change in which the new information is always accepted. Also it is possible to define a *consolidation* [Han97] that makes a contraction of a set of sentences by a contradiction, a *semi-revision* [Han97] that makes a non-prioritized revision of a set of sentences by a single sentence, and a *revision by sets of sentences* [FKIS02] that makes a non-prioritized revision of a set of sentences by a set of sentences.

All these operators have a particular property: every partial meet operator is a kernel operator. Since every partial meet operator is defined in terms of a selection function and every kernel operator is defines in terms of an incision function we study the relation of these functions, giving a a way to define incision functions from selection functions. Moreover, we present some properties for both kinds of functions and their relations.

2 Partial meet contractions

This kind of contraction is based on the concept of a remainder set.

Definition 2.1 (Alchourrón, Gärdenfors & Makinson [AGM85]): Let K be a set of sentences and α a sentence. Then $K^{\perp}\alpha$ is the set of all K' such that $K' \in K^{\perp}\alpha$ if and only if $K' \subseteq K$, $K' \nvdash \alpha$ and if $K' \subset K'' \subseteq K$ then $K'' \vdash \alpha$. The set $K^{\perp}\alpha$ is called the *remainder set* of K with respect to α, and its elements are called the *α-remainders of K*. ■

For instance, if $K = \{p, p \rightarrow q, r, r \rightarrow s, r \wedge s \rightarrow q, t \rightarrow u\}$ then the set of q-remainders is $\{\{p \rightarrow q, r \rightarrow s, r \wedge s \rightarrow q, t \rightarrow u\}, \{p, r \rightarrow s, r \wedge s \rightarrow q, t \rightarrow u\}, \{p \rightarrow q, r, r \wedge s \rightarrow q, t \rightarrow u\}, \{p, r, r \wedge s \rightarrow q, t \rightarrow u\}\}$. The set of v-remainders of K is equal to $\{K\}$ since $K \nvdash v$. The set of $(p \rightarrow p)$-remainders of K is $\varnothing$ because $p \rightarrow p \in Cn(\varnothing)$ and there is no subset of K failing to imply $p \rightarrow p$.

In order to define the a partial meet contraction operator, we need a selection function. This function selects sentences of K to be preserved and it is called selection function because it makes a selection among α-remainders.

Definition 2.2: Let K be a set of sentences. A *selection function for K* is a function "γ" ($\gamma : \mathbf{2^{2^{\mathcal{L}}}} \Rightarrow \mathbf{2^{2^{\mathcal{L}}}}$) such that for any sentence $\alpha \in \mathcal{L}$, it holds that:
1) $\gamma(K^{\perp}\alpha) \subseteq K^{\perp}\alpha$.
2) If $K^{\perp}\alpha = \varnothing$ then $\gamma(K^{\perp}\alpha) = K$. ■

For instance, given $K = \{p, p \rightarrow q, r, r \rightarrow q, \neg s\}$ and $\alpha = q$ then $K^{\perp}\alpha = \{\{p, r, \neg s\}, \{p, r \rightarrow q, \neg s\}, \{p \rightarrow q, r, \neg s\}, \{p \rightarrow q, r \rightarrow q, \neg s\}\}$ and some possible results of $\gamma(K^{\perp}\alpha)$ are $\{\{p, r, \neg s\}\}$, $\{\{p, r \rightarrow q, \neg s\}\}$, $\{\{p, r, \neg s\}, \{p \rightarrow q, r \rightarrow q, \neg s\}\}$ and $\{\{p, r \rightarrow q, \neg s\}, \{p \rightarrow q, r, \neg s\}, \{p \rightarrow q, r \rightarrow q, \neg s\}\}$.

Partial meet contractions are defined as follows: given a set of sentences K, a sentence α and a selection function γ for K, the *partial meet contraction* of K by α, noted by $K \div \gamma\alpha$, is equal to $\cap\gamma(K^{\perp}\alpha)$. That is, $K \div \gamma\alpha$ is equal to the intersection of the α-remainders of K selected by γ.

3 Kernel contractions

This kind of contraction is based on the concept of a kernel set.

Definition 3.1 (Hansson [Han93]): Let K be a set of sentences and α a sentence. Then $K^{\perp\!\!\!\perp}\alpha$ is the set of all K' such that $K' \in K^{\perp\!\!\!\perp}\alpha$ if and only if $K' \subseteq K$, $K' \vdash \alpha$, and if $K'' \subset K'$ then $K'' \nvdash \alpha$. The set $K^{\perp\!\!\!\perp}\alpha$ is called the *kernel set*, and its elements are called the *α-kernels of K*. ■

For instance, given $K = \{p, p \to q, r, r \to s, r \wedge s \to q, t \to u\}$ and $\alpha = q$ then the set of α-kernels is $K^{\perp\!\!\!\perp}\alpha = \{\{p, p \to q\}, \{r, r \to s, r \wedge s \to q\}\}$. If $K = \{p, p \to q\}$ then $K^{\perp\!\!\!\perp}(p \to p) = \{\varnothing\}$ because $p \to p \in Cn(\varnothing)$ and $K^{\perp\!\!\!\perp}\neg p = \varnothing$ since $K \nvdash \neg p$.

In order to define the operator of kernel contraction we need to use an incision function. This function selects sentences of K to be removed and it is called incision function because it makes an incision in every α-kernel.

Definition 3.2: Let K be a set of sentences. An *incision function for* K is a function "σ"(σ : $\mathbf{2}^{\mathbf{2}^{\mathfrak{L}}} \Rightarrow \mathbf{2}^{\mathfrak{L}}$) such that for any sentence $\alpha \in \mathfrak{L}$, the following hold:
1) $\sigma(K^{\perp\!\!\!\perp}\alpha) \subseteq \cup(K^{\perp\!\!\!\perp}\alpha)$.
2) If $X \in K^{\perp\!\!\!\perp}\alpha$ and $X \neq \varnothing$ then $(X \cap \sigma(K^{\perp\!\!\!\perp}\alpha)) \neq \varnothing$.
The limit case in which $K^{\perp\!\!\!\perp}\alpha = \varnothing$ then $\sigma(K^{\perp\!\!\!\perp}\alpha) = \varnothing$. ■

For instance, taking $K = \{p, p \to q, r, r \to s, r \wedge s \to q, t \to u\}$ and $\alpha = q$ then $K^{\perp\!\!\!\perp}\alpha = \{\{p, p \to q\}, \{r, r \to s, r \wedge s \to q\}\}$ and some possible results of $\sigma(K^{\perp\!\!\!\perp}\alpha)$ are $\{p, p \to q, r \to s\}$, $\{p \to q, r \wedge s \to q\}$ and $\{p, r\}$.

Kernel contractions are defined as follows: given a set of sentences K, a sentence α and a incision function σ for K, the *kernel contraction* of K by α, noted by $K \div \sigma\alpha$, is equal to $K \setminus (K^{\perp\!\!\!\perp}\alpha)$. That is, $K \div \sigma\alpha$ can be obtained erasing from K the sentences cutted by σ.

4 Incision functions from selection functions

Now we will present a formal definition of a incision function from a selection function.

Definition 4.1: Let γ a selection function for a set K. We may define an incision function σ_γ as follows: if $\beta \in K$ and $\beta \notin \cap\gamma(K^{\perp}\alpha)$ then $\beta \in \sigma(K^{\perp\!\!\!\perp}\alpha)$. ■

The intuition behind this definition is the following: α is selected by a incision function σ (over the set of α-kernels of K) if α does not belong to some α-remainder of K selected by γ. For instance, if $K = \{p, r, s, p \to q, r \to q, \neg t \to \neg q\}$ then $K^{\perp}q = \{\{p, r, s, \neg t \to \neg q\}, \{p, s, r \to q, \neg t \to \neg q\}, \{s, p \to q, r \to q, \neg t \to \neg q\}, \{r, s, p \to q, \neg t \to \neg q\}\}$. Suppose that $\gamma(K^{\perp}q) = \{\{p, r, s, \neg t \to \neg q\}, \{r, s, p \to q, \neg t \to \neg q\}\}$. Then the associated incision function is $\sigma_\gamma(K^{\perp\!\!\!\perp}q) = \sigma_\gamma(\{p, p \to q\}, \{r, r \to q\}\}) = \{p, p \to q, r \to q\}$.

Now we present some plausible properties for selection functions in order to define incision functions. Some of them have been modified from [FKIS02] to be applied in our framework.

Equitableness: If $K^{\perp\!\!\!\perp}\alpha = H^{\perp\!\!\!\perp}\alpha$ then $\beta \in K \setminus \cap\gamma(K^{\perp}\alpha)$ if and only if $\beta \in H \setminus \cap\gamma(H^{\perp}\alpha)$.

Weak Equitableness: If $X \in (K^{\perp\!\!\!\perp}\alpha) \cap (H^{\perp\!\!\!\perp}\alpha)$ and $\beta \in X$ then $\beta \in K \setminus \cap\gamma(K^{\perp}\alpha)$ if and only if $\beta \in H \setminus \cap\gamma(H^{\perp}\alpha)$.

The purpose of *equitableness* is that σ_γ be a well defined function. *Weak equitableness* tried to make "fair" to σ_γ, that is, if K and H share some α-kernel, then σ_γ will make the same cut in both contractions (contraction of K by α and contraction of H by α).

These two properties are needed when we are using *global selection functions* (called two-place selection function by Hansson [Han96]), that is, selection functions that can be applied to different sets of sentences. A global selection function γ has two arguments, one for the set

Definition 4.2: Let K and H be sets of sentences and γ a selection function for K and H. Then γ is an *equitable selection function* if $K^{\perp\!\!\perp}\alpha = H^{\perp\!\!\perp}\alpha$ implies that $K \setminus \cap\gamma(K^{\perp}\alpha) = H \setminus \cap\gamma(H^{\perp}\alpha)$. ■

The intuition behind this definition is that, if the set of minimally subsets of K implying α is equal to the set of minimally subsets of H implying α then β is erased in the selection of α-remainders of K if and only if β is erased in the selection of α-remainders of H.

For example, given $K = \{p, s, p \rightarrow q, s \rightarrow \neg t\}$, $H = \{p, t, u, \neg v, p \rightarrow q\}$ and $\alpha = q$, then:

$$K^{\perp}\alpha = \{\{s, p \rightarrow q, s \rightarrow \neg t\}, \{p, s, s \rightarrow \neg t\}\}$$
$$H^{\perp}\alpha = \{\{t, u, \neg v, p \rightarrow q\}, \{p, t, u, \neg v\}$$

We have that $K^{\perp\!\!\perp}\alpha = H^{\perp\!\!\perp}\alpha = \{\{p, p \rightarrow q\}\}$. Suppose that $\gamma(K^{\perp}\alpha) = \{\{p, s, s \rightarrow \neg t\}\}$. That is, γ selects only α-remainders that does not contain to $p \rightarrow q$. If γ is an equitable selection function $\gamma(H^{\perp}\alpha)$ must be equal to $\{\{p, t, u, \neg v\}\}$. Analogously, if $\gamma(K^{\perp}\alpha) = \{\{s, p \rightarrow q, s \rightarrow \neg t\}\}$ and γ is an equitable selection function then $\gamma(H^{\perp}\alpha)$ must be equal to $\{\{t, u, \neg v, p \rightarrow q\}\}$.

5 Conclusions and future work

We presented a way to define incision functions from selection functions. That means, if we have a partial meet contraction operator based on a selection function γ, we may define a kernel contraction operator based on a incision function σ_{γ} obtained from γ. The reverse process is not possible because there are some kernel contractions that are no partial meet contractions [Han93, Han96].

We defined some properties for these selection functions (equitableness, weak equitableness) and we hope to relate more properties such that transitivity, relationally [AGM85, Han96]) and maximality [Han96]. Moreover, we hope to define new properties to global selection/incision functions, that is, functions to be applied to more than one set of sentences.

References

[AGM85] Carlos Alchourrón, Peter Gärdenfors, and David Makinson. *On the Logic of Theory Change: Partial Meet Contraction and Revision Functions*. *The Journal of Symbolic Logic*, 50:510–530, 1985.

[FKIS02] Marcelo A. Falappa, Gabriele Kern-Isberner, and Guillermo R. Simari. *Belief Revision, Explanations and Defeasible Reasoning*. Submmited to *Artificial Intelligence Journal*, 2002.

[Han93] Sven Ove Hansson. *Kernel Contraction*. *The Journal of Symbolic Logic*, 1993.

[Han96] Sven Ove Hansson. *A Textbook of Belief Dymanics: Theory Change and Database Updating*. Uppsala University, Department of Philosophy, Uppsala, Sweden, 1996.

[Han97] Sven Ove Hansson. *Semi-Revision*. *Journal of Applied Non-Classical Logic*, in press, 1997.