Apéndice B

El sistema lineal en 3D

Mostramos en este apéndice la forma final del sistema lineal 12×12 a ser resuelto en cada uno de los subdominios Ω_{jkl} en los que dividimos al dominio original Ω, es decir, consideramos aquí la partición masiva del dominio. Suponemos, como antes, que tenemos $n_x \cdot n_y \cdot n_z$ subdominios. Para establecer claramente el problema algebraico, continuamos utilizando, como en la sección 4.2, tres subíndices para individualizar subdominios, y demás parámetros
Consecuentemente, también modificamos la notación para las caras Γjkl. Por ejemplo, para la cara frontal $\mathcal{F}$ de Ω_{jkl} la Ec. $(2.38c)$ deviene en

$$\lambda_{jkl}^{\mathcal{F},n+1} = -\lambda_{j-1kl}^{\mathcal{B},n} - \beta_{jkl}^{\mathcal{F}}(P_\tau \boldsymbol{E}_{jkl}^{n+1}(m_{jkl}^{\mathcal{F}}) - P_\tau \boldsymbol{E}_{j-1kl}^{n}(m_{jkl}^{\mathcal{F}})).$$

La matriz de coeficientes C, que se obtiene siguiendo los pasos detallados oportunamente en la sección 4.2 es simétrica, y tiene la estructura que mostramos a continuación.

$$C = \begin{pmatrix}
c_{11} & c_{12} & c_{13} & c_{14} & c_{15} & c_{16} & 0 & 0 & 0 & 0 & 0 & 0 \\
 & c_{22} & c_{23} & c_{24} & c_{25} & c_{26} & 0 & 0 & 0 & 0 & 0 & 0 \\
 & & c_{33} & c_{34} & 0 & 0 & 0 & 0 & 0 & 0 & c_{3\,11} & c_{3\,12} \\
 & & & c_{44} & 0 & 0 & 0 & 0 & 0 & 0 & c_{4\,11} & c_{4\,12} \\
 & & & & c_{55} & c_{56} & 0 & 0 & c_{59} & c_{5\,10} & 0 & 0 \\
 & & & & & c_{66} & 0 & 0 & c_{69} & c_{6\,10} & 0 & 0 \\
 & & & & & & c_{77} & c_{78} & c_{79} & c_{7\,10} & c_{7\,11} & c_{7\,12} \\
 & & & & & & & c_{88} & c_{89} & c_{8\,10} & c_{8\,11} & c_{8\,12} \\
 & & & & & & & & c_{99} & c_{9\,10} & 0 & 0 \\
 & & & & & & & & & c_{10\,10} & 0 & 0 \\
 & & & & & & & & & & c_{11\,11} & c_{11\,12} \\
 & & & & & & & & & & & c_{12\,12}
\end{pmatrix}$$

donde

$$\begin{aligned}
c_{11} &= \frac{781}{5040} h_j\, h_k\, h_l\, \sigma_{jkl} + \frac{1}{i\omega\mu}\left(\frac{1036}{784}\left(\frac{h_j\, h_k}{h_l} + \frac{h_j\, h_l}{h_k}\right) + \frac{h_j\, h_l}{h_k}\right) + \\
&\qquad h_j\, h_l (1-\delta_{k1})\beta^{\mathcal{W}}_{jkl} + (1-i)\delta_{k1} h_j\, h_l\, a_{jkl}, \\
c_{12} &= \frac{-59}{5040} h_j\, h_k\, h_l\, \sigma_{jkl} + \frac{1}{i\omega\mu}\left(\frac{1036}{784}\left(\frac{h_j\, h_k}{h_l} + \frac{h_j\, h_l}{h_k}\right) - \frac{h_j\, h_l}{h_k}\right), \\
c_{13} &= \frac{269}{5040} h_j\, h_k\, h_l\, \sigma_{jkl} - \frac{1}{i\omega\mu}\frac{1036}{784}\left(\frac{h_j\, h_k}{h_l} + \frac{h_j\, h_l}{h_k}\right), \\
c_{14} &= c_{13}, \\
c_{15} &= -\frac{1}{i\omega\mu}\, h_l, \\
c_{16} &= \frac{1}{i\omega\mu}\, h_l, \\
c_{22} &= \frac{781}{5040} h_j\, h_k\, h_l\, \sigma_{jkl} + \frac{1}{i\omega\mu}\left(\frac{1036}{784}\left(\frac{h_j\, h_k}{h_l} + \frac{h_j\, h_l}{h_k}\right) + \frac{h_j\, h_l}{h_k}\right) + \\
&\qquad h_j\, h_l (1-\delta_{kn_y})\beta^{\mathcal{E}}_{jkl} + (1-i)\delta_{kn_y} h_j\, h_l\, a_{jkl}, \\
c_{23} &= c_{13}, \\
c_{24} &= c_{14},
\end{aligned}$$

$$
\begin{aligned}
c_{25} &= c_{16}, \\
c_{26} &= c_{15}, \\
c_{33} &= \frac{781}{5040} h_j\, h_k\, h_l\, \sigma_{jkl} + \frac{1}{i\omega\mu}\Big(\frac{1036}{784}\big(\frac{h_j\, h_k}{h_l} + \frac{h_j\, h_l}{h_k}\big) + \frac{h_j\, h_k}{h_l}\Big) + \\
&\qquad h_j\, h_k (1-\delta_{l1})\beta^{\mathcal{S}}_{jkl} + (1-i)\delta_{l1} h_j\, h_k\, a_{jkl}, \\
c_{34} &= \frac{-59}{5040} h_j\, h_k\, h_l\, \sigma_{jkl} + \frac{1}{i\omega\mu}\Big(\frac{1036}{784}\big(\frac{h_j\, h_k}{h_l} + \frac{h_j\, h_l}{h_k}\big) - \frac{h_j\, h_k}{h_l}\Big), \\
c_{3\,11} &= -\frac{1}{i\omega\mu}\, h_k, \\
c_{3\,12} &= \frac{1}{i\omega\mu}\, h_k, \\
c_{44} &= \frac{781}{5040} h_j\, h_k\, h_l\, \sigma_{jkl} + \frac{1}{i\omega\mu}\Big(\frac{1036}{784}\big(\frac{h_j\, h_k}{h_l} + \frac{h_j\, h_l}{h_k}\big) + \frac{h_j\, h_k}{h_l}\Big) + \\
&\qquad h_j\, h_k (1-\delta_{l n_z})\beta^{\mathcal{N}}_{jkl} + (1-i)\delta_{l n_z} h_j\, h_k\, a_{jkl}, \\
c_{4\,11} &= c_{3\,11}, \\
c_{4\,12} &= c_{3\,12}, \\
c_{55} &= \frac{781}{5040} h_j\, h_k\, h_l\, \sigma_{jkl} + \frac{1}{i\omega\mu}\Big(\frac{1036}{784}\big(\frac{h_j\, h_k}{h_l} + \frac{h_k\, h_l}{h_j}\big) + \frac{h_k\, h_l}{h_j}\Big) + \\
&\qquad h_k\, h_l (1-\delta_{j1})\beta^{\mathcal{B}}_{jkl} + (1-i)\delta_{j1} h_k\, h_l\, a_{jkl}, \\
c_{56} &= \frac{-59}{5040} h_j\, h_k\, h_l\, \sigma_{jkl} + \frac{1}{i\omega\mu}\Big(\frac{1036}{784}\big(\frac{h_j\, h_k}{h_l} + \frac{h_k\, h_l}{h_j}\big) - \frac{h_k\, h_l}{h_j}\Big), \\
c_{59} &= \frac{269}{5040} h_j\, h_k\, h_l\, \sigma_{jkl} - \frac{1}{i\omega\mu}\frac{1036}{784}\Big(\frac{h_j\, h_k}{h_l} + \frac{h_k\, h_l}{h_j}\Big), \\
c_{5\,10} &= c_{59}, \\
c_{66} &= \frac{781}{5040} h_j\, h_k\, h_l\, \sigma_{jkl} + \frac{1}{i\omega\mu}\Big(\frac{1036}{784}\big(\frac{h_j\, h_k}{h_l} + \frac{h_k\, h_l}{h_j}\big) + \frac{h_k\, h_l}{h_j}\Big) + \\
&\qquad h_k\, h_l (1-\delta_{j n_x})\beta^{\mathcal{F}}_{jkl} + (1-i)\delta_{j n_x} h_k\, h_l\, a_{jkl}, \\
c_{6\,9} &= c_{59}, \\
c_{6\,10} &= c_{5\,10}, \\
c_{77} &= \frac{781}{5040} h_j\, h_k\, h_l\, \sigma_{jkl} + \frac{1}{i\omega\mu}\Big(\frac{1036}{784}\big(\frac{h_j\, h_l}{h_k} + \frac{h_k\, h_l}{h_j}\big) + \frac{h_j\, h_l}{h_k}\Big) + \\
&\qquad h_j\, h_l (1-\delta_{k1})\beta^{\mathcal{W}}_{jkl} + (1-i)\delta_{k1} h_j\, h_l\, a_{jkl}, \\
c_{78} &= \frac{-59}{5040} h_j\, h_k\, h_l\, \sigma_{jkl} + \frac{1}{i\omega\mu}\Big(\frac{1036}{784}\big(\frac{h_j\, h_l}{h_k} + \frac{h_k\, h_l}{h_j}\big) - \frac{h_j\, h_l}{h_k}\Big), \\
c_{79} &= -\frac{1}{i\omega\mu}\, h_j, \\
c_{7\,10} &= \frac{1}{i\omega\mu}\, h_j,
\end{aligned}
$$

$$
\begin{aligned}
c_{7\,11} &= \frac{269}{5040} h_j\, h_k\, h_l\, \sigma_{jkl} - \frac{1}{i\omega\mu}\frac{1036}{784}(\frac{h_j\, h_l}{h_k} + \frac{h_k\, h_l}{h_j}),\\
c_{7\,12} &= c_{7\,11},\\
c_{88} &= \frac{781}{5040} h_j\, h_k\, h_l\, \sigma_{jkl} + \frac{1}{i\omega\mu}(\frac{1036}{784}(\frac{h_j\, h_l}{h_k} + \frac{h_k\, h_l}{h_j}) + \frac{h_j\, h_l}{h_k}) + \\
&\qquad h_j\, h_l(1-\delta_{kn_y})\beta^{\mathcal{E}}_{jkl} + (1-i)\delta_{kn_y} h_j\, h_l\, a_{jkl},\\
c_{89} &= c_{7\,10},\\
c_{8\,10} &= c_{7\,9},\\
c_{8\,11} &= c_{7\,11},\\
c_{8\,12} &= c_{7\,12},\\
c_{99} &= \frac{781}{5040} h_j\, h_k\, h_l\, \sigma_{jkl} + \frac{1}{i\omega\mu}(\frac{1036}{784}(\frac{h_j\, h_k}{h_l} + \frac{h_k\, h_l}{h_j}) + \frac{h_j\, h_k}{h_l}) + \\
&\qquad h_j\, h_k(1-\delta_{l1})\beta^{\mathcal{S}}_{jkl} + (1-i)\delta_{l1} h_j\, h_k\, a_{jkl},\\
c_{9\,10} &= \frac{-59}{5040} h_j\, h_k\, h_l\, \sigma_{jkl} + \frac{1}{i\omega\mu}(\frac{1036}{784}(\frac{h_j\, h_k}{h_l} + \frac{h_k\, h_l}{h_j}) - \frac{h_j\, h_k}{h_l}),\\
c_{10\,10} &= \frac{781}{5040} h_j\, h_k\, h_l\, \sigma_{jkl} + \frac{1}{i\omega\mu}(\frac{1036}{784}(\frac{h_j\, h_k}{h_l} + \frac{h_k\, h_l}{h_j}) + \frac{h_j\, h_k}{h_l}) + \\
&\qquad h_j\, h_k(1-\delta_{ln_z})\beta^{\mathcal{N}}_{jkl} + (1-i)\delta_{ln_z} h_j\, h_k\, a_{jkl},\\
c_{11\,11} &= \frac{781}{5040} h_j\, h_k\, h_l\, \sigma_{jkl} + \frac{1}{i\omega\mu}(\frac{1036}{784}(\frac{h_j\, h_l}{h_k} + \frac{h_k\, h_l}{h_j}) + \frac{h_k\, h_l}{h_j}) + \\
&\qquad h_k\, h_l(1-\delta_{r1})\beta^{\mathcal{B}}_{jkl} + (1-i)\delta_{r1} h_k\, h_l\, a_{jkl},\\
c_{11\,12} &= \frac{-59}{5040} h_j\, h_k\, h_l\, \sigma_{jkl} + \frac{1}{i\omega\mu}(\frac{1036}{784}(\frac{h_j\, h_l}{h_k} + \frac{h_k\, h_l}{h_j}) - \frac{h_k\, h_l}{h_j}),\\
c_{12\,12} &= \frac{781}{5040} h_j\, h_k\, h_l\, \sigma_{jkl} + \frac{1}{i\omega\mu}(\frac{1036}{784}(\frac{h_j\, h_l}{h_k} + \frac{h_k\, h_l}{h_j}) + \frac{h_k\, h_l}{h_j}) + \\
&\qquad h_k\, h_l(1-\delta_{jn_x})\beta^{\mathcal{F}}_{jkl} + (1-i)\delta_{jn_x} h_k\, h_l\, a_{jkl}.
\end{aligned}
$$

La delta de Kronecker se introduce en los coeficientes para incluír en la misma expresión términos que contribuyen ó sobre el bosde Γ, ó dentro del dominio Ω. El coeficiente a_{jkl} es el ya definido, evaluando los parámetros en el dominio correspondiente, $a_{\mathrm{j}} = \left(\frac{\sigma_{\mathrm{j}}}{2\omega\mu}\right)^{\frac{1}{2}}$.
Para darle valores a β elegimos la misma idea que en el caso bidimensional,

es decir que tomando

$$\beta^{s}_{jkl} = \frac{1-i}{2\sqrt{2\omega\mu}}\left(\sqrt{\sigma^{*}_{jkl}} + \sqrt{\sigma_{jkl}}\right), \tag{B.4}$$

donde

$$\begin{cases} \text{para} \quad s = \mathcal{W} \quad \mathrm{j}^{*} = j\,k-1\,l, \\ \text{para} \quad s = \mathcal{E} \quad \mathrm{j}^{*} = j\,k+1\,l, \\ \text{para} \quad s = \mathcal{S} \quad \mathrm{j}^{*} = j\,k\,l-1, \\ \text{para} \quad s = \mathcal{N} \quad \mathrm{j}^{*} = j\,k\,l+1, \\ \text{para} \quad s = \mathcal{B} \quad \mathrm{j}^{*} = j-1\,k\,l, \\ \text{para} \quad s = \mathcal{F} \quad \mathrm{j}^{*} = j+1\,k\,l, \end{cases}$$

las Ecs. (2.38c) son condiciones de borde absorbentes para las fronteras interiores.

Finalmente, el vector de doce elementos que forma la el término independiente del sistema lineal, en la iteración n+1 es el siguiente:

$$\mathrm{b}^{n}_{jkl} = \begin{pmatrix} (1-\delta_{k1})\,h_j\,h_l\left(\beta^{\mathcal{W}}_{jkl}\varepsilon^{\mathcal{E}_x,n}_{j\,k-1\,l} - \lambda^{\mathcal{E}}_{j\,k-1\,l,x}\right) \\ (1-\delta_{kn_y})\,h_j\,h_l\left(\beta^{\mathcal{E}}_{jkl}\varepsilon^{\mathcal{W}_x,n}_{j\,k+1\,l} - \lambda^{\mathcal{W}}_{j\,k+1\,l,x}\right) \\ (1-\delta_{l1})\,h_j\,h_k\left(\beta^{\mathcal{S}}_{jkl}\varepsilon^{\mathcal{N}_x,n}_{j\,k\,l-1} - \lambda^{\mathcal{N}}_{j\,k\,l-1,x}\right) \\ (1-\delta_{ln_z})\,h_j\,h_k\left(\beta^{\mathcal{N}}_{jkl}\varepsilon^{\mathcal{S}_x,n}_{j\,k\,l+1} - \lambda^{\mathcal{S}}_{j\,k\,l+1,x}\right) \\ (1-\delta_{j1})\,h_k\,h_l\left(\beta^{\mathcal{B}}_{jkl}\varepsilon^{\mathcal{F}_y,n}_{j-1\,k\,l} - \lambda^{\mathcal{F}}_{j-1\,k\,l,y}\right) \\ (1-\delta_{jn_x})\,h_k\,h_l\left(\beta^{\mathcal{F}}_{jkl}\varepsilon^{\mathcal{B}_y,n}_{j+1\,k\,l} - \lambda^{\mathcal{B}}_{j+1\,k\,l,y}\right) \\ (1-\delta_{k1})\,h_j\,h_l\left(\beta^{\mathcal{W}}_{jkl}\varepsilon^{\mathcal{E}_z,n}_{j\,k-1\,l} - \lambda^{\mathcal{E}}_{j\,k-1\,l,z}\right) \\ (1-\delta_{kn_y})\,h_j\,h_l\left(\beta^{\mathcal{E}}_{jkl}\varepsilon^{\mathcal{W}_z,n}_{j\,k+1\,l} - \lambda^{\mathcal{W}}_{j\,k+1\,l,z}\right) \\ (1-\delta_{l1})\,h_j\,h_k\left(\beta^{\mathcal{S}}_{jkl}\varepsilon^{\mathcal{N}_y,n}_{j\,k\,l-1} - \lambda^{\mathcal{N}}_{j\,k\,l-1,y}\right) \\ (1-\delta_{ln_z})\,h_j\,h_k\left(\beta^{\mathcal{N}}_{jkl}\varepsilon^{\mathcal{S}_y,n}_{j\,k\,l+1} - \lambda^{\mathcal{S}}_{j\,k\,l+1,y}\right) \\ (1-\delta_{j1})\,h_k\,h_l\left(\beta^{\mathcal{B}}_{jkl}\varepsilon^{\mathcal{F}_z,n}_{j-1\,k\,l} - \lambda^{\mathcal{F}}_{j-1\,k\,l,z}\right) \\ (1-\delta_{jn_x})\,h_k\,h_l\left(\beta^{\mathcal{F}}_{jkl}\varepsilon^{\mathcal{B}_z,n}_{j+1\,k\,l} - \lambda^{\mathcal{B}}_{j+1\,k\,l,z}\right) \end{pmatrix}$$

La contribución del término fuente debe sumarse al vector b^{n}_{jkl}. Si se supone una polarización XY para la onda incidente, $-\frac{1}{4}\,h_j\,h_k\,h_l\,\sigma_{jkl}\,\boldsymbol{E}^{p}(z_j kl)$, donde $z_j kl$ es el centro de Ω_{jkl}, se suma a los elementos primero, segundo, tercero y cuarto. En el caso de una polarización YX, la contribución va a los elementos quinto, sexto, noveno y décimo.