Metrics for FIR Filters based on distributed arithmetic in FPGA

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ABSTRACT: In this paper, metrics regarding different architectures for distributed arithmetic based FIR filters in FPGA are presented. Main filter parameters are described as well as diverse design techniques applied: *pipelining*, *bit-serial*, *digit-serial* y *bit-parallel*. Each filter description was written in VHDL at RTL level. For achieving this goal no relative location (*rloc*) technique was used what redounds on more generic and expensive designs than those available through *Core Generator* tool. Implementation has been carried out over FPGAs belonging to Xilinx Virtex II family.

I. INTRODUCTION

Today's FPGAs high density has allowed a new field of application: single-chip systems design with embedded DSP algorithms [1,2]. Advantages of this solution are multiple: extra component usage is avoided, off-chip connections are reduced and *DSP-core* can be simplified and optimized with regard to the application, taking account of different aspects such as required data rate, precision, etc. Frequently *bit-parallel* circuits are used, but these implementations are expensive in terms of area yielding a greater speed than required. *Digit-serial* architectures are important choices for efficently implementing a wide range of circuits with real time signal processing. This solution allows the designer to get results expressed in terms of area-speed, ranging between both implementations: *bit-serial* and *bit-parallel*. [3]

Distributed arithmetic-based FIR filters are widely used for implementations that require high performance. Distributed arithmetic is basically a *bit-serial* operation that performs two vector product (one of which is constant). This technique avoids multiplications through the utilization of *lookup tables* (LUTs) taking highest advantage of technologies with plenty of memory elements such as RAM based FPGAs, where designers can use LUTs and adders to compute the product. [4]

In this paper a comparison between different *single-rate* FIR filter architectures based on distributed arithmetic in FPGA is shown. With this aim, RTL descriptions in VHDL and automatic synthesis tools were used. This study involves *bit-parallel, bit-serial*, *digit-serial* and *pipelining* architectures. For descriptions, relative location (*rloc*) techniques [5] were not used and this allows generic designs to be implemented in any kind of FPGA although at a higher cost because the place and route is performed by the automatic synthesis tool at ease.

II. DISTRIBUTED ARITHMETIC

A FIR filter with T taps constant coefficients is characterized by:

$$y[n] = \sum_{k=0}^{T-1} h[n].x[n-k] \qquad (1)$$

An efficient way for implementing this filter in FPGA consists in using distributed arithmetic [6]. The main idea underlying this technique is the computation of sum of products such as

$$y = \mathbf{h'}.\mathbf{x'} = \sum_{i=0}^{T-1} h_i.x_i \tag{2}$$

$$y = \sum_{I=0}^{T-1} h_i.(-x_{i\,0} + \sum_{k=1}^{N-1} x_{i\,k}.2^{-k}) \tag{3}$$

$$y = -\sum_{i=0}^{T-1} h_i.x_{i\,0} + \sum_{k=1}^{N-1} (\sum_{i=0}^{T-1} h_i.x_{i\,k}).2^{-k} \tag{4}$$

$$y = -A_0(x_{0\,0}, x_{1\,0},,,,x_{T-1\,0}) + \sum_{k=1}^{N-1} A_k(x_{0\,k}, x_{1\,k},,,,x_{T-1\,k}).2^{-k} \tag{5}$$

Where h_i stands for known values conforming filter coefficients, T the number of taps, N input value precision and x the input data.

$$A_k(x_{0\,k}, x_{1\,k},,,,x_{T-1\,k}) = \sum_{i=0}^{T-1} h_i.x_{i\,k} \tag{6}$$

$$y = -(h_0.x_{0\,0} + h_1.x_{1\,0} + ... + h_{T-1}.x_{T-1\,0}) + (h_0.x_{0\,1} + h_1.x_{1\,0} + ... + h_{T-1}.x_{T-11})\,2^{-1} + (h_0.x_{0\,1} + h_1.x_{1\,1} + h_2.x_{2\,1} + ... + h_{T-1}.x_{T-1\,1})\,2^{-2} + \; ... + (h_0.x_{0\,N-1} + h_1.x_{1\,N-1} + ... + h_T.x_{T-1\,N-1})\,2^{-(N-1)} \tag{7}$$

A_k only can have a finite number of values (2^T), what means that every single term in (7) is stored either in ROM or LUT. In fig. 1 is shown the way of implementing the sum of products where ROM size is $2^T x W_{ROM}$, with W_{ROM} the precision of output data.

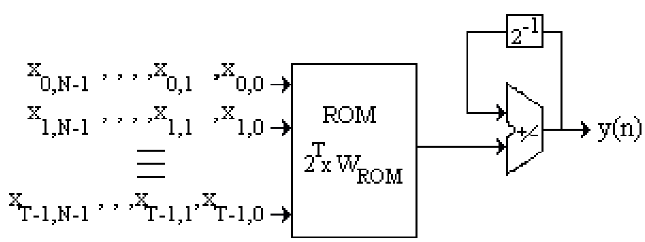

Fig. 1 - Sum of products with distributed arithmetic

II. Models Used in This Work

Operations to be performed by architectures based on distributed arithmetic consist in sequentially searching tables, adding, subtracting and shifting the input signal. All of these can be efficiently implemented through FPGAs.

As tables sizes can become huge, memory can be partitioned in T/k k-bit partitions. In this way it is possible to change from a memory with 2^T size to T/k memories with 2^k size, incorporating adders because of the partitioning .

As stated above, *bit-serial*, *digit-serial*, *bit-serial* with *pipelining, digit-serial* with *pipelining* and *bit-parallel* architectures are considered in this paper.

Distributed arithmetic based filters have a *bit-serial* pattern (fig. 2). For N-*bit* precision inputs N clock cycles are need to yield an output, so that if *CR* is the clock rate, filter sampling rate (*SR*) is CR/N.

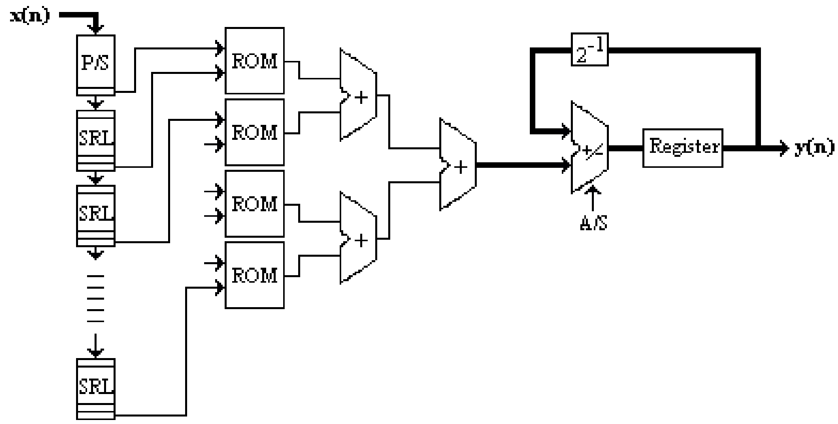

Fig. 2 – *Bit-serial* FIR filter

To indirectly increase the sampling rate, pipeline can be used. With this technique (*pipelining*) clock rate (*CR*) can be increased decreasing logical paths through the inclusion of registers. In fig. 3 is shown that the portion of segmented *bit-serial* architecture is that related with the cascaded adders that operate on the pre-stored values coming from ROMs or LUTs.

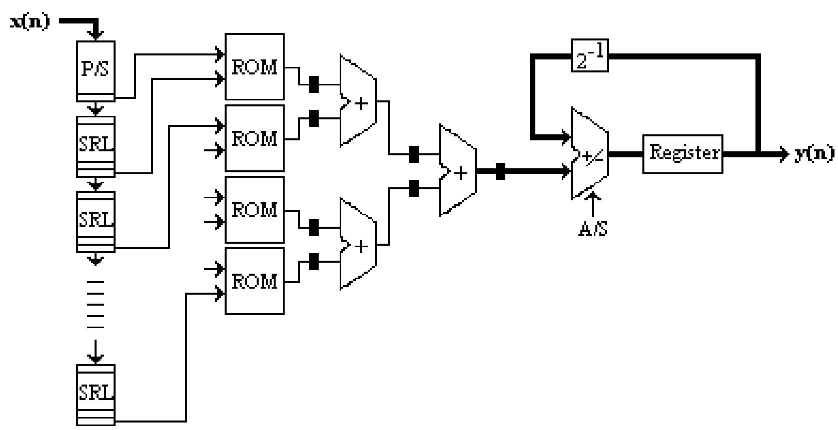

Fig. 3 – *Pipelined bit-serial* FIR filter

To directly improve filter sampling rate, more bits can be processed serially (fig. 4) using *digit-serial* and digit size D. In this way, the sampling rate is accelerated D times (SR=$D_x SR$ [*bit-serial*]). Furthermore, sampling rate can be increased by means of *pipelining* producing an increased clock rate. Maximum rate is obtained when D equals N, i.e. parallel situation.

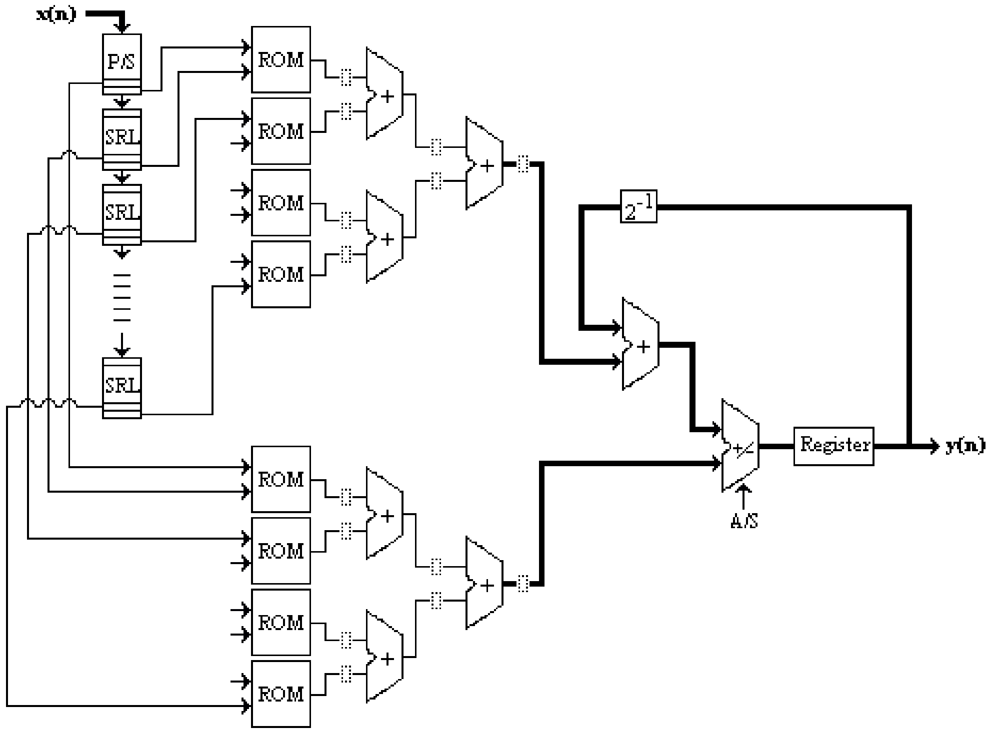

Fig. 4 – *Digit-serial* FIR filter (2-bit digit)

Let *CR* be the clock rate, *SR* the filter' sampling rate, N the input size, D the digit size, therefore the following assertions may be assumed as valid (theoretically):

SR[*bit-serial*] = CR [*bit-serial*]/N
SR[*digit-serial*] = D_xSR [*bit-serial*]
SR[*parallel*] = SR [*digit-serial*]
CR[*bit-serial* con *pipelining*] > CR[*bit-serial*] => **SR[*bit-serial* with *pipelining*] > SR[*bit-serial*]**
CR[*digit-serial* con *pipelining*] > CR[*digit-serial*] => **SR[*digit-serial* with *pipelining*] > SR[*digit-serial*]**

When FPGAs are used, it is not possible to certify these assertions because the implementation strongly depends on the successive stages of the automatic synthesis process [7]. As a matter of fact, in this experience neither relative location (*rloc*) technique nor physical or logical constraints were used. Therefore the place and route tool proceeded at ease when performing physical design for the different alternatives.

III. Experimental Results

Distributed-arithmetic FIR filters architectures with 8, 12, 16, 24, 32 and 64 taps were implemented using a Virtex 2 FPGA from Xilinx (XC2v2000-6bg575). Synthesis was provided with XST (Xilinx Synthesis Technology) Tools [8] running on Xilinx ISE (Xilinx System Environment) version 5.1 [9].
Different filters were tested with 8-bit and 12-bit inputs and coefficients. Precision and bit-size of output data were automatically computed taking account the number of taps, input data precision and coefficient values. Thus, calculations were performed in advance aiming the filter can compute the sums without *overflow* as well as an extension for successive shifts applied was considered.
The following 8-bit inputs and coefficients filters were implemented:

- FIR *bit-serial*
- FIR *bit-serial* with *pipelining*
- FIR *digit-serial* (digit size 2)
- FIR *digit-serial* (digit size 2) with *pipelining*
- FIR *digit-serial* (digit size 4)
- FIR *digit-serial* (digit size 4) with *pipelining*
- FIR *bit-parallel*

Taps	B-Serial	B-Serial Pipe	D-Serial (2)	D-Serial (2) Pipe	D-Serial (4)	D-Serial (4) Pipe	B-Parallel
8	101,94	149,48	85,32	133,69	62,03	73,26	43,63
12	92,25	144,30	73,21	123,91	54,79	81,57	36,79
16	80,71	138,12	68,68	112,99	51,02	70,97	32,16
24	73,69	142,04	63,01	117,92	40,75	77,34	30,07
32	69,98	131,93	55,01	106,72	39,81	71,17	29,58
64	53,05	92,51	46,30	90,01	36,35	63,86	29,88

Table 1 – Clock rate (*CR*) for filters with 8-bit inputs and coefficients [Mhz]

Table 1 shows that *bit-serial* filters with *pipelining* have the best *CR* for 8-bit inputs and coefficients but this is not what happened with the sampling rate (*SR*) because of the different digit sizes involved. In table 2 can be seen that, best *SR* filters (for all tap numbers, except for 8) were those implemented as *digit-serial* with *pipelining* and digit size equal 4.

Taps	B-Serial	B-Serial Pipe	D-Serial (2)	D-Serial (2) Pipe	D-Serial (4)	D-Serial (4) Pipe	B-Parallel
8	12,74	18,68	21,33	33,42	31,01	36,63	43,63
12	11,53	18,04	18,30	30,98	27,39	40,78	36,79
16	10,09	17,26	17,17	28,25	25,51	35,48	32,16
24	9,21	17,75	15,75	29,48	20,37	38,67	30,07
32	8,74	16,49	13,75	26,68	19,90	35,58	29,58
64	6,63	11,56	11,57	22,50	18,17	31,93	29,88

Table 2 – Sampling rate (*SR*) for filters with 8-bit inputs and coefficients [Mhz]

Tables 3 and 4 show *slices* and equivalent gates occupation for the 8-bit input filters. As expected, *parallel* filters are those more expensive in terms of *slices* and equivalent gate occupation.

Taps	B-Serial	B-Serial Pipe	D-Serial (2)	D-Serial (2) Pipe	D-Serial (4)	D-Serial (4) Pipe	B-Parallel
8	63 (1)	72 (1)	80 (1)	85 (1)	117 (1)	120 (1)	178 (1)
12	92 (1)	106 (1)	120 (1)	135 (1)	184 (1)	201 (1)	298 (2)
16	122 (1)	135 (1)	161 (1)	174 (1)	244 (2)	261 (2)	395 (3)
24	188 (1)	209 (1)	259 (2)	287 (2)	408 (3)	447 (4)	680 (6)
32	239 (2)	252 (2)	324 (3)	335 (3)	494 (4)	506 (4)	825 (7)
64	477 (4)	508 (4)	660 (6)	699 (6)	1030 (9)	1089 (10)	1718 (15)

Table 3 – *slices* occupation for FIR with 8-bit inputs and coefficients
(Percentage over a total of 10752 slices)

Taps	B-Serial	B-Serial Pipe	D-Serial (2)	D-Serial (2) Pipe	D-Serial (4)	D-Serial (4) Pipe	B-Parallel
8	1296	1660	1574	2014	2178	3124	3156
12	1860	2512	2372	3470	3486	5518	5426
16	2338	3028	2988	4156	4336	6544	6700
24	3536	4832	4736	7108	7244	11774	11720
32	4448	5830	5890	8434	8822	13756	14056
64	8706	11508	11786	17156	18006	28572	29234

Table 4 – Equivalent gate occupation for FIR with 8-bit inputs and coefficients

The following 12-bit inputs and coefficients filters were designed and implemented:

- FIR *bit-serial*
- FIR *bit-serial* with *pipelining*
- FIR *digit-serial* (digit size 2)
- FIR *digit-serial* (digit size 2) with *pipelining*
- FIR *digit-serial* (digit size 4)
- FIR *digit-serial* (digit size 4) with *pipelining*
- FIR *digit-serial* (digit size 6)
- FIR *digit-serial* (digit size 6) with *pipelining*
- FIR *bit-parallel*

Table 5 shows that *bit-serial* filters with *pipelining* have the best *CR* for 12-bit inputs and coefficients (as happened with 8-bit precision). Regarding the sampling rate (*SR*), in table 6 can be seen that, best *SR* filters were those implemented as *digit-serial* with *pipelining* and digit size equal 6.

Taps	B-Serial	B-Serial Pipe	D-Serial (2)	D-Serial (2) Pipe	D-Serial (4)	D-Serial (4) Pipe	D-Serial (6)	D-Serial (6) Pipe	B-Parallel
8	111,60	133,15	75,07	131,93	61,05	70,67	44,31	53,30	25,99
12	92,98	131,06	73,80	122,10	53,99	75,81	40,92	51,33	25,32
16	85,11	129,03	69,83	115,07	51,39	74,40	38,94	50,45	24,08
24	63,86	129,87	59,84	118,48	43,76	71,63	34,47	49,07	21,89
32	64,68	122,25	48,68	110,99	43,29	72,41	31,54	48,17	21,71
64	47,71	100,20	43,94	94,61	34,78	57,50	29,66	46,32	20,25

Table 5 – Clock rate (*CR*) for filters with 12-bit inputs and coefficients [Mhz]

Taps	B-Serial	B-Serial Pipe	D-Serial (2)	D-Serial (2) Pipe	D-Serial (4)	D-Serial (4) Pipe	D-Serial (6)	D-Serial (6) Pipe	B-Parallel
8	9,30	11,09	12,51	21,99	20,35	23,56	22,15	26,65	25,99
12	7,75	10,92	12,30	20,35	17,99	25,27	20,46	25,66	25,32
16	7,09	10,75	11,64	19,18	17,13	24,80	19,47	25,22	24,08
24	5,32	10,82	9,97	19,75	14,59	23,88	17,23	24,53	21,89
32	5,39	10,19	8,11	18,50	14,43	24,14	15,77	24,08	21,71
64	3,97	8,35	7,32	15,77	11,59	19,17	14,83	23,16	20,25

Table 6 – Sampling rate (*SR*) for filters with 12-bit inputs and coefficients [Mhz]

Tables 7 and 8 show *slices* and equivalent gates occupation for the 12-bit input filters. As with 8-bit, *parallel* filters are those more expensive in terms of *slices* and equivalent gate occupation.

Taps	B-Serial	B-Serial Pipe	D-Serial (2)	D-Serial (2) Pipe	D-Serial (4)	D-Serial (4) Pipe	D-Serial (6)	D-Serial (6) Pipe	B-Parallel
8	91 (1)	102 (1)	115 (1)	121 (1)	167 (1)	171 (1)	221 (2)	224 (2)	368 (3)
12	133 (1)	150 (1)	173 (1)	191 (1)	261 (2)	286 (2)	345 (3)	380 (3)	589 (5)
16	174 (1)	189 (1)	229 (2)	242 (2)	344 (3)	362 (3)	459 (4)	481 (4)	793 (7)
24	266 (2)	293 (2)	357 (3)	396 (3)	561 (5)	616 (5)	761 (7)	837 (7)	1328 (12)
32	339 (3)	355 (3)	454 (4)	466 (4)	689 (6)	704 (6)	927 (8)	940 (8)	1617 (15)
64	668 (6)	706 (6)	910 (8)	957 (8)	1398 (13)	1476 (13)	1887 (17)	1918 (18)	3309 (30)

Table 7 – *slices* occupation for FIR with 12-bit inputs and coefficients
(Percentage over a total of 10752 slices)

Taps	B-Serial	B-Serial Pipe	D-Serial (2)	D-Serial (2) Pipe	D-Serial (4)	D-Serial (4) Pipe	D-Serial (6)	D-Serial (6) Pipe	B-Parallel
8	1840	2356	2246	2996	3130	4474	4044	6010	6420
12	2628	3524	3364	4866	4938	7754	6522	10680	10930
16	3282	4252	4204	5848	6120	9244	8052	12678	13456
24	4956	6744	6606	9908	10090	16440	13570	23004	23506
32	6192	8116	8194	11726	12270	19150	16362	26612	28008
64	11998	15850	16168	23542	24592	39124	33002	54744	57074

Table 8 – Equivalent gate occupation for FIR with 12-bit inputs and coefficients

Let δ be the relationship between the sampling rate (throughput) and the area or number of equivalent gates. This quantity was computed for every single *filter(i,j)*, where *i* stands for the coefficient number or taps (row of tables) and *j* stands for the technique applied (column of tables). Hence:

$$\delta\,[\text{filter}(i,j)] = [SR(i,j) / EG(i,j)]^{*}\, r(i) \qquad (8)$$

where EG is the equivalent gate number for the design and *r(i)* a factor used for normalizing *SR*(i,j) as well as EG(i,j):

$$r(i) = \max[EG(i,j)] / \max[SR(i,j)] \qquad (9)$$

maximums are computed for all *j* or filters designed for a certain tap (*i* row).

Tables 10 and 11 show speed (rate)-area ratio for filters with 8-bit and 12-bit inputs respectively. Values were computed through (8)

Taps	B-Serial	B-Serial Pipe	D-Serial (2)	D-Serial (2) Pipe	D-Serial (4)	D-Serial (4) Pipe	B-Parallel
8	0,711	0,814	0,980	1,200	1,030	0,848	1
12	0,825	0,956	1,026	1,188	1,045	0,983	0,902
16	0,815	1,076	1,085	1,284	1,111	1,024	0,906
24	0,789	1,113	1,008	1,257	0,852	0,995	0,778
32	0,776	1,117	0,922	1,250	0,891	1,021	0,831
64	0,697	0,920	0,900	1,201	0,924	1,023	0,936
Avg.	**0,769**	**0,999**	**0,987**	**1,23**	**0,975**	**0,982**	**0,892**

Table 10 – Speed-area ratio (δ) for filters with 8-bit inputs and coefficients

Taps	B-Serial	B-Serial Pipe	D-Serial (2)	D-Serial (2) Pipe	D-Serial (4)	D-Serial (4) Pipe	D-Serial (6)	D-Serial (4) Pipe	B-Parallel
8	1,218	1,134	1,342	1,768	1,566	1,269	1,319	1,068	0,975
12	1,256	1,320	1,557	1,781	1,552	1,388	1,336	1,023	0,987
16	1,153	1,349	1,477	1,750	1,493	1,431	1,290	1,061	0,954
24	1,029	1,537	1,446	1,910	1,336	1,392	1,217	1,021	0,892
32	1,012	1,460	1,151	1,836	1,368	1,466	1,121	1,052	0,901
64	0,815	1,298	1,116	1,651	1,161	1,207	1,107	1,043	0,874
Avg.	**1,080**	**1,350**	**1,348**	**1,782**	**1,413**	**1,359**	**1,232**	**1,045**	**0,930**

Table 11 – Speed-area ratio (δ) for filters with 12-bit inputs and coefficients

IV. CONCLUSIONS

A distributed arithmetic based *single-rate* FIR in FPGAs study was presented. 8-bit and 12-bit inputs and coefficients filters were analyzed as well as a wide coefficient diversity.

As no relative location (*rloc*) technique was used, designs are generic, i.e. can be implemented over any FPGA.

Although the synthesis tool had the maximum freedom degree for implementing at ease, results obtained have a close correspondence with that expected through theoretically considerations. *Bit-parallel* filters are more expensive in terms of area but are faster (in terms of sample rate) than *digit-serial* filters, which in turn are more expensive and faster than *bit-serial* filters. Through the application of *pipelining* techniques speed and area were increased.

Finally, was concluded that *digit-serial* with *pipelining* and digit size 2 implementation has the best speed-area ratio (δ) among all filters considered in this work.

V. REFERENCES

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