Matrix Estimation using Matrix Forgetting Factor and Instrumental Variable for Nonstationary Sequences with Time Variant Matrix Gain

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Abstract. Consider us the problem of time-varying parameter estimation. The most immediate and simple idea is to include a discounting procedure in an estimation algorithm i.e., a procedure for discarding (forgetting) old information. The most common way to do is to introduce an exponential forgetting factor (*FF*) into the corresponding estimation procedure (to see: Ljung and Gunnarson (1990)).

In this paper, the authors going to describe a good enough estimator considering a system with nonstationary time variant properties with respect to input and output qualities. The techniques used are Instrumental Variable (*IV*) and Matrix Forgetting Factor (*MFF*). The results previously obtained by (Poznyak and Medel 1999_a, 1999_b) were the basis of this paper. The theoretical description illustrates the advantages with respect to others filters below cited.

Keywords: *Filtering, simulation, estimation, signal processing.*

1 INTRODUCTION

In many papers used a *constant scalar Forgetting Factor (FF)* for to filter a non-stationary system in SISO case, for example:

Marco Campiy (1994) exposed in his paper, that the systems with unknown time-varying parameters and subject to stochastic disturbances have a problem for tracking parameters because in each parameter evolution, resorting to a class of adaptive recursive least squares algorithms, equipped with variable *FF*. The basic assumption in the analysis is that the observation vector, the noise and the parameter drift are stochastic processes satisfying a mixing condition. Furthermore, the observation vector satisfies an excitation condition imposed on its minimum power. In this paper, the author shown that the algorithm estimates with bounded error whenever the so-called covariance matrix of the algorithm keeps bounded. Finally, the size of such a matrix by a suitable choice of the feasible range for *FF* is possible to control.

George V. Moustakides (1997) investigated the convergence properties of the *FF* into RLS algorithm by stationary data environment. He used the settling time as a performance measure and shown that the algorithm exhibits a variable performance depending to the particular combination of the initialization and noise level. Specifically when the observation noise level is low, the RLS had a matrix with small norm and it has an exceptional convergence, i.e., that the convergence speed decreases as we increase the norm of the initialization matrix. Now if the observation noise level is high, he shown that it is preferable to initialize the algorithm with a matrix of large norm.

Xue and Liu (1991) shown that, when apply a *FF* to the past data, the convergence is bounded exponentially, and prediction coefficients fluctuate around the least squares estimates in the steady state, in probability sense.

The studies carried out by us with small *FF* into standard least square algorithm, increases the convergence rate and the set of results have a larger fluctuation around of real values, and Guo, Ljung and Priouret (1992) developed the analytical results.

Tsakalis and Limanond (1992) considered to apply the adaptive techniques for to smooth the trajectories of the space station setting in orbit with moving payload, and used an adaptive least square algorithm with adjustable *FF*.

On the other hand, Bittanti and Campi (1994_a) studied the performance of the recursive least squares method with constant *FF* for to estimate of time-varying parameters in a stochastic systems.

Continuing with their studies Bittanti and Campi (1994_b) exposed the properties of this class of estimator into stochastic system, showed that if use the standard least square method with a large enough *FF* for tracking, the error keeps bounded, and has in according to them “an interesting expression”. In addition, they conclude that the estimation error has two terms: One depending on the parameter drift and the other depending on the noises.

The method suggested by Goto, Nakamura and Uosaki (1995) for to estimate on-line the set of parameter, considered a linear representation into recursive least squares estimation algorithm with ladder *FF,* and bounded it by the unit zone.

Xue and Liu (1991_b) evaluated the asymptotic convergence of the least squares algorithm with *FF* for stochastic inputs and concluding that the convergence is a function of input-output noises variance and observed that a small *FF* the rate of convergence is very “good”, in other case generate a set of fluctuations.

In a nonlinear multiaxial thrust vectoring, Ward, Barren and Carley (1994) presented a prediction using a sequential least squares estimator and they observed a “good” results. In a few moths later, they prove the sequential least squares estimator with *FF* and obtained the “best” results.

Ting and Chiders (1990) described a recursive least-squares algorithm with a *variable FF,* and introduced for speech signal analysis. The *variable FF* was a function of the state changes of the estimator. Pahalawatha *et al* (1990) used to *variable FF* in the recursive least squares algorithm. Bittanti and Campi (1994_a) worked a constant *FF* to parameter tracking with recursive least square algorithm in a fully stochastic framework.

Avanzolini, Barbino, Cappello and Cevenini (1995) considered two algorithms:

a. The least squares algorithm with *variable FF: the variable* FF is expressed as a function of covariance modifications related with to noise around of the system and the noise inside of it, and b.The least squares algorithm with *constant FF* previously selected by Monte Carlo Method.

Park, Jun and Kim (1991) used to least squares algorithm with innovative *variable FF* into a unity zone.

Poznyak and Medel (1999_a 1999_b) suggested a new approach based on the use of the recursive *Instrumental Variable Method* (IVM) with a constant *Matrix FF* (MFF) for input and output noises into the system, uncorrelated. This tool gave a two times better estimations with respect to previous results above cited.

In addition, C. F. So and et. al. (2003) obtained a new variable *FF* recursive least-square adaptive algorithm. They showed that the theoretical analysis and the simulation results are close to each other. The adaptive FF use a function generated on the dynamic equation of the gradient of mean square error. Their results had compared with other types of variable *FF* algorithms, and their algorithm provides fast tracking and small mean square error, described it by second probability moment.

2 MODEL DESCRIPTION AND STATEMENT PROBLEM

Matrix ARMA-model of fixed order n_d and noises of ζ_τ of the Moving Average type with the same order, disturbing the state vector $x_\tau \in \mathbf{R}^N$ described by time variant stochastic model:

$$\begin{aligned} x_\tau &= \left[A_{1,\tau},\ldots,A_{n_d,\tau}\right]\left[x_{1,\tau},\ldots,x_{n_d}\right]^T + \zeta_\tau , \\ \zeta_\tau &= \left[D_{0,\tau},\ldots,D_{n_e,\tau}\right]\left[\xi_{1,\tau},\ldots,\xi_{n_d,\tau}\right]^T . \end{aligned} \tag{1}$$

Where $\{\xi_{i,\tau}\} \in \Re^N, i = \overline{1,n_d}$ is a white noise vector, centered random variables with distribution and fourth bounded moments and $\{D_{i,\tau}\} \in \Re^{N \times N}, i = \overline{1,n_d}$, unknown and deterministic bounded matrices. All random sequences $\{\xi_{i,\tau}\}$ are into a *filtered probability space* $(\Omega, \{\Im_i\}, \Im, P)_\tau$ in agreement to Ash (1972), into interval measurable in symbolic form expressed by τ.

Let us also assume that a linear algebraic relation gives the output model in discrete time, which also contains a white noise vector: $\{\upsilon_{j,\tau}\} \in \Re^M, i = \overline{1,n_d}$, disturbing in additive sense, the measured output signal vector $y_\tau \in \Re^M$:

$$y_\tau = C_\tau x_\tau + \upsilon_\tau \tag{2}$$

If we considering that $C_\tau \in \Re^{M \times N}$ is a known and has full rank matrix $\left(C_\tau{}^T C_\tau\right) > 0$. The state vector is described as: $x_\tau = C_\tau{}^+ \left(y_\tau - \upsilon_\tau\right)$, where $C_\tau{}^+ := \left(C_\tau{}^T C_\tau\right)^{-1}$ is knowing as pseudoinverse matrix (to see: Rao 1965). Replacing x_τ in (1):

$$x_\tau = \left[A_{1,\tau},\ldots,A_{n_a,\tau}\right]\left[\alpha_{1,\tau},\ldots,\alpha_{n_a,\tau}\right]^T + \zeta_\tau ,$$

with

$$\begin{aligned} \alpha_{1,\tau} &= (C_{1,\tau})^+ \left(y_{1,\tau} - \upsilon_{1,\tau}\right) \\ &\vdots \\ \alpha_{n_d,\tau} &= (C_{n_1,\tau})^+ \left(y_{n_d,\tau} - \upsilon_{n_d,\tau}\right). \end{aligned}$$

In addition, substituting this result into output signal vector expressed in (2):

$$y_\tau = \beta_{1,\tau} y_{1,\tau} + \ldots + \beta_{n_d,\tau} y_{n_d,\tau} + \nu_\tau \tag{3}$$

Considering that: $\{\beta_{i,\tau} = C_\tau A_{i,\tau} (C_{i,\tau})^+\}$, $i = \overline{1,n_d}$, and "*the generalized noise-signal vector*" ν_τ described as: $\nu_\tau = C_\tau \zeta_\tau - \Theta_\tau + \upsilon_\tau$ with $\Theta_\tau = \sum_{i=\overline{1,n_d}} \beta_{i,\tau} \upsilon_{i,\tau}$.

The sequence υ_τ has a moving average structure. Throughout this paper use us the standard vector-form; then, considering in (3) these properties, expressed in vector-form:

$$y_\tau = A_\tau z_\tau + \nu_\tau \ , \tag{4}$$

Where are defined the vector $z_\tau \in \Re^{Mn_d}$ as "*generalized inputs*" and $A_\tau \in \Re^{M \times Mn_d}$ as the extended matrix of nonstationary parameters:

$$\begin{aligned} z_\tau^{\ T} &= \left(\left[C_{\tau-1}^{\ T}, \cdots, C_{\tau-n_d}^{\ T}\right]\left[y_{\tau-1}, \cdots, y_{\tau-n_d}\right]\right), \\ A_\tau^{\ T} &= \left(\left[C_{\tau-1}^{\ T}, \cdots, C_{\tau-n_d}^{\ T}\right]\left[A_{\tau-1}, \cdots, A_{\tau-n_d}\right]\right). \end{aligned} \tag{5}$$

To deal with nonstationary models (to see: 4) containing a nonstationary unknown matrix A_τ as well as nonstationary random disturbances expressed in symbolic vector form by ν_τ. Will be require to introduce: An exponential discounting mathematical expression knowing as *Forgetting Factor Matrix* (FFM) denoted as R and defined positive; i.e.: $0 < R$ and $R = R^T \in \Re^{Mn_d \times Mn_d}$, and Instrumental Variable Method (IVM) expressed in vector form as $\vartheta_\tau \in \Re^{Mn_d}$. Considering the experience of previous result of estimation of nonstationary systems and estimation of nonstationary unknown matrix parameters, we suggest to use both in combined form as a tool, with the next properties:

Multiplying both sides of (4) by $\vartheta_\tau^T \in \Re^{n_d-\tau}$, exposed in (Poznyak and Medel 1999$_a$), and averaging it in time $(\tau = \overline{1,n_d})$:

$$\begin{aligned} &\textstyle\sum_{i=\overline{1,n_d}} y_\tau \vartheta_\tau^{\ T} R^{n_d-\tau} = \\ &\textstyle\sum_{i=\overline{1,n_d}} A_\tau z_\tau \vartheta_\tau^{\ T} R^{n_d-\tau} \\ &+ \textstyle\sum_{i=\overline{1,n_d}} \nu_\tau \vartheta_\tau^{\ T} R^{n_d-\tau} \end{aligned} \tag{6}$$

Selecting the instrumental variable set $\{\vartheta_\tau\}$ in such a way that:

$$\textstyle\sum_{i=\overline{1,n_d}} \nu_\tau \vartheta_\tau^{\ T} R^{n_d-\tau} \xrightarrow{a.s} l_{n_d} \ . \tag{7}$$

With l_{n_d} as element of filtered probability space:

$$l_{n_d} \subseteq \left(\Omega, \{\Im_i\}, \Im, P\right)_{n_d}$$

The contraction *a. s.*, significate: almost surely or with probability 1 with respect to the measure P (to see: Ash 1972).

To define the *parameter estimator matrix* $\hat{A}_{n_d}$ in the time n_d with respect to the matrix $A_{n_d} \in \Re^{M \times Mn_d}$ and the perturbations around the process, previously expressed in (4):

$$\hat{A}_{n_d} = \left(\textstyle\sum_{\tau=1,n_d} y_\tau \vartheta_\tau^{\ T} R^{n-\tau} - I_{n_d}\right) \Gamma_{n_d} \tag{8}$$

Where the *gain matrix* Γ_{n_d} has form:

$$\Gamma_{n_d} = \left(z_{n_d} \vartheta_{n_d}^{\ T} + \left(\textstyle\sum_{\tau=1,n_d-1} z_\tau \vartheta_\tau^{\ T} R^{(n_d-1)-\tau}\right) R\right)^{-1} \tag{9}$$

The expression (8) in recursive description, the *gain matrix* Γ_{n_d}:

$$\Gamma_{n_d} = \left(z_{n_d} \vartheta_{n_d}^{\ T} + \left(\Gamma_{n_d-1}\right)^{-1} R\right)^{-1} . \tag{10}$$

Applying the inversion matrix lemma (Ljung 1987 and Rao 1965), finally, the *gain matrix* Γ_{n_d} has the form:

$$\Gamma_{n_d} = R^{-1}\Gamma_{n_d-1} - S_{n_d}\ , \qquad (11)$$

With S_{n_d}, to express as a quotient:

$$S_{n_d} = \left(R^{-1}\Gamma_{n_d-1}z_{n_d}\vartheta_{n_d}{}^{T}R^{-1}\Gamma_{n_d-1}\right)\left(1+\vartheta_{n_d}{}^{T}R^{-1}\Gamma_{n_d-1}z_{n_d}\right)^{-1}.$$

Where the *gain matrix* Γ_{n_d} is valid for any

$$n_d \geq n_0(\omega_\tau) := inf_{\tau\geq 0}\left\{\tau : det(\Gamma_\tau)^{-1} \neq 0\right\}, \omega_\tau \in \Omega\ .$$

In agreement to (7) the recursive form, and remembered that (6) has stationary conditions:

$$l_{n_d} \cong l_{n_d-1} + v_{n_d}\vartheta_{n_d}{}^{T}\ . \qquad (12)$$

Taking into account that (to see: (10) in

$$\left(\Gamma_{n_d-1}\right)^{-1}R\Gamma_{n_d} = I - z_{n_d}\vartheta_{n_d}{}^{T}\Gamma_{n_d}$$

The recurrent estimator, using the combined tool formed by IVM and MFF, and considering that matrix estimator described by (8) have stationary properties:

$$\hat{A}_{n_d} := \hat{A}_{n_d-1} + \left(\Psi_{n_d} - R_{n_d-1}z_{n_d}\right)\vartheta_{n_d}{}^{T}\Gamma_{n_d} \qquad (13)$$

Where Ψ_{n_d} has the form:

Remark 1. *When (7) in almost surely tend to zero, the algorithm obtained was described by (Poznyak and Medel 1999$_a$).*

Remark 2. *In the partial case when* $\vartheta_{n_d} = z_{n_d}$ *(we have a Least Squares Method) and R=ρI (a scalar forgetting factor). The system obtained has been intensively studied from different points of view (see the list of references, for example, Poznyak 1980, Bittanti and Campi 1994, Guo, Ljung and Prioret 1993, Lindoff and Holst 1995$_a$, Lindoff and Holst 1995$_b$, Parkum, Poulsen and, Holst 1992, Porat 1995, Poznyak & Medel 1999$_a$, and 999b and Medel & Poznyak 2001, etc.*).

The aim of this investigation is to describe the properties of this algorithm considering preselected a MFF R which minimizes the estimation error $\left\|\hat{A}_{n_d} - A\right\| \in R^{M\times Mn_d}$ in some average probabilistic sense.

3 ASYMPTOTIC ANALYSIS

To analyze the properties of the estimation procedure of equation (13) let us introduce the matrix $\Delta_{n_d} \in R^{M\times Mn_d}$ characterizing the error of this estimating process:

$$\Delta_{n_d} := \hat{A}_{n_d} - A_{n_d} \qquad (14)$$

The next lemma going to expose the analytical expression for the matrix Δ_{n_d} defined along the trajectories of the random process in equation (13).

Theorem 1. For the matrix Δ_{n_d} defined by equation (14) for $n_d \geq n_0$, where the estimates are generated by equation (13), the following presentation holds:

$$\Delta_{n_d} = \Delta_0\pi_{n_d} + \sum_{i=\overline{1,n_d}}\Lambda_i\left(\pi_{n_d-(i-1)}\right)^{-1}\pi_{n_d} \qquad (15)$$

Where the matrix $\Lambda_i = A_{n-i} - A_{n-(i-1)}$, , and $\pi_{n_d} \in R^{M_{nd}} \times R^{M_{nd}}$, :

$$\pi_{n_d} := \prod_{i=\overline{1,n_d}}\left(I - z_i\vartheta_i{}^{T}\Gamma_i\right)\ . \qquad (16)$$

Proof (Theorem 1). Substituting (13) in (14) we derive:

$$\Delta_{n_d} = \Delta_{n_d-1} + \left(\Psi_{n_d} - \hat{\Lambda}_{n_d-1} z_{n_d}\right)\vartheta_{n_d}{}^T \Gamma_{n_d} - \Lambda_{n_d}$$

Using then expression (4) in the previous result:

$$\Delta_{n_d} = \Delta_{n_d-1} + \left(\Lambda_{n_d} - \Delta_{n_d-1}\right) z_{n_d} \vartheta_{n_d}{}^T \Gamma_{n_d} - \Lambda_{n_d}$$

Reducing the previous expression:

$$\Delta_{n_d} = \left(\Delta_{n_d-1} + \Lambda_{n_d}\right) E_{n_d} \tag{17}$$

Denoting $E_{n_d} = I - z_{n_d} \vartheta_{n_d}{}^T \Gamma_{n_d}$, $E_{n_d} \in R^{Mn_d \times Mn_d}$.

Considering that (17) is described as stationary process, then, one interval before:

$$\Delta_{n_d-1} = \left(\Delta_{n_d-2} + \Lambda_{n_d-1}\right) E_{n_d-1}$$

And substituting this in (17)

$$\Delta_{n_d} = \left(\left(\Delta_{n_d-2} + \Lambda_{n_d-1}\right) E_{n_d-1} + \Lambda_{n_d}\right) E_{n_d}$$

This procedure is developed (n_d *-1*) times. Describing the error matrix as:

$$\Delta_{n_d} = \Delta_{n_0} \prod_{i=\overline{1,n_d}} \mathrm{E}_i - \sum_{i=\overline{1,k}} \left(-\Lambda_i\right) \prod_{k=\overline{n_d-(i-1),n_d}} \mathrm{E}_{n_k}$$

Taking in to account that

$$\prod_{k=\overline{n_d-(i-1),n_d}} E_{n_k} = \left(\pi_{n_d-i}\right)^{-1} \pi_{n_d} \tag{18}$$

And finally, obtain us (15). ■

CONCLUSIONS

In this paper, deal with the class of models with varaying parameters and subjected to random disturbances of the moving-average type. We suggested a general version of (Instrumental Variable Method) IVM with Matrix Forgetting Factor (MFF) in agreement to the dynamical properties of the real system. A recursive version of this procedure has a basic description. The main issue of this work is the combination of MFF with IVM for multimatrices. Observing that a Digital Filter can be implemented into embedded systems using micro controllers, DSP's, and others electronic technologies, considering a recursive structure.

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